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CONTENTS

Power spectrum analysis of geomagnetic indices	<i>R P Kane</i>	1
Wave packet structure and phase jumps in low latitude Pc3 geomagnetic pulsation	<i>I A Ansari, B J Fraser and K J McLean</i>	13
Spectral characteristics of low latitude Pc3 geomagnetic pulsations in south-east Australia	<i>I A Ansari, B J Fraser and K J McLean</i>	25
Sub-seasonal scale fluctuations of the ITCZ over the Indo-Pacific region during the summer monsoon season. Part I: Features over the Indian region <i>D R Sikka, D K Paul, V R Deshpande, V R Mujumdar and P V Puranik</i>		47
Quasi-specular sea scatter with modified reflection coefficient	<i>Raj Kumar and Abhijit Sarkar</i>	75
The effect of filtering on drift and anisotropy parameters determined by full correlation analysis	<i>B M Vyas, D V Sardesai, R K Rai, H Chandra and G D Vyas</i>	83
Geochemistry of coexisting hornblende and biotite from the Ambalavayal granite, Kerala.....	<i>M Santosh</i>	91
Radioelement geochemistry of alkali granites of the Kerala region, south-west India	<i>M Santosh and K Vijaya Kumar</i>	103
The petrology and geochemistry of massif anorthosites and gabbros of the Bavali fault zone, North Kerala	<i>G R Ravindra Kumar</i>	117
Base metal exploration by gravity surveys in a part of the Cuddapah basin (India) — a case study ...	<i>G Ramadass, S M Varaprasada Rao and N L Mohan</i>	131
Evolution of the landscapes in the Calcutta-Sundarban coastal areas of South Bengal, India.....	<i>Dilip K R Das, Jay P Majumdar and Aniruddha Ganguly</i>	143
Numerical hysteresis model for intermittent studies in unsaturated soils	<i>M Banerjee</i>	153
Radionuclide depth profiles in Dhajala chondrite	<i>M B Potdar, N Bhandari and K M Suthar</i>	169
Cosmogenic isotope and track records in meteorites and their implications to cosmic ray fluxes	<i>Narendra Bhandari</i>	183
Absorption of radio waves during a solar eclipse	<i>D B Patel, K M Kotadia, P D Lele and K G Jani</i>	193

Effect of linear trend and mean value on maximum entropy spectral analysis	<i>R P Kane and N B Trivedi</i>	201
Physico-chemical conditions of ore deposition in Malanjikhand copper sulphide deposit	<i>S Jaireth and M Sharma</i>	209
Inverse problem for the determination of the bounds on the mantle heat flow	<i>R N Singh and G Vasseur</i>	223
Effect of waves in the redistribution of sediments along the Konkan coast	<i>G Victor Rajamanickam, P Vethamony and A R Gujar</i>	237
Preferential solution of ^{234}U from recoil tracks and $^{234}\text{U}/^{238}\text{U}$ radioactive disequilibrium in natural waters	<i>N Hussain and D Lal</i>	245
The effect of whitecaps and foam on wind speed extraction with a pulse limited radar altimeter	<i>R M Gairola and P C Pandey</i>	265
Gravity anomalies of a trapezoidal model with quadratic density function	<i>D Bhaskara Rao</i>	275
Origin and depositional environment of the china clay deposits of south Kerala	<i>K Soman and Terry Machado</i>	285
Some theoretical aspects of electric field and precipitation growth in a finite thundercloud	<i>Pratap Singh, T S Verma and N C Varshneya</i>	293
Global features of sea-level pressure distribution in April and July associated with contrasting situations of Indian summer monsoon	<i>K Rupa Kumar and L S Hinghane</i>	299
<i>In situ</i> stress measurements using hydraulic fracturing method	<i>T N Gowd, S V Srirama Rao, K B Chary and F Rummel</i>	311
Computation of synthetic seismograms for plane layered earth models including absorption and dispersion phenomena	<i>K N S Yadav and Avadh Ram</i>	321
Magnetic anomalies over thin plates and their analysis	<i>H V Rambabu and G D J Sivakumar Sinha</i>	331
Analysis of gravimagnetic anomalies by horizontal gradients	<i>G Ramadass, S M Varaprasada Rao and A Raja Mohammed</i>	343
Palaeo-tectonic environment of the precambrian basaltic mafic dykes on the application of chemical discriminants	<i>T Ramamohana Rao, B V Satyanarayana Raju and M V Rama Raju</i>	351
Lineament analysis of Cannanore District and their implication on the tectonic evolution of the area	<i>Venkatesh Raghavan and Narayanaswamy</i>	363
An interpretation of induced magnetic variations near Shillong and Gulmarg	<i>A K Agarwal and H B Naik</i>	373
On the thermal and redox history of the Yamato diogenite Y-74013	<i>Amalbikash Mukherjee and T A Viswanath</i>	381

Natural radioactivity distribution studies in Trivandrum District, Kerala, India	<i>G K Raju, John Mathai, G R Ravindra Kumar and N G K Nair</i>	397
Seasonal and solar cycle variations of night-time anomalous enhancements in total electron content	<i>N Balan, P S Rao and K N Iyer</i>	409
Estimation of evaporation rates over the Arabian Sea from satellite data	<i>M Venkateswara Rao, V Ramesh Babu, L V Gangadhara Rao and J S Sastry</i>	417
Wind gradients at meteor heights over mid-latitude stations	<i>P C S Devara, G Chandrasekhar and M I Ahmed</i>	427
Sea surface temperature variability over North Indian Ocean — A study of two contrasting monsoon seasons	<i>M R Ramesh Kumar, S Sathyendranath, N K Viswambharan and L V Gangadhara Rao</i>	435
An overview of climate models	<i>M Sankar-Rao</i>	447
A review of stochastic dynamic method of weather prediction	<i>P C Joshi and D R Sikka</i>	485
Subject Index		505
Author Index		511

Power spectrum analysis of geomagnetic indices

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Abstract. Using the method of maximum entropy spectral analysis (MESA) by Burg and the least-squares linear prediction (LSLP) (also called FABNE) by Barrodale and Erickson, the spectra of geomagnetic indices A_p , A_N , A_S , A_E , A_U , A_L , Dst and cosmic ray neutron intensity at Deep River are obtained for 1965 and 1969 from daily means and for longer periods from monthly means. A large number of peaks of periodicities from 2 days to several years is obtained, many of which are shown by all the indices. Some of these are probably harmonics of the 27 days solar rotation period and the 20–22 years double sunspot cycle period. Comparison is made with results of earlier workers who reported fewer peaks.

Keywords. Power spectrum analysis; geomagnetic indices; maximum entropy spectral analysis; least-squares linear prediction; periodicity peaks.

1. Introduction

For detecting the periodicities in a given time series, the most popular method adopted till recently was that of Blackman and Tukey (1958). It is based on the autocorrelation function and one has to choose a lag factor m and the power is finally obtained for periods $2m$, $2m/2$, $2m/3$ etc. Recently Burg (1967) proposed the method of maximum entropy spectral analysis (MESA) which gives a very good resolution in frequency even when the periods are almost comparable to the data length (see review by Ulrych and Bishop 1975). In the MESA, one chooses the length of the prediction error filter (LPEF). For low LPEF of the order of a few percent of the data length, only higher harmonics (smaller periodicities) start getting resolved. As LPEF is increased, larger and larger periodicities are revealed.

In spite of the superior resolution of the Burg method (Burg 1972), there are some serious drawbacks of this method viz, (i) a shift in the spectral lines, (ii) the occurrence of multiples, (iii) the difficulty in knowing the variance of the spectral density estimator and (iv) choice of the optimal length of the prediction error filter (LPEF). The problem of the proper choice of LPEF is the most tricky one. At very low values of LPEF, no worthwhile resolution is obtained. As LPEF is increased, more and more peaks in the low frequency side are revealed but the spectra become unstable, with the "explosion" of many peaks into multiples (Chen and Stegan 1974). For determining the optimum LPEF, Ulrych and Bishop (1975) suggested the use of the Akaike's (1969) final prediction error (FPE) criterion, and, if this failed (as happens often), an LPEF of about 50 % of the data length was suggested to be generally adequate. Our own studies (Kane 1977, 1979) indicate that for samples containing peaks in a wide range of frequency, an LPEF of about 50 % of data length was adequate to resolve frequencies exceeding the fifth harmonic, while for lower harmonics, LPEF even as high as 90 % was sometimes needed,

with the danger of peak-splitting and frequency shifts (specially in low frequencies) ever present.

The limitations of the Burg algorithm are caused by its imposition of a Toeplitz structure on the matrix of the system of equations which yield the AR (auto regressive) parameters. This procedure is responsible for the computational efficiency of the Burgs algorithm; but there is enough empirical evidence to show that the resulting spectra so obtained are inferior (in accuracy of frequency determination) to those obtained by using least-square (LS) solutions to the AR model (Ulrych and Clayton 1976).

Estimation of AR parameters by LS methods has previously been unpopular because of the large computational effort involved, besides other reasons. However, Barrodale and Erickson (1980a, b) have recently developed an algorithm for solving the least-squares linear prediction (LSLP) problem directly (without forcing a Toeplitz structure on the AR model). This algorithm, termed by them as FABNE, is claimed to be computationally efficient and numerically stable. Kane and Trivedi (1982) compared the Burg and FABNE spectra for some artificial samples and confirmed the superiority of FABNE over Burg spectra. Recently, Marple (1980) has also reported a new algorithm which is similar to that of Barrodale and Erickson except for some numerical sophistication and swiftness in calculation. In the present communication, we study the spectra of geomagnetic indices, by both the Burg algorithm (as given by Anderson 1974) and the FABNE method, mainly to *locate* the various periodicities T_n (giving more weightage to the FABNE), using LPEF = 50% or less for periodicities $T_n < T/5$ and using LPEF exceeding 50% for periodicities $T_n > T/5$ where T = data length. Having picked up the possible T_n , we obtain the best estimates of the amplitudes r_n and their standard errors σ_{r_n} using the expression

$$\begin{aligned} f(t) &= \sum_n^n (a_n \sin 2\pi(t/T_n) + b_n \cos 2\pi(t/T_n)) \\ &= \sum_n^n r_n \sin(2\pi(t/T_n) + \phi_n), \end{aligned} \quad (1)$$

where $f(t)$ is the observed series, expressed as deviation from mean, and t runs from 1 to data length T , and adopting standard statistical procedures involving a least-square fit (Johnston 1960; Bevington 1969). Amplitudes r_n larger than $2\sigma_{r_n}$ are supposed to be significant at a 95% level.

Geomagnetic disturbances are graded by various indices viz, mid-latitude indices Kp, Ap, AN, AS (Mayaud 1968), Dst (Sugiura and Poros 1971) and auroral region indices AE, AU, AL. Periodicities of 11 and 22 years have been reported (Chernsky 1966; Russell and McPherron 1973). Mayaud (1970a, b) reported annual, semi-annual and diurnal variations. Shapley (1947) reported a 27-day recurrence tendency. Fraser-Smith (1972) obtained very prominent peaks at 35.6, 16.1, 10.2 and 0.5 years, prominent peaks at 7.04, 5.14, 4.10, 1.47, 1.09 years and 27.2, 27.6 days, moderate peaks at 54.0, 30.5, 26.9, 18.7, 14.1, 13.7, 13.6 and 9.39 days and very weak peaks for 2–9 days, from a fast fourier transform analysis of 38 years data of Ap. Abdel-Wahab and Goned (1974) reported periods of 27 and 13.5 days. Burch (1973) showed that the auroral AU and AL indices had a seasonal as well as sector structure effect. It would be interesting to see what Burg and FABNE spectra for these parameters reveal. Currie (1976) had applied MESA to 100

with and without trend removals and showed (besides other features) that the removal of trend affected longer periods considerably. We now present our results.

2. Data analysis

Geomagnetic indices are known to have year-to-year variations. Hence, for the study of periodicities of a few days, it may be desirable to remove the long-term trends by using proper filters. We used daily mean values for the various parameters and obtained Burg and FABNE spectra from the original values as also after applying a high-pass (cutoff at 20 days) 51 point sine-termination filter (Behannon and Ness 1966). The results shown by both were almost similar except that peaks for periods exceeding about 20 were missing in the filtered data, as expected. Recently, Courtillot *et al* (1977) have also shown that Burg spectra of say a 100-year series with and without removing a linear or parabolic trend are the same for low periodicities (about 7 years or less) only. Hence, only analysis of *unfiltered* data is reported in what follows. Burg and FABNE spectra using 365 daily mean values for years of low and high solar activity (1965 and 1969 respectively) were obtained for the geomagnetic indices Dst, Ap, AN, AS, AE, AU, AL as also for the cosmic ray (CR) neutron monitor intensity at Deep River, Canada. Figure 1 shows the sample plot for the AN index for 1965. For $LPEF = 50$, very few peaks are revealed. For $LPEF = 100$, many more peaks are revealed; but the number of peaks is

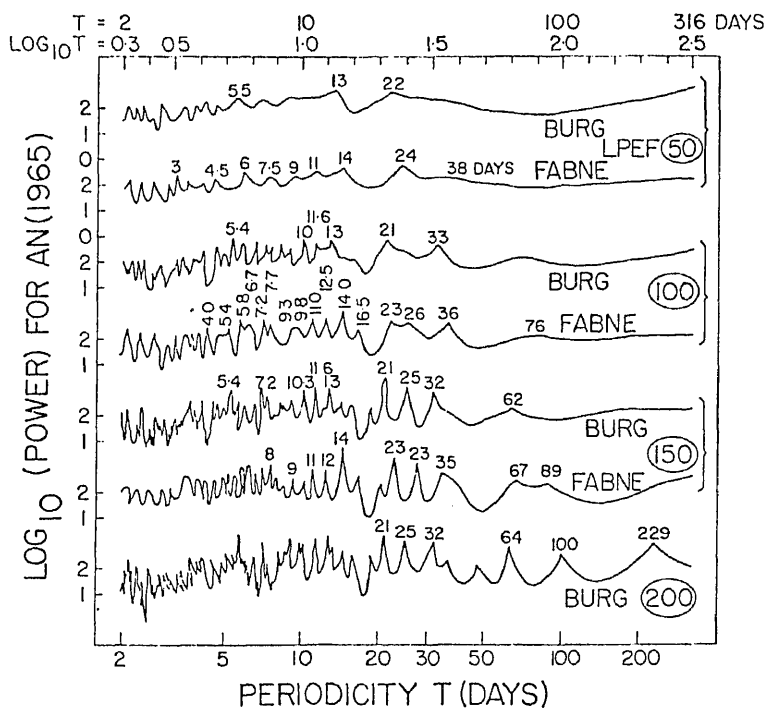


Figure 1. Burg and FABNE spectra for AN index for 365 days of 1965 for $LPEF = 50, 100, 150$.

lesser in the FABNE spectra. Probably there is some peak splitting in the Burg spectra. For LPEF = 150, longer periodicities are revealed; but shorter periodicities show peak splitting in both Burg and FABNE spectra. Hence, for periodicities lesser than $T = 20$ days, peaks were chosen from FABNE spectra for LPEF = 100. For periodicities exceeding $T = 20$ days, peaks were chosen from FABNE spectra for LPEF = 100 and 150. For LPEF = 200 or more the FABNE program failed. The Burg spectra for LPEF = 200 are shown at the bottom in figure 1, only for illustration. These were not used for choosing peaks.

Having chosen the peaks T_n as mentioned above, these were used in (1) for estimating the amplitudes r_n and their standard errors σ_{rn} . Periodicities (in days) for which r_n exceeded $2\sigma_{rn}$ are listed in table 1, for 1965 in the left half and for 1969 in the right half. In the middle column, the results of Fraser-Smith (1972) are also given for comparison. As can be seen, all the peaks seen by Fraser-Smith are revealed in our analysis also. In addition, our analysis reveals many more peaks. Thus, in 1965, significant peaks common to four or more indices were at about 4.0, 5.4, 5.8, 6.7, 7.2, 7.6, 9.3, 11.0, 12.5, 14.0, 16.5, 23, 26, 36, 50 and 66 days. In 1969, the common peaks were at about 4.0, 9.9, 11.0, 12.5, 13.5, 14, 17, 20, 29, 35, 48 and 60 days. Peaks near 27 days are probably related to the solar rotation period and higher harmonics of the same at about 13.5, 9.0, 6.7, 5.5 days etc. seem to exist, as indicated in the last column of table 1. Other peaks could be harmonics of the 90 days (3 months) and 180 days (6 months) periods. The double solar rotation peak at 54 days reported by Fraser-Smith is not seen clearly in our analysis. Peaks near 7–8 days are indicated for many parameters. These peaks may be related to the sector structure of the interplanetary magnetic field (Burch 1973). AN and AS represent disturbance indices for the northern and southern hemisphere. For these, most of the peaks seem to be common in 1965. (For 1969, data for AN and AS were not available). For the auroral indices AE, AL, AU, most of the peaks were common but not all, indicating a possibility of different physical mechanisms for AL and AU (Burch 1973). Whereas the significance of all these individual peaks cannot be guaranteed, their occurrence in more than one (often 3 or more) parameters leads us to believe that these peaks are genuine.

Table 1 and figure 1 indicate peaks in the long periodicity region ($T = 100$ days or more). However, in a 365 day data length, such long periodicities would be somewhat unreliable. These could be studied better by spectral analysis of monthly means. Such an analysis gave significant periodicities (in months) which are listed in table 2. The peaks observed by Fraser-Smith (1972) for 38 years of Ap data (1932–1970) by the fast fourier transform (FFT) method are given in the last column of table 2. Figure 2 shows the locations of these spectral peaks in the periodicity range 2–120 months. For Ap (full dots) and Dst, data used are monthly means for about 14 years (1957–70), for cosmic rays data are for 16 years (1958–73), for AL, AU, AE, data are for about 6 years (1965–70) and for AN, AS, data are for 4 years (1964–67). Periodicities are seen near 12 months and 6 months, in agreement with the annual and semi-annual waves reported by earlier workers (Mayaud 1970a; Fraser-Smith 1973) and also at 3 months. However, there seem to be many other peaks also common to many parameters. There is a prominent peak at about 15 months. Prominent peaks seem to occur at about 27 months also, similar to the quasibiennial oscillation (QBO) in meteorological parameters (Angell and Korshover 1964) and in the H component of the geomagnetic field (Yacob and Bhargava 1968). Yet another prominent peak is at about 48 months (4 years). For Ap, we had data for a longer period also (1943–1970, 28 years). Spectral analysis gave

Table 1. Periodicities (days) significant at 2σ level, obtained from analysis of daily means, during 1965 and 1969.

Year 1965										Ap Fraser- Smith (1972)	Year 1969					Harmonics of the 27-day period				
Ap	AN	AS	Dst	AE	AL	AU	CR						Ap	Dst	AE	AL	AU	CR		
	4-0	4-0		4-0	4-0										4-0	4-0	4-0	4-0		
5-5	5-4	5-4		5-4	5-4											5-4		5-5	V	
5-8	5-8	5-8	5-8																	
6-7	6-7	6-7	6-7	6-8	6-8	6-8							5-9							
7-2	7-2	7-2	7-2	7-2	7-2	7-2							(2-9)	6-5		6-8		6-9	IV	
7-6	7-7	7-6	7-6										7-8		7-9					
9-3	9-3	9-3	9-3	9-3	9-3								9-0	9-2				9-4	III	
	9-8	9-8													9-9	9-9	9-8			
11-0	11-0	11-0	11-0	11-0	11-0	11-0	11-0						10-0	11-0		11-0		11-0		
12-5	12-5	12-5	12-5	12-5	12-5	12-5							12-5	12-5	12-5	12-5				
													13-6	13-5	13-5	13-5	13-5	13-0	II	
14-0	14-0	14-0	14-0	14-0	14-0	14-0	14-0						14-1	15-0	15-0	15-0	15-0	15-0		
16-5	16-5	16-5	16-5	16-5		17-0	17-0						17-0	17-0	17-0	17-0	17-0	18-0		
			19-0			19-0	19-0						18-7	20-0	20-0	20-0	19-0	19-0		
23	23	23	23		23		23							24				23		
26	26	26	26	26														26		
													26-9							
	28	28					27						27-2			27		28	I	
													27-6	28						
							31						30-5	30	29	29	29			
36	35	35	36	35	36		35						37		35	35	35	35		
														48	49	50	50	49		
65	67	66	64	68	66	68							54-0	67	60	61	62			
97	89		78	101	100	98	108						132	111	104				107	

Table 2. Periodicities (months) significant at a 2σ level, obtained from analysis of monthly means of several years data.

Ap (1943-70) 28 yrs	Ap (1957-70) 14 yrs	AN (1964-67) 4 yrs	AS (1964-67) 4 yrs	Dst (1957-70) 14 yrs	AE (1965-70) 6 yrs	AL (1965-70) 6 yrs	AU (1965-70) 6 yrs	CR (1958-73) 16 yrs	Ap Fraser- Smith (1972) (1932-70) 38 yrs
		2.1	2.1		2.1				
		2.5	2.4		2.4	2.5	2.5		
		3.0	3.0		3.0	3.0	3.1		
		3.4	3.4			3.4			
		3.8	3.8	3.7	3.9	3.8	3.9		
		4.5	4.6	4.5	4.6	4.6	4.6		
5.5		5.6	5.6	5.6					
	6.0			6.1	6.0	6.0	6.0		6.0
6.8		6.5	6.5	6.6			6.5		
7.9	7.7	7.7	7.5	7.7	7.5	7.4			
	12.9	13.2	12.2	12.3	12.2	12.2	12.0		13.1
15.0			15.5	14.3	14.8	14.8	14.0		
17.0				16.6					17.6
					27.4	26.9		25.4	
33.5	33.1			33.8			36.7		
50	45	46	49	44	42	47			49
61				69					62
81									84
123	116			112				115	122

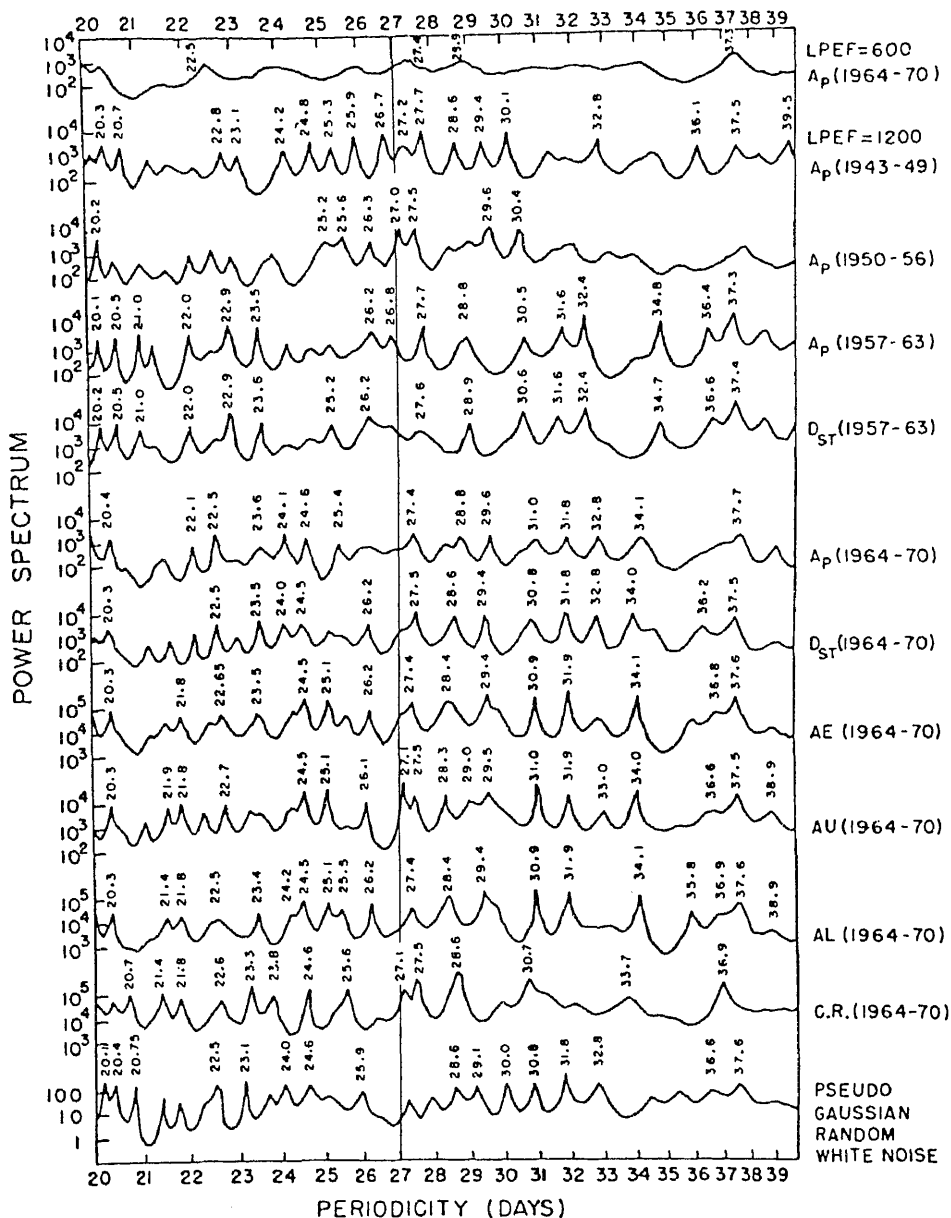


Figure 3. Spectra from about 2500 consecutive daily mean values of the various geomagnetic indices for LPEF of about 50% of total data length, for the restricted periodicity range $T = 20-40$ days (which includes the 27-day periodicity band) for several 7-year intervals as indicated. The top curve is for LPEF = 25% only for A_p (1964-70). The bottom curve is for LPEF = 50% for a 2500 data point sample of pseudo-Gaussian random noise.

days are expected. The top plot shows spectra for A_p for the 7 year period 1964-70 for an LPEF = 600, i.e., about 25% of the total data length. Only some broad peaks are revealed and we concluded that 25% LPEF was inadequate. Hence, we chose LPEF = 1200, i.e., about half data length for all further analysis. The resulting spectra are shown in

figure 3, each plot representing a 7-year period. Even in the restricted range of 20–40 days, several peaks are noticed. Near 27 days, Ap shows peaks at 27.2 and 27.7, 27.0 and 27.5, 26.8 and 27.7 and 27.4 days for the four successive 7-year periods. This could be because of a *solar cycle control* of the main solar rotation period of 27–37 days as suggested by Fraser-Smith (1973). In all other parameters, a prominent peak occurs at 27.55 ± 0.15 but only a small peak at $T = 27.1 \pm 0.1$ is seen for some parameters. Thus, a major peak at 27.5 and a minor peak at 27.1 is indicated. However, equally (or sometimes more) prominent peaks are noticed at other values of T . The question of reliability of these again becomes important. We prepared a 2500 data point sample of pseudo-Gaussian random white noise and the spectra for the same for $LPEF = 1200$ are shown as the bottom plot of figure 3. Many peaks are observed here too! Jensen and Ulrych (1973) calculated the probability distribution of the power density of such peaks. We noticed that in this sample of pseudo-Gaussian random noise for which the standard deviation chosen was 5 (variance 25), all peaks had a power less than about 140. For the 4 seven-yearly samples of Ap which we used, the standard deviations were 18, 16, 21 and 12 and thus their variances were 13, 10, 18 and 6 times the variance of the random samples. Thus, power densities up to about 1800, 1400, 2300 and 900 for 1943–49, 1950–56, 1957–63 and 1964–70 respectively for Ap could be considered as suspect. Peaks having power exceeding these limits could be considered as genuine. Using these criteria, the peaks for which periodicities are labelled in figure 3 could be considered reliable. As can be seen, even these are many in number. In particular, peaks at about $T = 20.7 \pm 0.3$, 22.8 ± 0.3 , 24.8 ± 0.3 , 25.9 ± 0.3 , 27.0 ± 0.2 , 27.55 ± 0.15 , 28.7 ± 0.3 , 29.5 ± 0.1 , 30.7 ± 0.3 , 31.7 ± 0.2 , 32.9 ± 0.1 , 34.0 ± 0.1 , 34.7 ± 0.1 , 36.5 ± 0.3 , and 37.5 ± 0.2 days are observed prominently in many parameters.

It is tempting to identify $T = 29.5$ days with the lunar synodic period. It must be noted, however, that some of these peaks tally rather well with the values observed in the random sample. Hence, there is some doubt about their genuineness.

In principle, one could do an analysis for 28 years data of continuous daily means. However, for a computer this would be a colossal task. As a naive alternative, we obtained an average plot for Ap by superposing the four plots for Ap in figure 3. The average showed peaks in bands as indicated above with a broad *plateau* in the region 26.6–27.8 days with a small peak only at about 27.6 days. We also tried using 7-day i.e. weekly means of Ap. From the 52 values per year, we obtained spectra for Ap for 1965, 1966, 1967, 1968 and 1969. The significant periodicities are shown in figure 4. There is a slight indication of a systematic shift of some periodicities from smaller to larger values from quiet sun to disturbed sun years. However, this could be a highly subjective judgement.

3. Summary and conclusions

The results of the present analysis may be summarized as follows:

(1) Burg and FABNE spectra of geomagnetic disturbance indices Ap, AN, AS, Dst, AE, AU, AL and cosmic ray intensity CR were obtained for the quiet and disturbed solar activity years 1965 and 1969 respectively. Several significant periodicities (given in table 1) common to many parameters were noticed though these were not exactly identical for 1965 and 1969. In particular, whereas a 26–27 day peak was prominent in

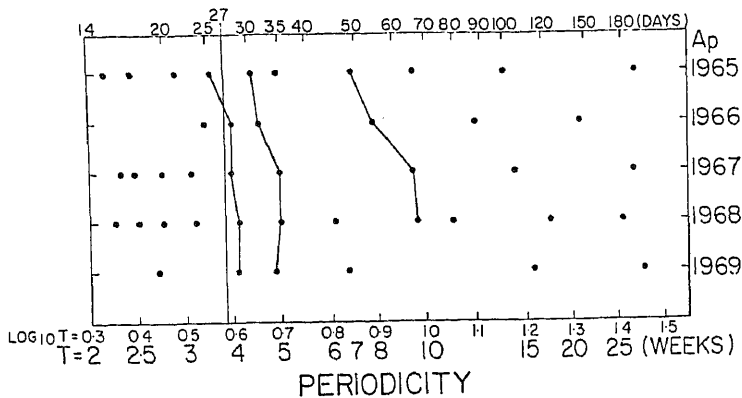


Figure 4. Spectral peaks (significant at a 2σ level), from 52 weekly mean values of Ap for 1965, 1966, 1967, 1968 and 1969.

1965, it was obscure in 1969 when a 28–29 day periodicity was more prominent. This might be related to the observation of Svalgaard and Wilcox (1975) that whereas during most of the solar cycle a four-sector structure of interplanetary magnetic field is seen with a recurrence period of 27 days, near sunspot maximum, a superposed structure with polarity towards sun and a recurrence period of 28–29 days is observed. Periodicities could be identified as harmonics of the 27-day solar rotation period and possibly of the 90-day, 180-day and 360-day waves. Periodicities near about 8 days or less could be related to the sector structure of the interplanetary magnetic field. Occurrence of the same periodicities in cosmic ray intensity perhaps indicates modulation due to geomagnetic effects or a common cause in interplanetary space.

(2) Whereas AN and AS representing mid-latitude disturbances in northern and southern hemisphere show almost similar periodicities, AU and AL in the auroral zone show some differences indicating a partly different origin for the two (Burch 1973).

(3) Analysis of about 7 years data of continuous daily means revealed several peaks in the 20–40 days periodicity interval at about 20.7, 22.8, 24.8, 25.9, 27.0, 27.6, 28.7, 29.5, 30.7, 31.7, 32.9, 34.0, 34.7, 36.5, and 37.5 days within a spread of ± 0.3 days. Fraser-Smith (1972) reported lines at 26.9, 27.2, 27.6, 30.5 and 37.4 days which tally with our list except that we observed a strong peak at 27.6 days for several parameters but only a smaller peak at 27.2 ± 0.2 days for some parameters only. Our 29.5 days matches the synodic period of the moon, which Fraser-Smith seems to have missed.

(4) Analysis of weekly means of Ap for 1965, 1966, 1967, 1968 and 1969 separately showed a slight possibility of a shift of some periodicities from lower to higher values from quiet sun to disturbed sun years.

(5) Analysis of continuous monthly means for several years revealed several significant peaks common to many parameters notably at 48, 27, 15, 12–13, 7.6, 6.6, 5.6, 4.6, 3.8, 3.4, 3.0, 2.5 and 2.1 months. More extended data for Ap (28 years) gave 10.2, 6.8, 5.1 and 4.2 years as additional periodicities and these tally with those obtained by Fraser-Smith. These peaks seem to be harmonics of a 20–22 year double sunspot cycle.

changes in Ap activity are due to a direct influence of sunspot activity. He has referred to the possibility of some frequencies occurring as side lobes of some frequencies. From some samples of artificial single sinusoids that we examined, there was no evidence of any side lobes. We presume, therefore, that the Burg and FARNBERG analysis does not give any significant side lobe effects. If so, the frequencies we used must have real physical causes. The line near 4.1 years observed by us and by Smith has no matching line in the sunspot spectrum; but there could be a connection through the coronal 5305 Å line which seems to be well correlated to Ap (Pisyshev 1967).

The MESA analysis by Currie (1976) of the 100 annual values of the *aa* magnetic index of Mayaud after removing the mean and linear trends showed a 79 year cycle which he correlated with the Gleissberg sunspot cycle of 80 years, Waldmeier, the fundamental solar magnetic cycle (SMC) at 22 years, the fundamental solar cycle (SC) at 11 years and several other peaks, most of which fitted into harmonic relationships of the SC and SMC. The MESA analysis of the same *aa* index series by Courtillot (1977) showed that the larger periods were considerably affected when trends were removed. Also, the choice of LPEF seemed to play an important role e.g. splitting of the 11 year peak) for LPEF exceeding 30. Whether this peak splitting indicates a genuine solar process (Radoski *et al* 1975) or is an artifact of the MESA method (Courtillot *et al* 1977) is a moot question. Our analysis of artificial samples (Kane 1979) showed that closely spaced doublets or triplets could be resolved only with LPEF exceeding 50% of the period length (if at all).

The present analysis shows many more lines than those reported by the above-mentioned authors. Probably, some of our lines are due to spectral instabilities. Some are probably representative of transient powers in particular frequency bands in data of short intervals and hence disappear in longer data sets. However, a possible relationship between the periodicities in solar activity and periodicities in geomagnetic indices seems reasonably certain.

Acknowledgements

These studies are due to the various people and agencies responsible for producing and maintaining the geomagnetic indices and other data. This work was partially supported by the CNPQ, Brazil, under contract FINEP-537/CT.

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Wave packet structure and phase jumps in low latitude Pc3 geomagnetic pulsation

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Abstract. An array of four low latitude induction coil magnetometer stations has been used to study the spatial and temporal characteristics of Pc3 pulsations over a longitudinal range of 17° at $L = 1.8$ to 2.7 in south-east Australia. A preliminary study of individual Pc3 wave packet structure at the azimuthal stations has established the existence of phase jumps between wave packets at low latitudes similar to those observed at synchronous orbit and at higher latitude ground stations. However there did not appear to be any obvious pattern in phase jump occurrences between stations or signal components.

Keywords. Pc3 geomagnetic pulsations; wave packet; phase jump; magnetosphere; field line resonance.

1. Introduction

Pc3 geomagnetic pulsations are quasisinusoidal variations in the earth's magnetic field in the period range 10 to 45 sec (Jacobs 1970). These pulsations are characterized by wave-train envelopes which show a regular beat-like modulation. Herron (1966) investigated the phase characteristics of Pc3-4 signal wave packets at mid-latitudes and observed discrete phase changes in the pulsation signal when the amplitude of the signal between wave packets became small. In their study of longitudinal phase variations in high latitude Pc4 pulsations Olson and Rostoker (1978) observed sudden phase changes but did not study them in detail. Using data from three synchronous satellites and ground data, Mier-Jedrzejowicz and Hughes (1980) have shown that the phase jumps occur between wave packets in dayside pulsation signals. Signals in a single wave packet from two stations may drift apart in phase and are brought back into phase as a result of phase jumps. They suggested that the changes in Pc3 wave polarization are due to phase skips in either the H or the D component. They further found that these phase skips seldom occur simultaneously at all latitudes.

Phase skipping is an important property in Pc3-5 pulsations. It may provide an insight into the impulsive, temporal or spatial variations associated with the wave generation mechanism.

2. Data and analysis

An array of four low latitude induction coil magnetometer stations was established in south-east Australia to accomplish the study. The array is shown in figure 1 and station

locations and interstation distances are listed in tables 1a and b respectively. A geomagnetic longitude range of 17° was covered at $L = 1.8^+$ and a latitudinal range of 10° over $L = 1.8$ to 2.7 .

The block diagram of the recording system is shown in figure 2. Fluctuations in the geomagnetic field induced a small voltage in the detector coil which was then amplified

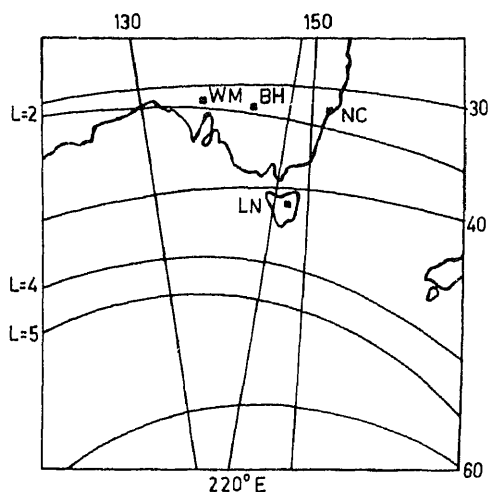


Figure 1. The University of Newcastle Pc3 recording station network.

Table 1a. Coordinate details of four Pc3 recording stations in Australia

Stations	Geographic		Geomagnetic		L value
	$\varphi(^{\circ}\text{S})$	$\lambda(^{\circ}\text{E})$	$\varphi(^{\circ}\text{S})$	$\lambda(^{\circ}\text{E})$	
Woomera (WM)	31.1	136.7	41.7	209.1	1.79
Broken Hill (BH)	32.0	141.5	42.4	214.5	1.81
Newcastle (NC)	32.6	151.7	42.0	226.3	1.81
Launceston (LN)	41.7	147.2	52.4	231.1	2.69

Table 1b. Interstation distances of the four Pc3 recording stations in Australia

Station pairs	Interstation distances (km)
WM-BH	425
BH-NC	1065
WM-NC	1485
NC-LN	1120

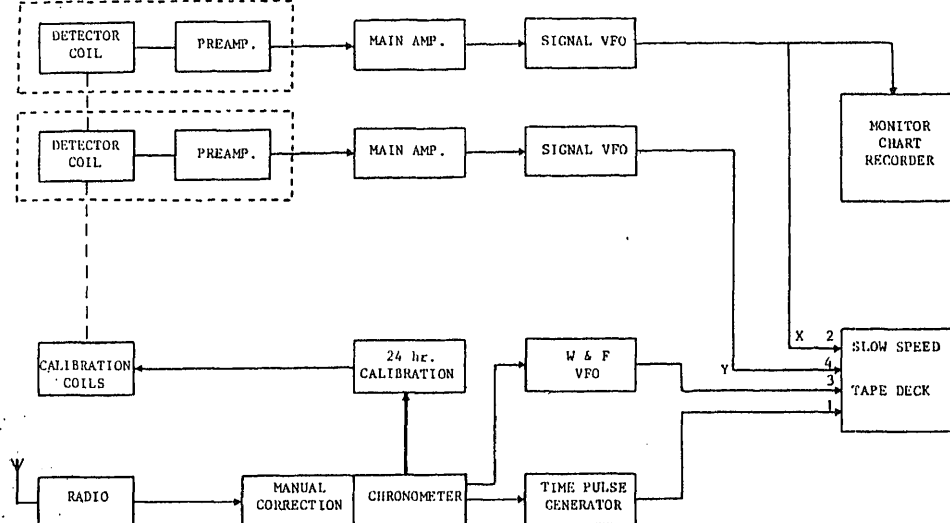


Figure 2. Block diagram of Pc3 recording system.

by a solid-state preamplifier situated within the coil containers and a main amplifier 100 m away in the equipment caravan. A frequency modulated recording system with a variable frequency oscillator (VFO) centre frequency of 6.25 Hz was employed to record the geomagnetic north-south (X) and east-west (Y) components of Pc3-4 wave signals in the 5–100 mHz (milli Hertz) band using slow speed magnetic tapes. In addition to the two signal VFO, a 6.25 Hz square wave signal was recorded to provide wow and flutter correction during analysis. The minimum detectable signal was 0.1 nT at 50 mHz. The timing system consisted of a chronometer accurate to 10 millisecond day $^{-1}$ which provided regular 5 sec pulses with 1 and 20 min interval pulses delayed by 1.5 and 3.5 sec respectively. At 24-hour intervals a range of four frequencies from 25 to 60 mHz were fed to the detector coil to provide frequency calibration over the operating bandwidth of the system. The slow speed tape deck operated with a transport speed of approximately 57 cm hr^{-1} and had four recording channels on a 1/4 inch tape. These were the 5 sec time pulses, the X channel, the wow and flutter signal and the Y channel respectively. An Easterline Angus chart recorder operating at 3.8 cm hr^{-1} provided a visual monitor of the X channel.

Both analog and digital analysis procedures were applied to the recorded data (figure 3). Data from 25 March to 21 September 1982 were digitized in the laboratory with a 5 sec sample rate using the recorded time channel pulses and providing a Nyquist frequency of 100 mHz (Ansari 1984). The resulting $X(t)$ and $Y(t)$ time series (see for example, figure 4) for each station were used to construct digital sonagrams over one day (25-hr) intervals using the maximum entropy method (MEM) on 10 min subsets overlapping by 5 min.

In order to obtain a simpler and averaged visual representation of frequency-time structure, contours at four different power spectral levels were plotted. A program CONTPT (see appendix) which uses the FFT method of calculating the power spectrum and employs the interactive graphics library (IGL) plotting routines, was used. The plots were normally obtained on a Tektronic-4025 graphics terminal and a 4612 hard

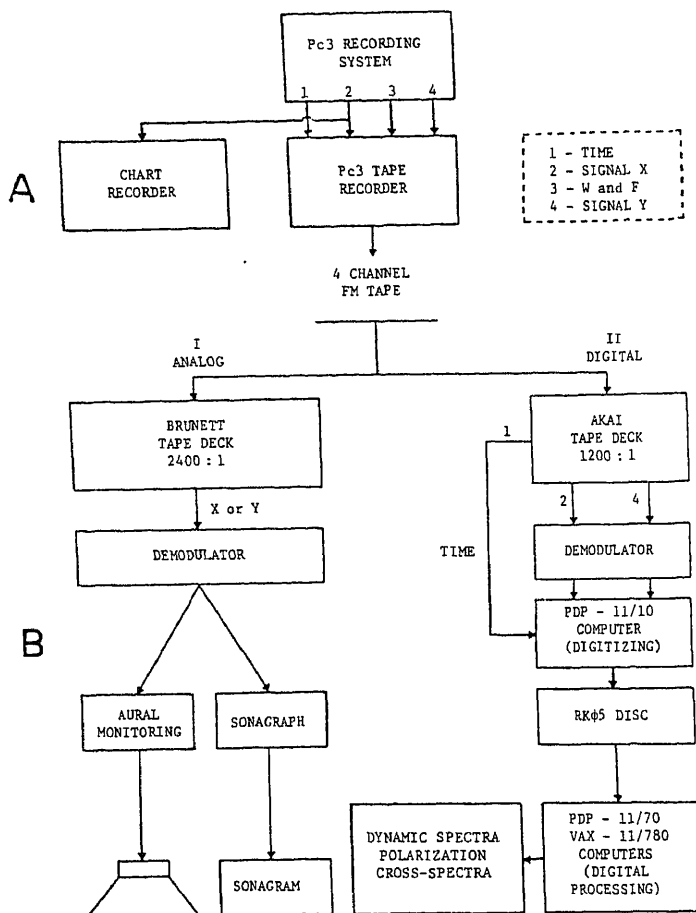


Figure 3. Pc3 data processing sequences. (A) Field recording instrumentation. (B) Laboratory analysis instrumentation.

copy unit or, were constructed on the plotter at the University of Newcastle computing centre. Plots were obtained for the duration of the Pc3 activity over four- and eight-hour time intervals. An example of contour sonagrams representing four power contour levels on May 30, 1982 at all four stations WM, BH, NC and LN is shown in figure 5. From these plots more precise time durations of Pc3 activity were measured to the nearest 10 min. These segments are indicated by horizontal lines on the time axis. Time is always stated in local time which is conveniently close to the Australian eastern standard time (AEST = UT + 10 hr).

3. Results

The properties of individual Pc3 wave packet structures are important since these may relate to the phase properties of the wave generation mechanisms. Wave packet

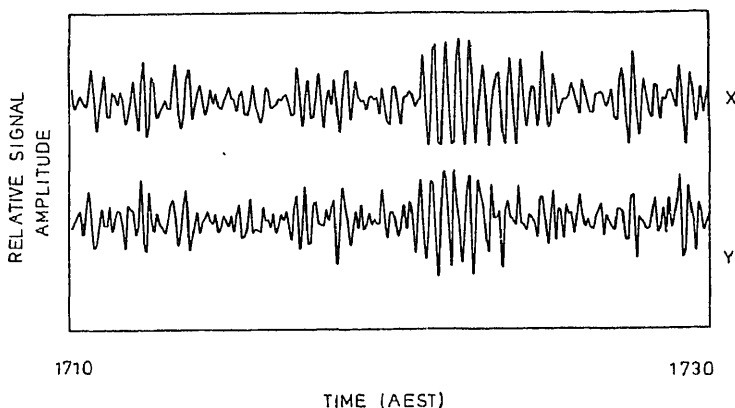


Figure 4. An example of an amplitude time record of a series of Pc3 wave trains recorded at Broken Hill on September 20, 1982 between the time intervals of 1710 to 1730 hr AEST.

structure and phase jumps were studied for March 25, 26, 27, 28, 30 and May 27, 30, 1982 in all selected time intervals at individual and pairs of stations for both *X* and *Y* components. The study was also carried out for WM data of August 31, 1982. The programs WPS and PSHIFT (see appendix) were developed to carry out the analysis. The results of a few examples are considered here.

Figure 6 shows the variations in the *X* component at WM between 1410–1430 hr AEST on August 31, 1982. Packet structure can be seen in the upper plot and clearly identifiable wave packets are labelled sequentially. In the lower plot the vertical axis is the wave cycle number counts and the times of the peaks are plotted horizontally. A constant phase difference between cycles will appear as a straight line segment in the plot. A discontinuity will indicate a sudden phase change while a change in gradient will indicate a change in frequency. It can be seen that the five wave packets have constant phase over their duration and their frequencies are the same as indicated by similar slopes. Phase jumps can be seen clearly at B, D, F and G in figure 6.

It is of interest to compare the cycle-by-cycle signal phase difference between stations for individual wave packets. Figure 7a shows the interstation phase comparison for WM and BH *X*-components over four wave packets. Here the time difference between the corresponding signal peaks is plotted on the vertical axis in the bottom graph. A horizontal line of plotted points indicates constant phase between the two stations while vertical points indicate variable phase. A sudden discontinuity relates to a phase jump. The absolute value of the time differences have little meaning since they depend on the relative alignment of the nearest corresponding peaks from the two stations. In figure 7a four wave packets are identified. The phase difference for packet 1 increases gradually and then drops to zero at B. Constant phase is seen between B and C over wave packet 2. Packet 3 shows a variable phase difference similar to packet 1 while the short wave packet 4 is constant in phase difference. Phase jumps are seen at C and D.

Figure 7b shows the phase differences over the same data interval and wave packets for BH and NC *X*-components. Variable phase differences are seen for packets 1 and 2

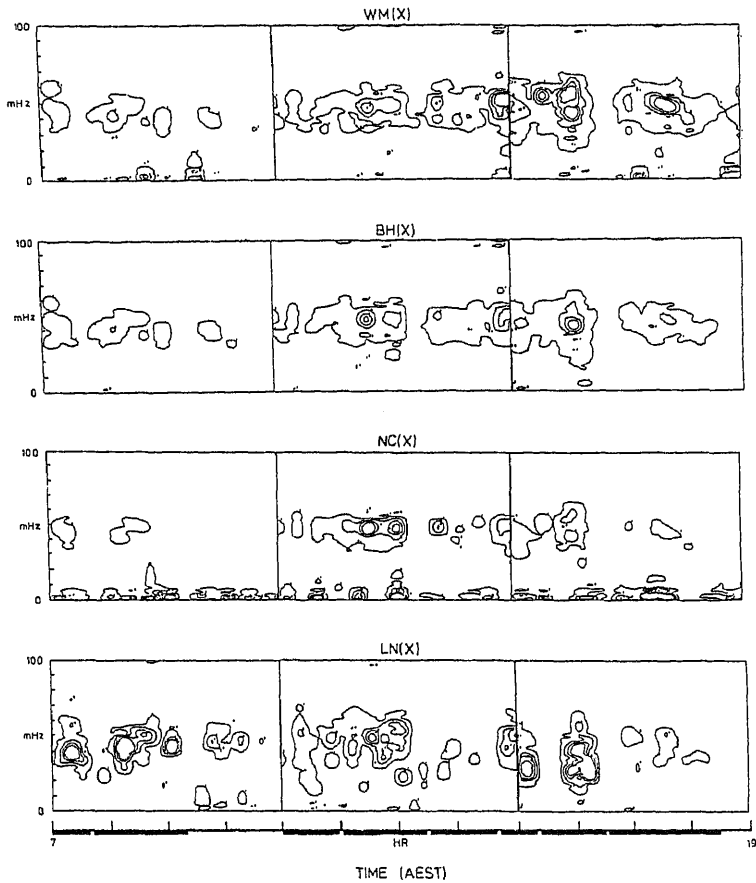


Figure 5. Contour sonagram showing Pc3 activity on the X component at the four stations WM, BH, NC and LN on May 30, 1982 between 0700–1900 hr AEST. The segments marked with a horizontal line on the time axis show continuous pulsation activity.

with an apparent periodicity of about 80 sec while packet 3 is constant. Packet 4 has a low amplitude at NC and the measurements are probably unreliable. Here phase jumps are seen at C and possibly D in agreement with the WM-BH station

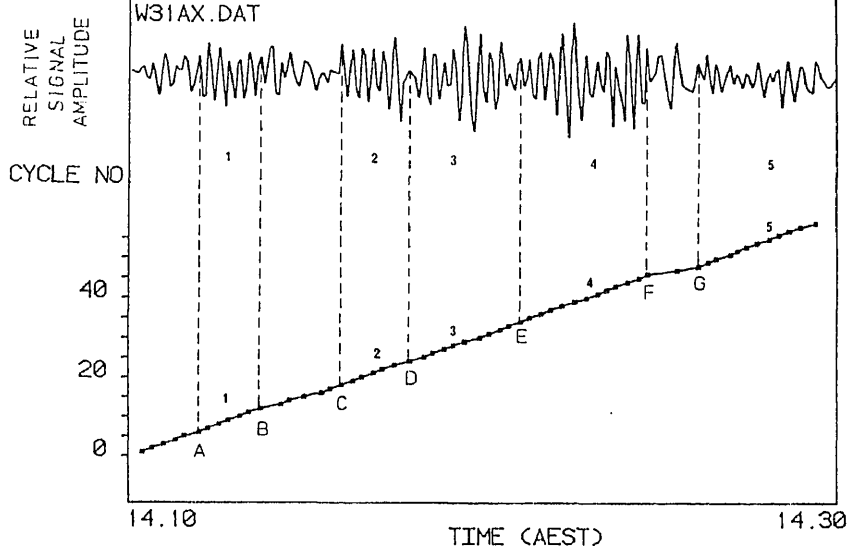


Figure 6. The amplitude-time data at the top of the figure indicates the wave packet structure in the WM X component on August 31, 1982 (W31AX.DAT) between 1410–1430 hr AEST. The plotted dots at the bottom of the figure correspond to amplitude peaks in the magnetic field data.

at C and at different times at BH-NC namely at B and E. The X components show a different packet structure with the NC data, the boundaries D and E shifted to D' and E' respectively. Phase differences are reasonably constant over packets 2 and 4. Data are variable in all packet time intervals at the WM-BH pair. Phase jumps occur near D' and possibly B at both station pairs but at F for only the BH-NC pair.

This example shows that there is more similarity between longitudinal X components at spaced stations and also the Y components than between the X and Y components at a single station. Furthermore, no consistent pattern of phase jumps is seen between components or station pairs.

Another example of interstation wave packet phase structure is shown in figure 9 for the afternoon interval 1800–1820 hr AEST. Wave packets 1 to 6 were identified using the WM and BH X -component data as reference, since these showed the clearest packet structure. For the WM-BH station pair constant phase difference on the X components is seen over packets 1, 2, 4 and 6 while for the BH-NC pair constant phase is seen only in packet 2. On the Y components packets 1 and 2 show reasonably constant phase differences at both pairs of stations. Variable phase is seen over all other packets at both pairs of stations. It should be noted that the wave amplitude is reasonably low on the NC Y component and this may contribute to some of the variability seen in the BH-NC data for this component. The largest phase jump is seen between packets 2 and 3 at B, C but only on the Y component at the WM-BH stations and the X component at the BH-NC stations. The only other significant jump is at E for the WM-BH stations X component signal but it is not accompanied by a corresponding jump on the Y component or seen at the other station pair.

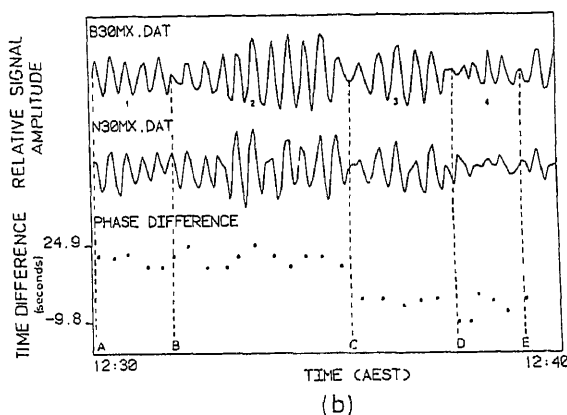
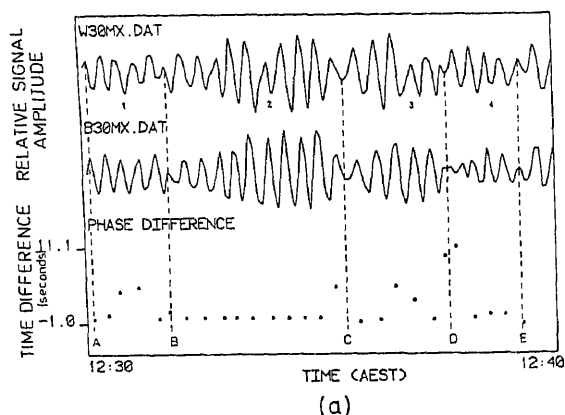
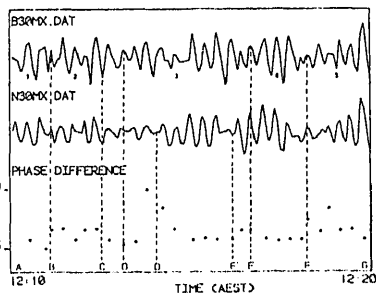
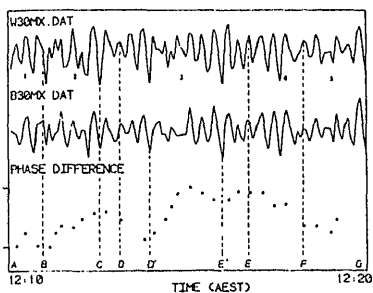


Figure 7. Interstation wave packet phase structure for May 30, 1982 between 1230–1240 AEST. The two *X* wave forms show four wave packets at (a) WM and BH, (b) BH and NC. The bottom plot is the time difference between nearest corresponding peaks at the two stations. The time difference is positive if the station at the top of the panel leads in phase and negative if it lags.

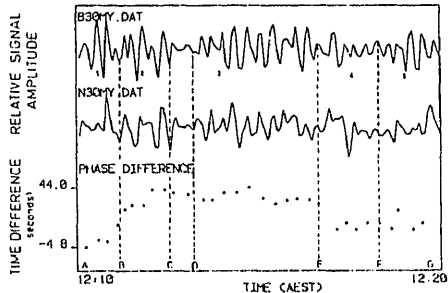
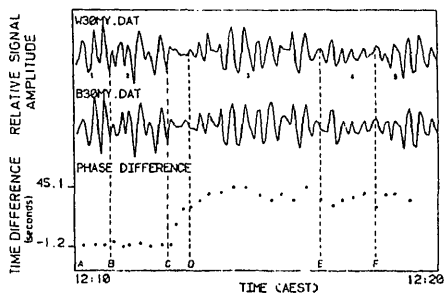
4. Discussion and conclusion

These results on individual Pc3 wave packet structure show that phase jumps are observed at low latitudes but the data do not show any systematic temporal or spatial patterns. More cases of constant phase within individual wave packets are seen for the WM-BH station pair. It may be significant that these stations are spaced only half the distance apart compared to the BH-NC stations.

With field line resonance when the driving force in the magnetosphere is removed each resonant shell will oscillate at its own resonant frequency as the signal decays (Southwood 1975). Mier-Jedrzejowicz and Hughes (1980) while explaining the

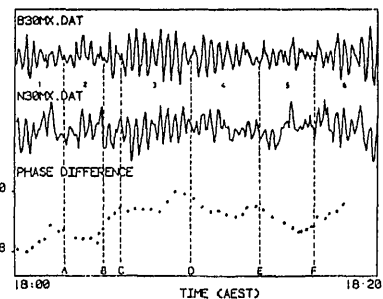
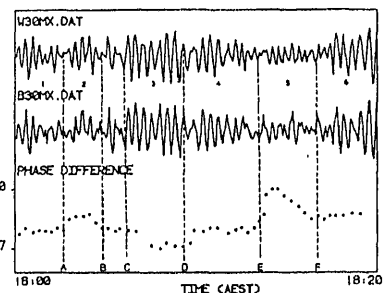


(a)

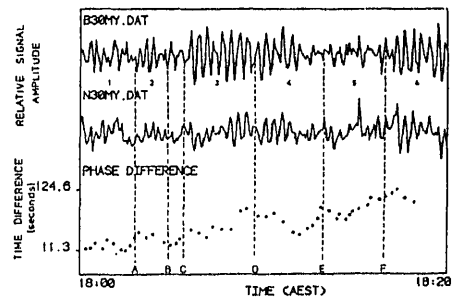
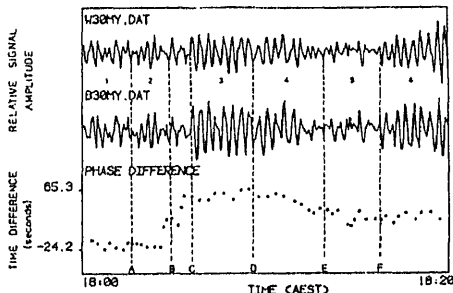


(b)

Figure 8. Interstation wave packet structure for 1210–1220 hr AEST on 30 May, 1982 for the WM-BH and BH-NC pairs of stations. (a) *X* component data, (b) *Y* component data.



(a)



(b)

Figure 9. Interstation wave packet structure for 1800–1820 hr AEST on 30 May, 1982 for the WM-BH and BH-NC pairs of stations. (a) *X* component data, (b) *Y* component data.

geometry, the field lines at different stations could have slightly different natural frequencies and, as a consequence of this, the signals drift out of phase and they also damp out because of either collisionless or ionospheric damping (Newton *et al* 1978). In order to provide a new impulse for each wave packet these impulses must occur every three minutes or so. This impulsive process suggested by Mier-Jedrzejowicz and Hughes could not explain our observations as phase shifts seldom occur simultaneously at all latitudes.

A second explanation by Mier-Jedrzejowicz and Hughes suggests that there may be sharply delineated regions, each with its own field line resonance, which may differ in frequency and phase. These regions may be moving in longitude relative to the station. According to this explanation the boundary of the resonance region should move sufficiently fast so as to reach all the three longitudinal station instruments. Taking into consideration the resolution of the recording instrument and the distance between stations, the azimuthal speed comes out to be about 150 km/sec which is a reasonable speed for an impulse, but not for the boundary of a region which then resonates for three minutes. Hence this explanation is also not suitable to explain our observations.

According to the third explanation by Mier-Jedrzejowicz and Hughes the motion of a resonance region in latitude past the station would cause a sudden change in polarization which in turn would move power among the components of a signal. A slow rotation of the polarization ellipse would produce an attenuation of the signal in one component and a growth in the other. Because of the gradual change in the relative phase of two components, it would appear that the frequency of the signal was changing when only one component was observed. A rapid change in polarization would, therefore, cause apparent phase skips.

This interpretation also cannot explain our observed phase skips because a phase skip in one component due to a sudden change of polarization must be accompanied with a corresponding skip in one or both of the other components and this does not always happen as is obvious from our results.

Lanzerotti *et al* (1981) also observed many sudden phase shifts when studying data from three low latitude Californian stations at $L \approx 2$. The phase shifts were found to be variable between components and stations. They also argue against the impulsive source process.

The results for the azimuthal stations at WM, BH and NC show similar properties to those found by Lanzerotti *et al* (1981). It was difficult to define distinct wave packets which showed the same time occurrence pattern at all three stations. It appears that corresponding signal components between different stations often show a greater similarity in wave packet structure than do the two components at a single station (figure 8). Sudden changes in phase or phase jumps are seen at all three stations in both components but like Lanzerotti's and coworkers' data no systematic pattern was observed between components or stations. The phase differences between pairs of stations were mostly variable and no consistently linear frequency drifts such as those expected as a consequence of a free resonant shell oscillation in the absence of a driving force were observed. On many occasions a constant phase was observed over individual packets but again no consistent pattern between stations and components was evident. More examples of stable phases were observed between WM and BH

974). It is therefore conceivable that some of the phase difference patterns in experimental data may be caused by amplitude and phase variations arising from low and middle latitude ionospheric currents and irregularities. Furthermore, irregularities observed on the ground may arise from a combination of individual L -shell irregularities which could also produce confusing spatial and temporal phase patterns.

Acknowledgements

This research was supported by grants from the Australian Research Grants Scheme of the University of Newcastle. One of the authors (IAA) was supported by a University of Newcastle Scholarship and is also thankful to the Department of Electrical and Computer Engineering of the University of Wollongong for assistance.

Computer programs

Descriptions of the computer programs are given here.

Contour plots

The program consists of a single channel (X or Y) of data. The power spectrum is calculated using the FFT method. A three-point smoothing routine SMOOTH was also used to smooth power spectral densities for better representation. There are two versions of the main program, one is employed for eight hour data blocks and the other for four hour data blocks. The subroutine CNTPLT was written to arrange the power spectral density to be stored in the form of equispaced grid points in a two dimensional array.

CONTOUR is a subroutine package available for use on the VAX-11/780 computer in conjunction with IGL. The subroutine CONTOUR plots contours of the power spectral density function $P = f(fr, t)$ in a two-dimensional graph when values of P are stored at grid points which are equispaced in the fr (frequency) and t (time) directions. Therefore the total size of the rectangular array is determined by IDATA and the number of grid points in the fr and t directions, respectively.

Linear interpolation is employed to plot a continuous contour line. The maximum and the minimum values of the power spectral densities are specified and contour levels are computed. The output to be plotted is sent to an IGL Capture/Replay file CONTOUR.DAT. The plots are then obtained on a Tektronix-402 graphics terminal and on the plotter at the University of Newcastle Engineering centre.

The packet structure

The program plots a time series (X or Y) data file. It also plots the wave cycle number on the vertical axis using the IGL plotting routines. The output to be plotted is sent to an IGL Capture/Replay file. The plots were normally obtained on the Tektronix-402 graphics terminal and on the plotter at the University of Newcastle Engineering centre.

PSHIFT: phase shift

The program plots two time series (X and Y) data files. It also plots the time difference between the corresponding signal peaks on the vertical axis by employing the IGL plotting routines.

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Spectral characteristics of low latitude Pc3 geomagnetic pulsations in south-east Australia

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Abstract. Geomagnetic pulsations in the Pc3 period range (10–45 sec) were recorded simultaneously in south-east Australia using an array of four low latitude induction coil magnetometers over a longitudinal range of 17° at $L = 1.8$ to 2.7 . Geomagnetic data from March 25 to September 21, 1982 were digitized and spectral studies were carried out from Grey scale digital (GSD) dynamic spectra and contour sonagrams. Similar wave spectra were generally seen at the three stations WM, BH and NC in the same latitude with Pc3 waves mostly being observed in the 40–45 mHz band. In addition the higher latitude station (LN) also showed lower frequency Pc4 bands (~ 10 mHz) on some days but higher frequency bands (60–80 mHz) on others. Sometimes completely different wave frequencies were observed on the X and Y components with the stations in the same latitude generally showing similar spectra on the corresponding components. Dynamic spectra also indicated that wave energy often turned off and on abruptly over short intervals of time implying solar wind control and Pc3 pulsation activity. A band harmonic structure was occasionally seen in wave spectra at the azimuthal stations mainly on the X component during the local afternoon with a harmonic spacing (Δf) of ~ 10 mHz.

Keywords. Geomagnetic pulsations; Grey scale dynamic spectra; contour sonagrams; induction coil magnetometers.

1. Introduction

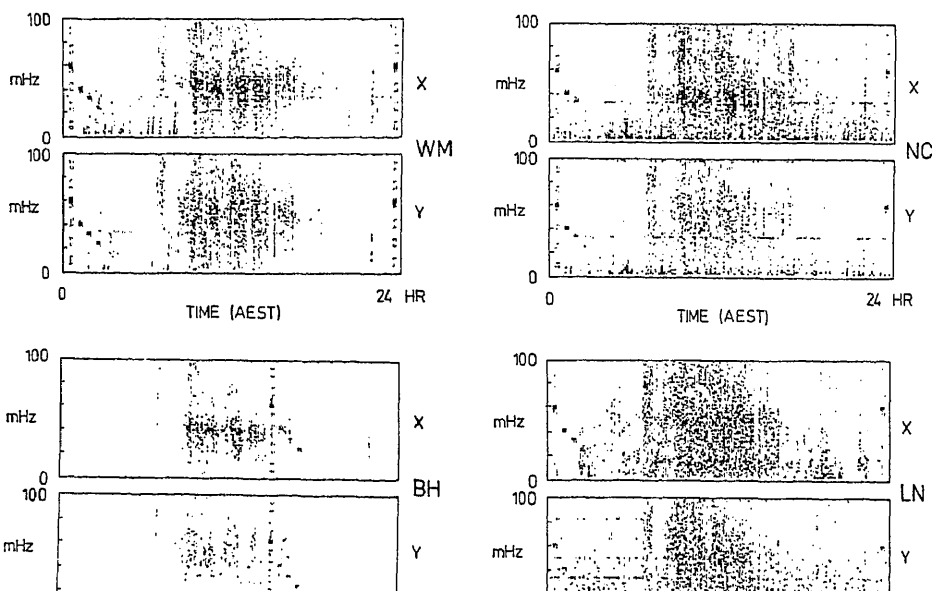
Geomagnetic pulsations of the Pc3 type are daytime phenomena with a maximum occurrence around noon (Saito 1964). At low geomagnetic latitudes the amplitude of a typical Pc3 event is 0.1 nT (Orr 1973). Dynamic spectra in the frequency-time domain provide the most convenient method for examining the general properties and interstation similarities of geomagnetic pulsations. This technique has been widely employed within the Pc1 pulsation range (Kokubun 1970) but the difficulty in obtaining the large record to replay tape recorder speed-up ratio has restricted its use in the dynamic spectral studies of the longer period (Pc3–5) pulsations. However Saito *et al* (1979) developed the P-HISSA (power-spectrum-type high speed spectrum analyzer) which is capable of producing dynamic spectra over all the Pc3–5 bands. The present paper describes the spectral studies of low latitude ($L \leq 3$) Pc3 geomagnetic pulsations (period range 10–45 sec) data recorded in south-east Australia.

2. Data and analysis

and the two separated in latitude along a longitude, were used. A geomagnetic longitudinal range of 17° was covered at $L = 1.8$ and a latitudinal range of 10° over $L = 1.8$ to 2.7 . The stations were situated at Woomera (WM) (41.7°S , 209.1°E geomagnetic), Broken Hill (BH) (42.4°S , 214.5°E), Newcastle (NC) (42.0°S , 226.3°E) and Launceston (LN) (52.4°S , 231.1°E).

The geomagnetic north-south (X) and east-west (Y) components of Pc3-4 wave signals in the 5–100 mHz were recorded using slow speed analog magnetic tapes employing a frequency modulated recording system. Brief details of the recording and analysis instrumentation have been given by Ansari (1984). Both analog and digital analysis procedures were applied to the recorded data (Ansari 1984). Data from March 25 to September 21, 1982 was digitized in the laboratory with a 5s sample rate using the recorded time channel pulses and providing a Nyquist frequency of 100 mHz (Ansari 1984). Digital sonagrams from the $X(t)$ and $Y(t)$ time series of all the data for each station over one day (24 hour) intervals were constructed using the maximum entropy method (MEM) on 10 minute subsets overlapping by 5 minutes. A detailed analysis of the digital sonagram characteristics was then carried out and ten days of data were selected for detailed analysis. These are March 25–28, 30; May 27, 29–31 and June 1, 1982.

For all these days Grey scale digital (GSD) dynamic spectra (see figure 1) were constructed using a 64 K RAM CP/M based microcomputer system and a CITHO 1550 dot matrix printer with graphics capability (see appendix). In order to obtain a simpler and averaged visual representation of frequency-time structure, contours (see for example figure 2) of four-hour duration at four different power spectral levels were



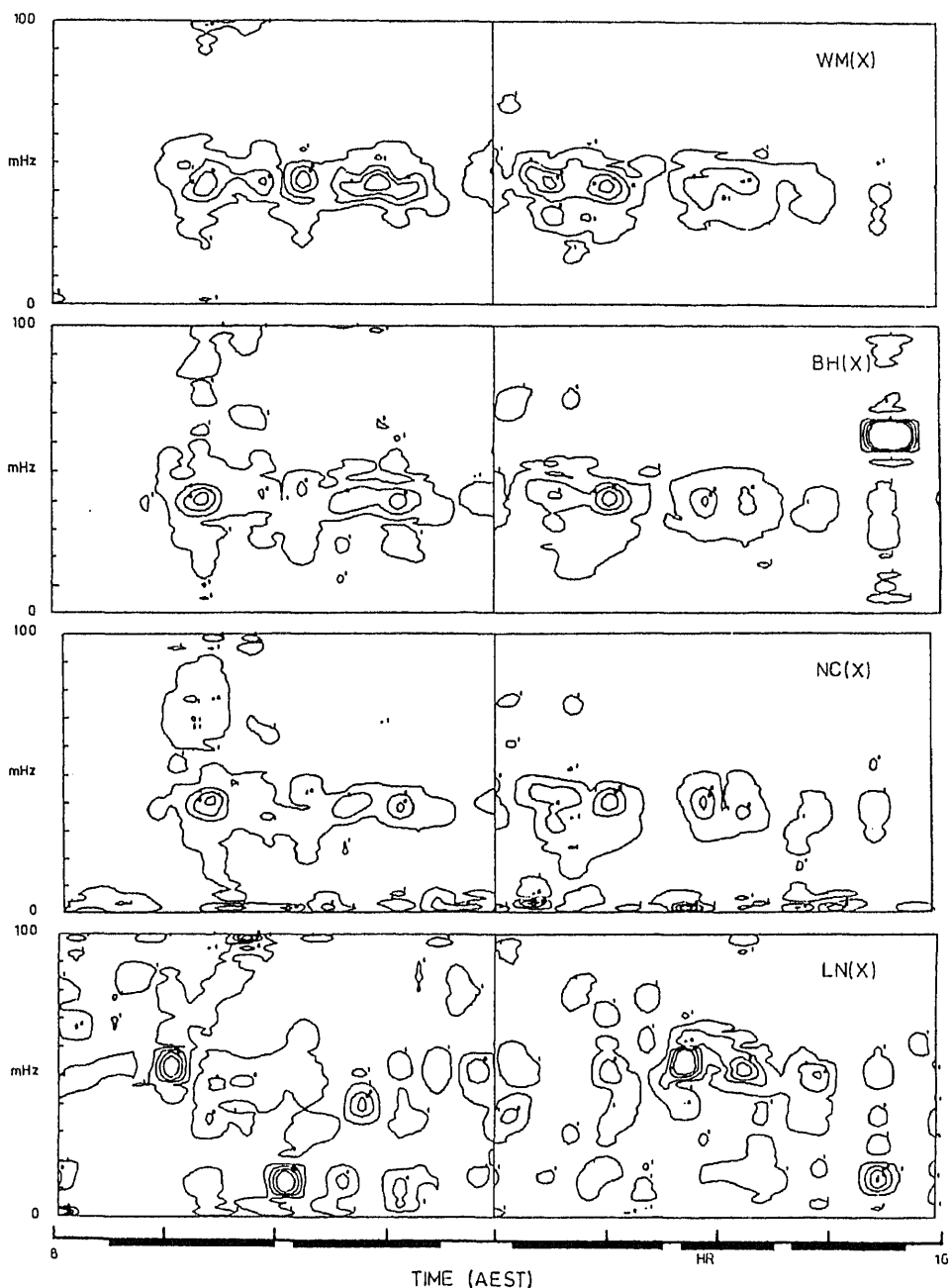


Figure 2. Contour sonagram showing Pc3 activity on the X component at the four stations WM, BH, NC and LN on March 25, 1982 between 0800–1600 hr AEST. The segments marked with a horizontal line on the time axis show continuous pulsation activity.

plotted employing the FFT method on a Tektronix 4025 graphics terminal and a 4612 hard copy unit (for details see Ansari 1984).

Although the frequency response of Pc3 recording equipment (Ansari 1984) is identical at all the four recording stations, the amplifier gains at the four stations were not necessarily the same and consequently the dynamic spectra presented are not normalized with respect to amplitude. Therefore it is not possible to compare the power spectral intensity between the stations from the spectrograms. However at a given station the *X* and *Y* signal channel gains are always identical and the spectral intensity between components can be directly compared. In general, there appears to be a higher signal amplitude on the *X* component than on the *Y*.

In obtaining the GSD spectra fixed levels (0–256) of PIXEL data were used (appendix). Therefore the GSD technique provides a uniform response for all the spectra from all the four stations. However the 256 levels are scaled to range between the maximum and minimum signal intensities for the contour sonagrams and therefore different stations and different days will be portrayed at different intensity levels. This is done in order to emphasize the frequency time structure. All the GSD dynamic spectra are obtained over a time range of 25 hours (0000 hr AEST of the current day to 0100 hr AEST of the next day) and a linear frequency span of 0 to 100 mHz. Time is always stated in Australian Eastern Standard Time (AEST). AEST = UT + 10 hr. The contour sonagrams are separately obtained in four-hour data sections. These sections have the same intensity range and are joined to form a total data span of eight or twelve hours (see for example figure 2). Again a frequency range of 0 to 100 mHz is employed. As the signal intensity level is higher in the *X* component, the contour sonagrams of the *X* component only are shown. The horizontal bars at the bottom of each contour sonagram figure show the durations of Pc3 activity on that day.

Interstation comparison using cross-spectral analysis techniques were carried out on WM-BH, BH-NC and WM-NC station pairs to give longitudinal wave properties and on the NC-LN pair to give latitudinal wave properties. Plots of cross-spectral phase, coherency, auto-power and cross-power spectra at pairs of stations for *X* and *Y* components in all data intervals for all selected days were also constructed (see for example figure 15, referred to later).

3. Results

Detailed spectral studies were carried out for all the ten selected days. The results relating to some days of data are presented here.

3.1 *March 25, 1982*

The GSD dynamic spectra and the contour sonagrams showing Pc3 activity on March 25, 1982 are shown in figures 1 and 2 respectively. The LN *Y* component spectra in figure 1 show interference bands because these data were not filtered. This day is an example of a moderately disturbed day with $\Sigma Kp = 28$ – and the range of *Kp* values during the interval of Pc3 occurrence is from 3 – to 4 –. Figure 2 clearly shows that almost continuous Pc3 activity occurred on this day from about 0845 hr AEST to 1515 hr AEST at a constant frequency centred around 40 mHz at WM, BH and NC. Activity is more variable

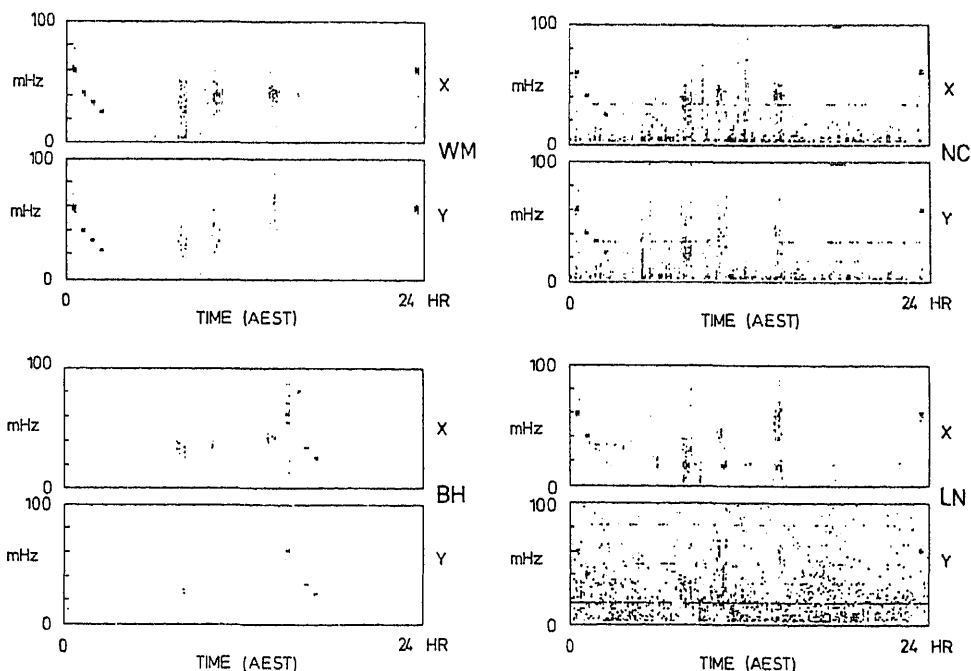
at LN. A higher frequency band centred around 50 mHz and of greater intensity than the 40 mHz band is also observed at LN. The 60 mHz signal (figure 1) occurring at BH between 1520 to 1540 hr AEST is the calibration signal which is normally recorded at all other stations.

3.2 March 26, 1982

The spectrograms for this day are shown in figures 3 and 4 respectively. It is again a moderately disturbed day with $\Sigma Kp = 21$ and the Kp value during the interval of Pc3 occurrence ranges from 2+ to 3. This day is an example of fairly low Pc3 activity with only three bursts occurring. The frequency bands at WM, BH and NC are centred about 30–40 mHz (figure 4). At LN the higher frequency band centred on 40 mHz is present between 1000 hr and 1100 hr AEST while a higher frequency band centred on 60 mHz and not seen at the other stations is predominant at LN between 1400 hr and 1500 hr AEST. Figures 3 and 4 clearly show low frequency (0–10 mHz) instrumentation noise at NC.

3.3 May 27, 1982

Shown in figures 5 and 6 are the GSD dynamic spectra and the contour sonagrams for May 27, 1982. The activity pattern on this day is similar to that of March 25. The range



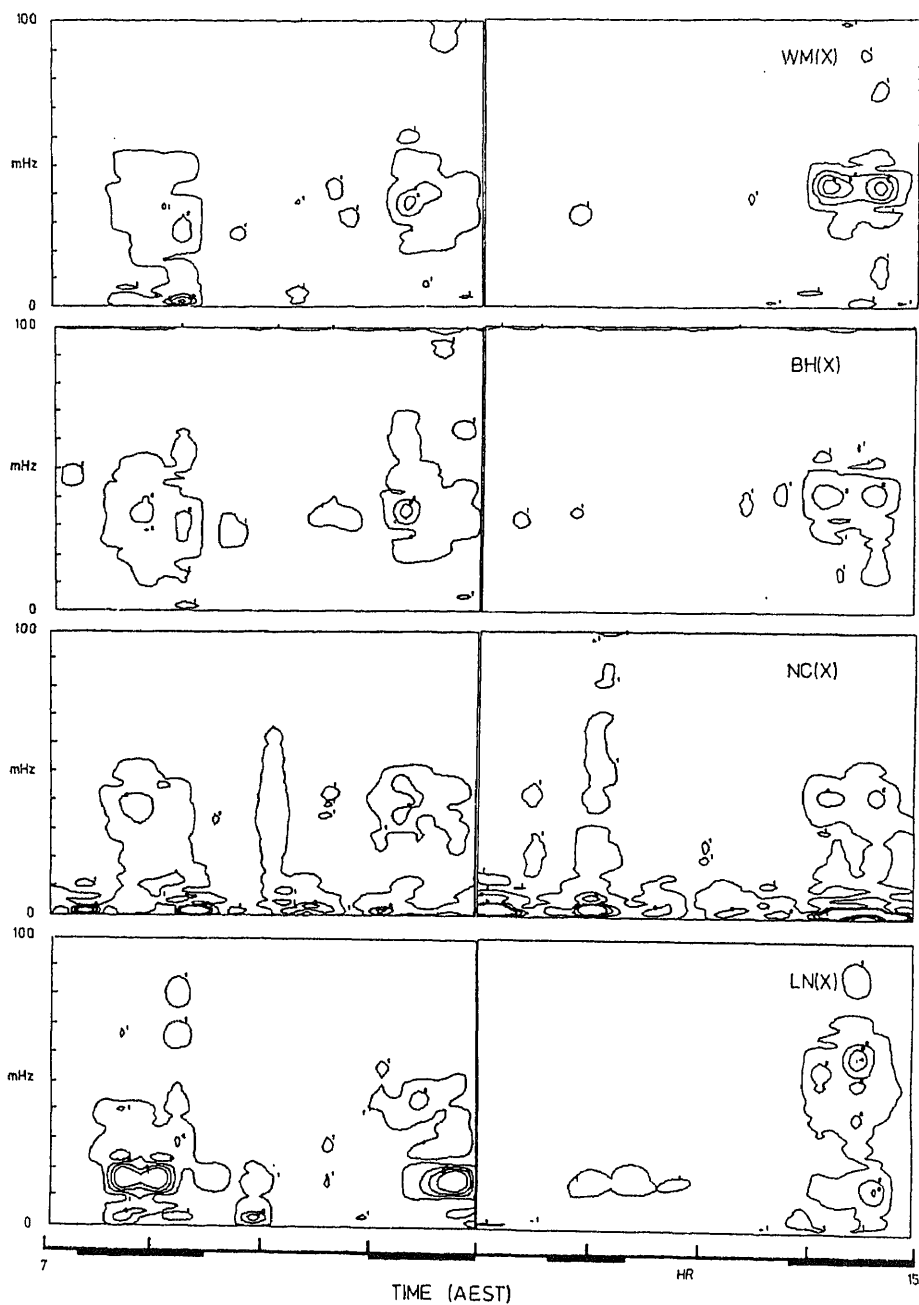


Figure 4. Contour sonagram showing Pc3 activity between 0700–1500 hr AEST on March 26, 1982.

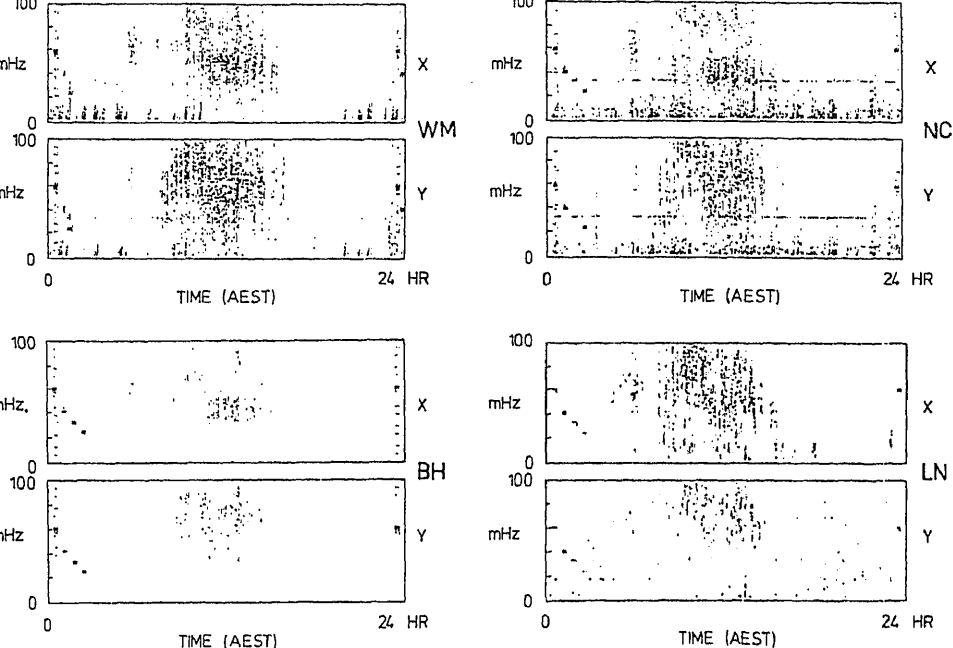


Figure 5. GSD dynamic spectra depicting Pc3 activity on May 27, 1982.

of Kp values during the Pc3 occurrence period is 3+ to 5-, and $\Sigma Kp = 39$ - indicating a magnetically disturbed day. Figure 5 clearly shows that there is a broader band signal on the Y channel than on the X channel. The contour sonagrams show that at WM the X component band is centred at about 50 mHz. At BH between 1130-1330 hr AEST there are two frequency bands centred on 40 and 50 mHz. At NC, however, the lower frequency band predominates. At LN there is activity in both bands with additional activity centred around 50 mHz. Higher frequency signals around 80 mHz are also seen at LN and to a lesser extent at the other stations. These may be higher order harmonics.

3.4 May 29, 1982

The GSD dynamic spectra and the contour sonagrams for this day are shown in figures 7 and 8 respectively. This is a magnetically active day with $\Sigma Kp = 38$ + and the range of Kp values during the Pc3 occurrence is 4+ to 6. It is obvious from the spectrograms that Pc3 activity continues throughout the day over a wide range of frequencies. At WM there is a dominant activity centred around 30-50 mHz between 0630 hr and 1800 hr AEST and showing a gradual decrease in frequency with time. Activity consisting of a double frequency band centred between 40 and 60 mHz persists between 0900 hr and 1100 hr AEST. The pulsation activity at BH and NC is essentially the same where a frequency band centred around 40 mHz dominates between 0630 hr and 1800 hr AEST. At LN the Pc3 activity is of a broader band (40-70 mHz) between 1000 hr and 1400 hr

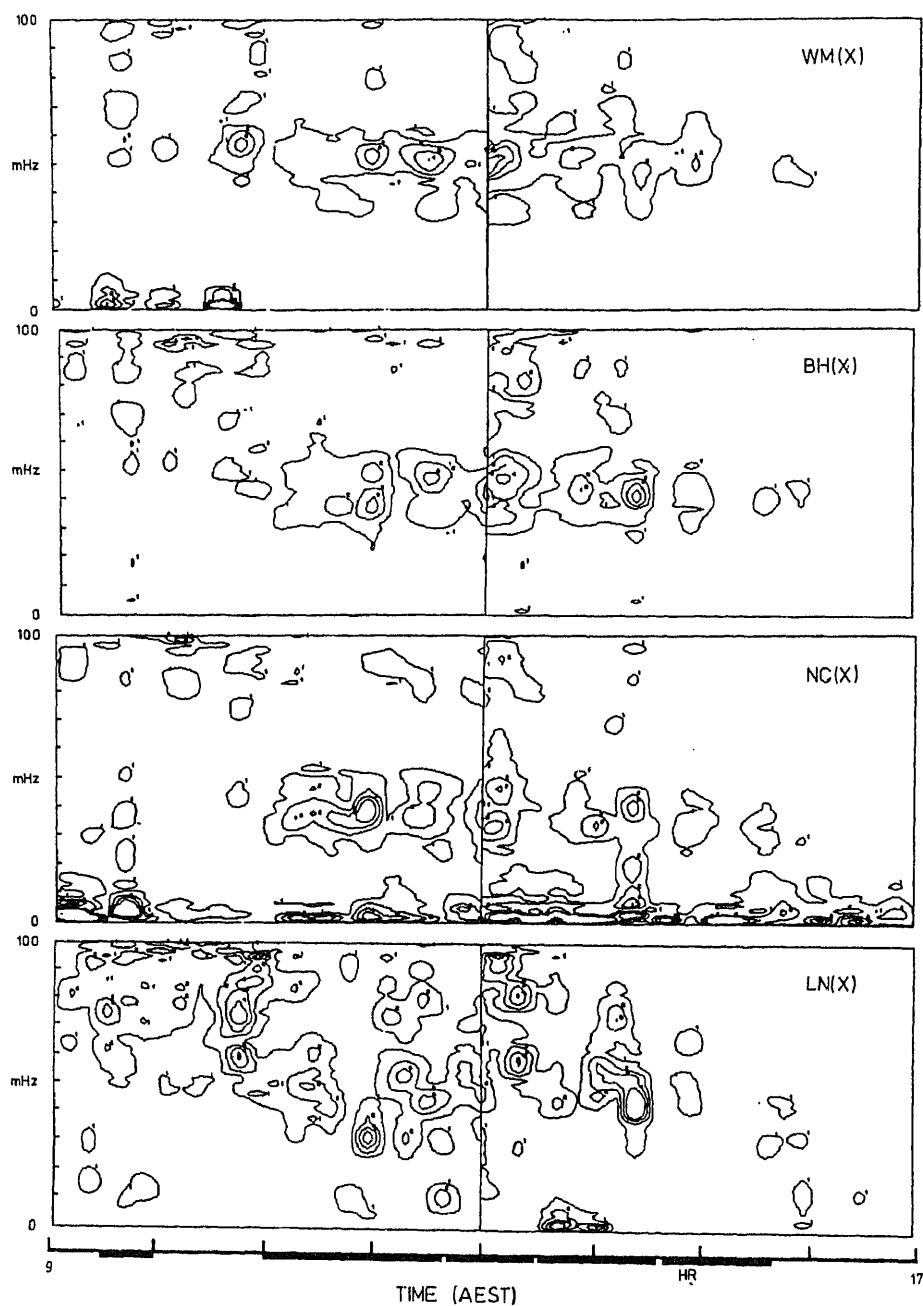


Figure 6. Contour sonagram showing Pc3 activity between 0900–1700 hr AEST on May 27, 1982.

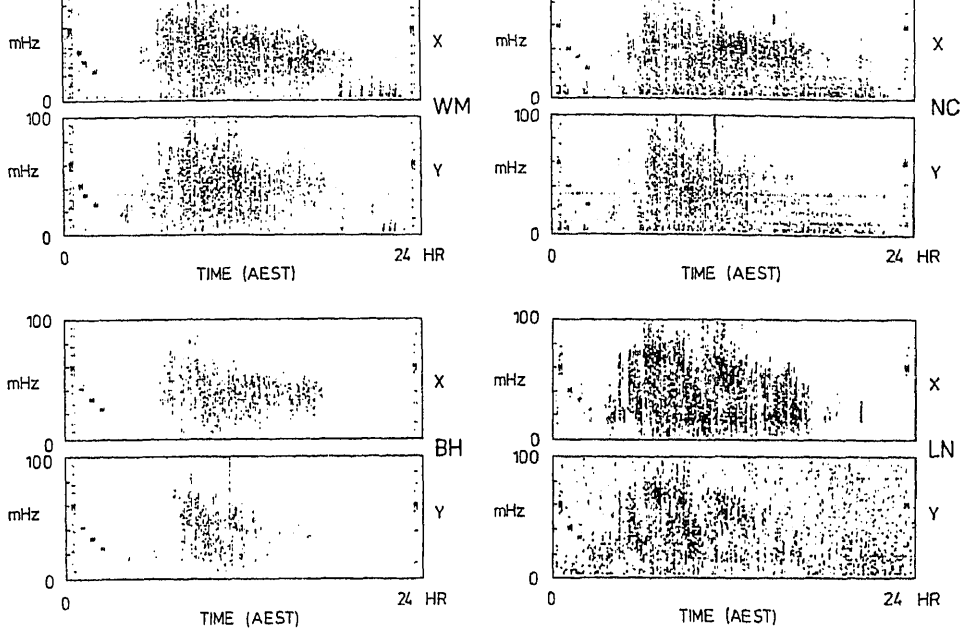


Figure 7. GSD dynamic spectra depicting Pc3 activity on May 29, 1982.

AEST and shows common activity with the other stations only at the lower frequencies. There is a 10 mHz Pc4 band present at LN between 0900–1000 hr AEST. Between 0600 hr and 0930 hr AEST high frequency activity in the 60–80 mHz band is also seen.

3.5 May 30, 1982

Figures 9 and 10 depict the GSD dynamic spectra and the contour sonagrams of May 30, 1982 from the data recorded at all the four stations. The range of Kp values during the Pc3 occurrence period is 4 – to 4 +, and $\Sigma Kp = 34$. The spectrograms show that Pc3 pulsation activity is strongest in the afternoon at WM, BH and NC. The same frequency bands in the 30–60 mHz range are seen at all the four stations. However additional activity is seen at LN at 20 mHz.

3.6 May 31, 1982

The spectrograms for this day are shown in figures 11 and 12. This is also a magnetically disturbed day with $\Sigma Kp = 35$ and the range of Kp values during the Pc3 occurrence is 4 to 6 –. The spectrograms show that at WM, BH and NC most of the pulsation activity during the period of occurrence lies between 40 and 60 mHz. This band does not show any significant energy at LN until after 1300 hr AEST. However pulsation activity consisting of a lower frequency Pc4 band at 10–30 mHz is apparent after 1030 hr AEST.

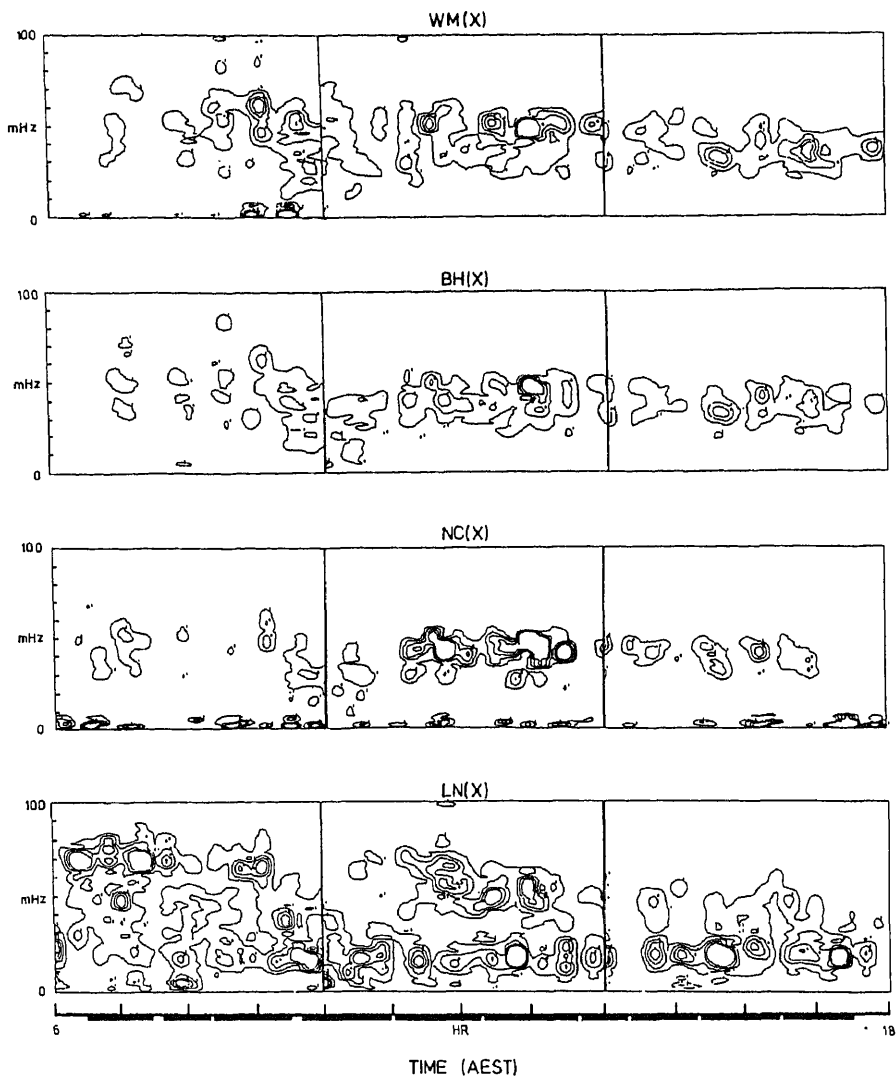


Figure 8. Contour sonogram showing Pc3 activity between 0600–1800 hr AEST on May 29, 1982.

Furthermore, variable frequency activity is seen at LN between 0700 hr and 1000 hr AEST. It does not relate to any great extent to activity at the other stations.

3.7 June 1, 1982

The GSD dynamic spectra and the contour sonagrams for this day are shown in figures 13 and 14 respectively. This is an example of a moderately disturbed day with

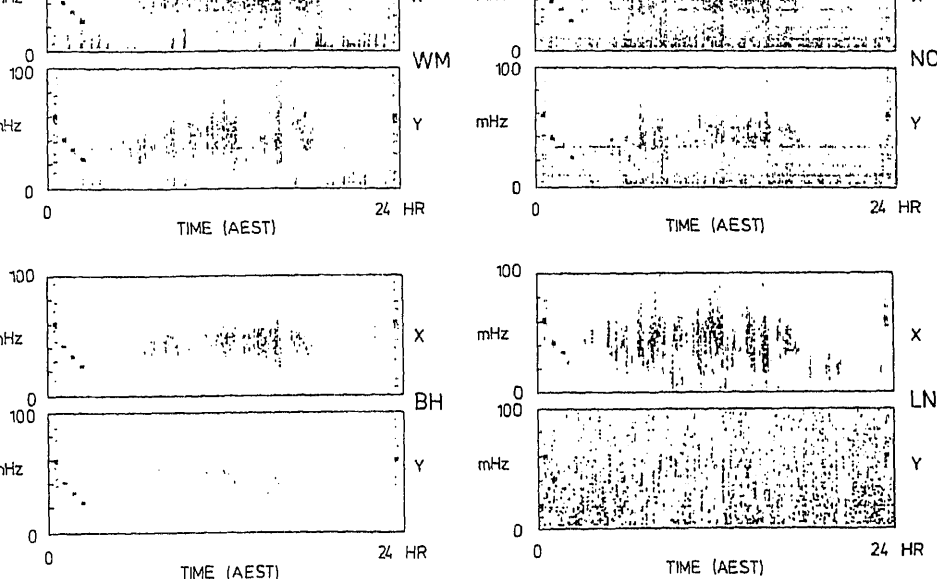


Figure 9. GSD dynamic spectra depicting Pc3 activity on May 30, 1982.

$EKp = 27+$ and the range of Kp values during the interval of Pc3 occurrence is 3 to 4. Activity is similar to that of the previous day (figures 11 and 12). The spectrograms show that at WM, BH and NC the Pc3 pulsation activity consists of a broad frequency band over the range 20–60 mHz. Activity between 30–60 mHz and similar to WM and BH is observed at LN between 0700 hr and 1100 hr AEST. However after 1030 hr AEST Pc4 activity at 10–20 mHz predominates at LN. This pattern of activity is not observed at other stations.

4. Discussion and conclusion

4.1 General features

It is generally accepted that at a given station there appears to be a higher signal amplitude on the X component than on the Y component. This has been found to be so in the majority of cases and examples can be seen in the GSD spectra in figures 1, 3, 7, 9, 11 and 13. However, exceptions sometimes do occur. An example is seen in the spectra of May 27 (figure 5) where greater power on the Y component is a general feature all day. Although the powers plotted between stations are relative, LN generally shows the greatest signal power. This is supported by the generally higher upper limit used for GSD spectra and contour sonagrams and is expected since this is the higher latitude station. This is in agreement with the observations that the signal amplitude of Pc3 pulsations increases with latitude at middle latitudes (Saito 1969).

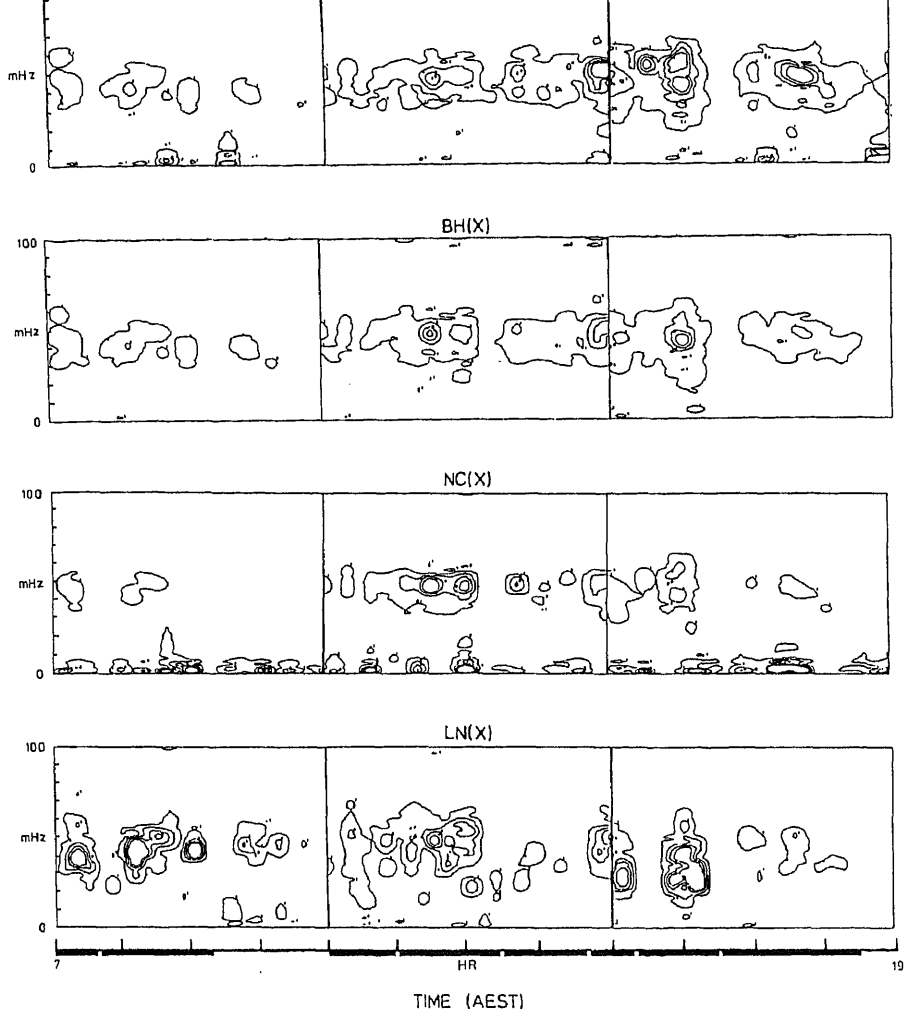


Figure 10. Contour sonagram showing Pc3 activity between 0700–1900 hr AEST on May 30, 1982.

It can be seen from the spectrograms that Pc3 pulsations are daytime phenomena with the activity occurring in the daylight hours from 0600 to 1800 hr AEST. This occurrence behaviour totally agrees with the classical Pc3 results reported in the literature (see for example Saito 1969; Jacobs 1970; Orr 1973 and others). However Pc3 activity at LN often starts earlier in the day. An example of this behaviour can be seen in the spectra of May 19 (figure 8). This behaviour also appears in the statistical results (Ansari 1984). Generally the same type of continuous pulsation activity has been observed throughout the day at all the four stations. On some days activity during the morning hours appears to be more continuous than the afternoon activity which may

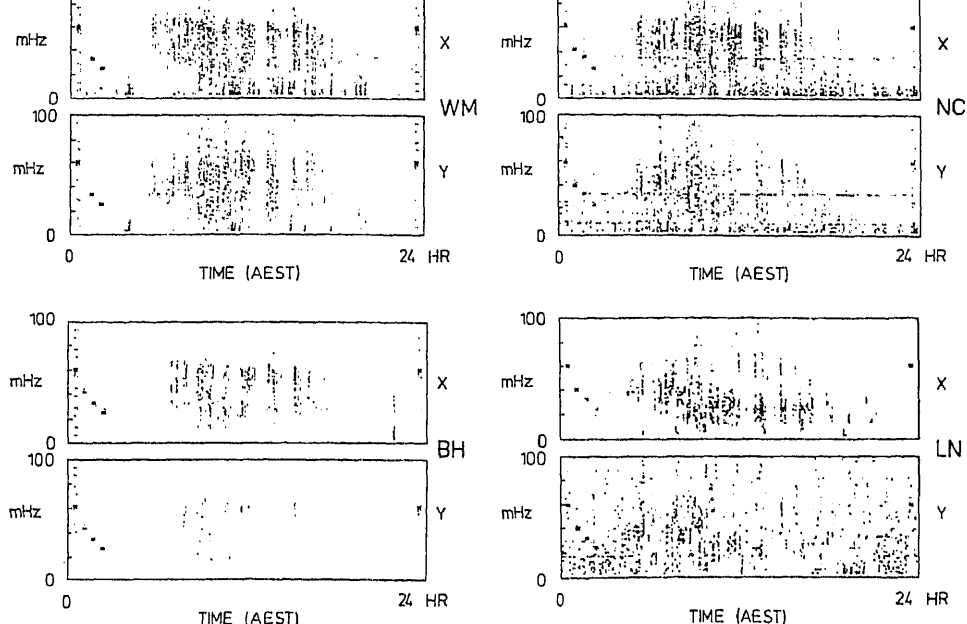


Figure 11. GSD dynamic spectra depicting Pc3 activity on May 31, 1982.

appear in bursts. An example is seen on May 31 in figure 12 at WM and LN. On other occasions activity is reasonably continuous throughout the day (March 25) or is burst like (March 26). The latter has low magnetic activity ($\Sigma Kp = 21$) and such activity is typical for relatively quiet days.

An important feature which has been noticed in the spectra is that Pc3 activity appears and disappears at almost the same time at all the stations and often there are very abrupt signal cessations and commencements. Such behaviour is easily seen in the GSD dynamic spectra of May 30 (figure 9) in the 40–60 mHz band at 0730–0740, 0815–0840, 1010–1030, 1100–1140, 1245–1330, 1420–1515, 1600–1625, 1640–1645 and 1700–1730 hr AEST. It is also of interest to note that the 20 mHz band at LN also follows the same pattern in the afternoon. There appears to be some overriding mechanism which can control the wave generation resonances in all pulsation bands and which is independent of station location. It could be associated with solar wind control of pulsation activity and in particular a change in the θ_{XB} (angle of the interplanetary magnetic field from the sun-earth line) which appears to control some Pc3 occurrence activity. Other examples of this signal dropout at single closely spaced stations have been observed at low latitudes by Medford *et al* (1982).

Finally the general feature emerging from the interstation comparison of the spectrograms is that stations along a latitude (WM, BH and NC) show greater frequency-time similarities than stations (NC and LN) along a longitude which are often quite different.

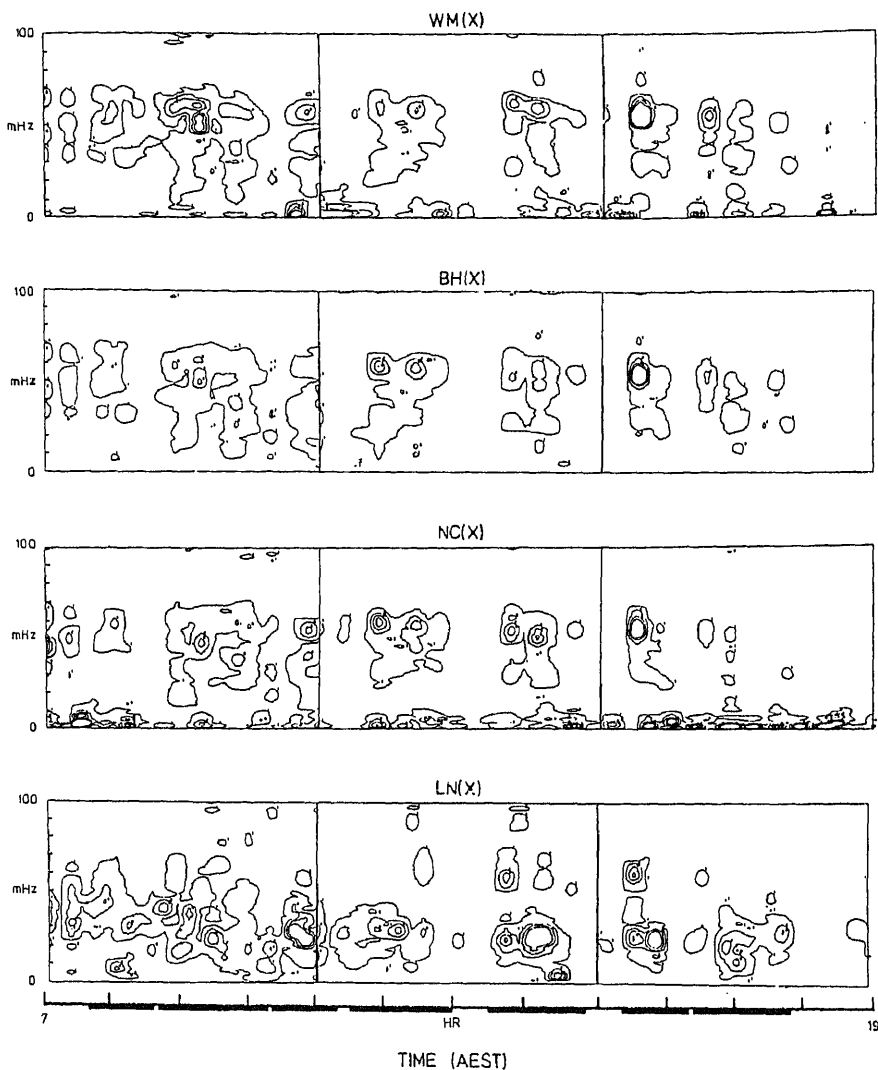


Figure 12. Contour sonagram showing Pc3 activity between 0700–1900 hr AEST on May 31, 1982.

4.2 Frequency structure

In general Pc3-4 spectra consist of one or more frequency bands. The spectra of May 21 and June 1 present excellent examples of multiple frequency bands. On close examination the GSP dynamic spectra and the contour sonagrams (figures 11 to 14) show four frequency bands for each of these two days. These bands can be seen more

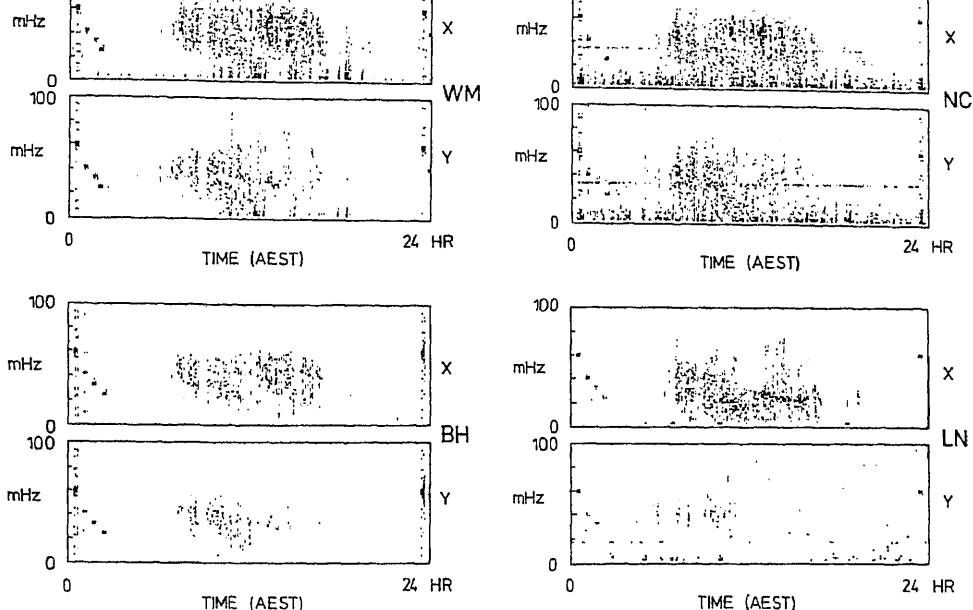


Figure 13. GSD dynamic spectra depicting Pc3 activity on June 1, 1982.

between 0940–1020 hr AEST. There are two large peaks at LN corresponding to the 25 and 44–46 mHz bands but the higher bands are absent at this station. The Y component at the longitudinal stations shows completely different activity over a broader frequency range. Peaks can be identified from the cross-spectra at 15 and 30 mHz. At LN these bands are also present but the greatest power is seen at 44 and 50 mHz in agreement with the X component higher frequency peaks. Minor peaks at these frequencies are seen at the other three stations. On June 1, peaks occur on the X components at 25, 35, 45 and 55 mHz between 1400–1450 hr AEST. Large power is also seen in the lower band at LN but high frequencies are absent. A Pc4 band at 8 mHz appears at LN. On the Y component the lowest band (25 mHz) is seen at the longitudinal stations but not at LN. Earlier band structure results have been reported at ground based stations (Duncan 1961; Tonegawa *et al* 1983) and in space (Takahashi and McPherron 1982).

Another general feature emerging from the study of the pulsation spectra is that Pc3 frequencies seen at longitudinal stations (WM, BH and NC) are generally centred about 40–45 mHz. The majority of data analyzed show this pattern. Examples are found in the spectra of March 25, 27, 28, 30 and May 29 and 30. By comparison Lanzerotti *et al* (1981) observed Pc3 frequencies in the range 40–50 mHz at $L \sim 1.9$. Arthur *et al* (1973) and Arthur and McPherron (1975) have reported average peak frequency of 40 mHz for Pc3 waves observed on the ground and on the ATS-1 satellite at $L = 6.6$. Also significant power in the frequency band of 40 mHz can easily be seen in the D

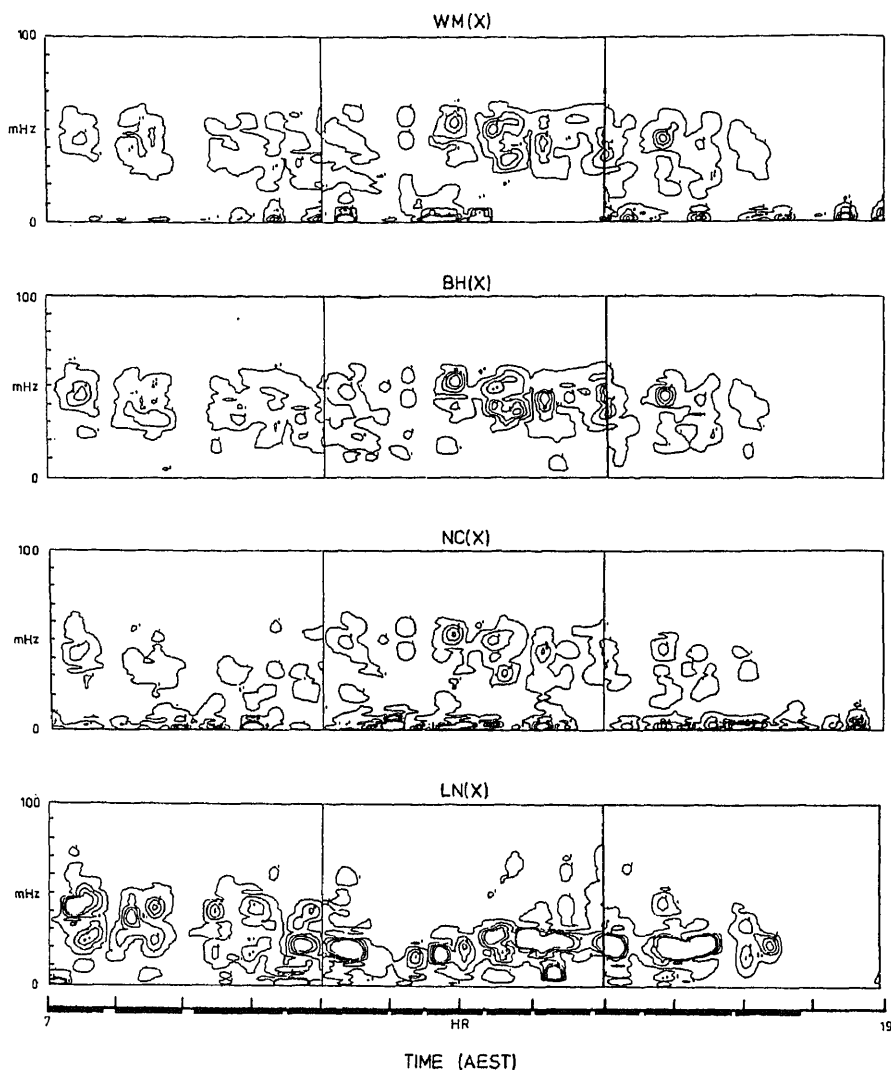


Figure 14. Contour sonagram showing Pc3 activity between 0700–1900 hr AEST on June 1, 1982.

component of the digital dynamic spectra of Pc3-4 data recorded at Sanae ($L \approx 4$) as reported by Barker (1977).

Sometimes Pc3 activity in lower and higher frequency bands has also been observed. The spectra of the data of March 26 (figures 3 and 4) are good examples of a lower frequency band where the frequency is centred about 30 mHz. The spectra reported by Barker (1977) also show significant power at a frequency of about 30 mHz. An example of a higher frequency band in the present study is seen in the WM spectra of May 27 (figure 6) where the frequency is centred at 50 mHz. The dynamic spectra of ULF X

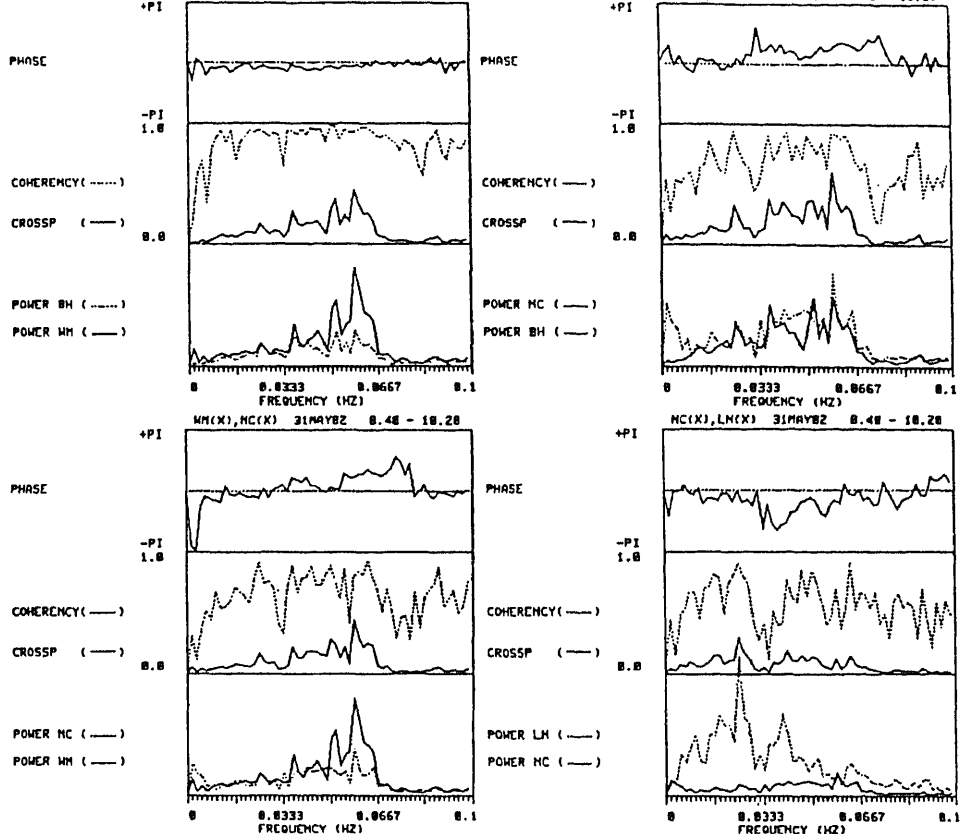


Figure 15a. Cross-spectral analysis for May 31, 1982 between 0840–1020 hr AEST, X component data.

average frequency of 50 mHz at all three Japanese stations. It is interesting to note that the Pc3 pulsation spectral properties at low and higher latitudes are the same.

The spectra of LN usually consist of slightly lower Pc3 frequencies centred about 35–40 mHz and often include a strong Pc4 band. However sometimes higher frequencies can also be seen at LN. An example is May 27 (figure 5) where broad band activity is seen between 40–100 mHz.

A detailed comparison of peak signal frequencies among the azimuthal components shows that the average pulsation frequently is slightly higher at WM, the most westerly station, than at BH and NC. An example can be seen in the data of May 27 (figure 5). This behaviour was observed in four of the ten selected days, namely March 27, 28, 30 and May 27.

The frequency-time structure of the data under investigation has shown that all days and stations show either constant or gradually falling frequencies with time during the day. This is a common feature previously identified in Pc3 studies at all latitudes (Barker 1977).

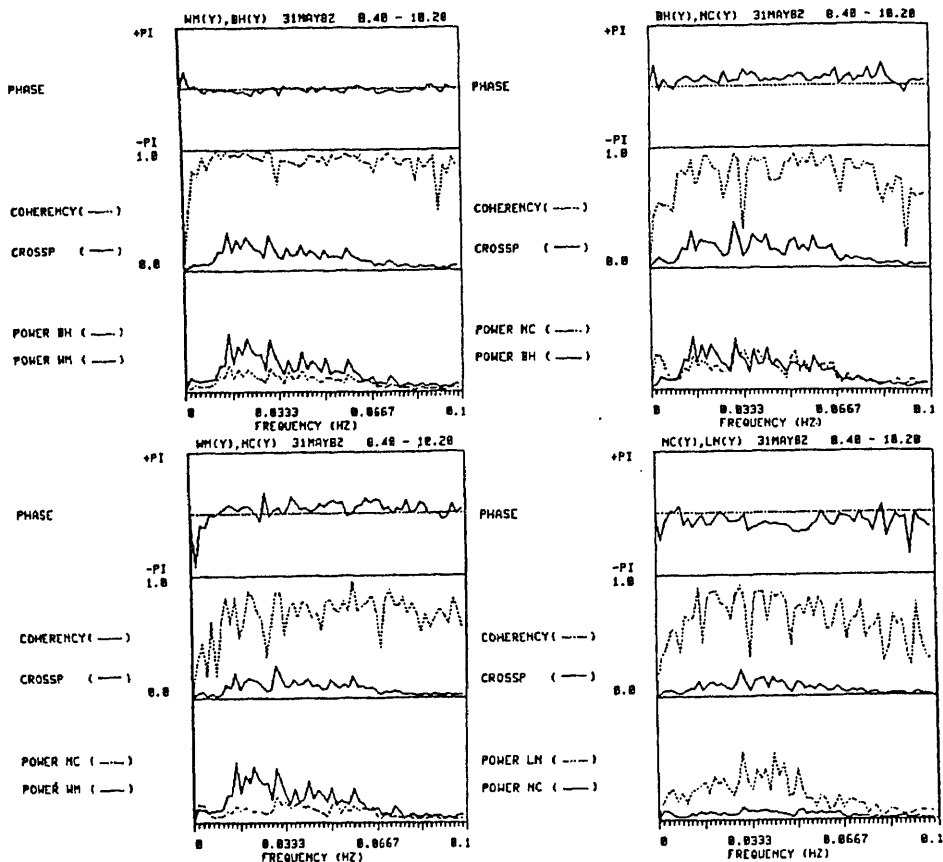


Figure 15b. Cross-spectral analysis for May 31, 1982 between 0840–1020 hr AEST, Y component data.

It can be seen from the cross-spectral plots that there are often different frequencies observed on the X and Y components. An example of this is seen in the cross spectral plot of June 1 between 1400–1450 hr AEST (figure 16). Furthermore the frequency band and the cross power spectrum of the Y signal generally are restricted in the longitudinal pairs of stations and sometimes multiple frequency bands are observed on the X component but not on the Y (for example see figure 16).

4.3 Harmonic structure

Another very interesting feature of the Pc3 spectra observed is that they sometimes show multiple frequency peaks with nearly constant separation between successive peaks. Takahashi and McPherron (1982) have reported this harmonic structure in their study of Pc3-4 from the ATS-6 satellite and have explained the results as the higher order

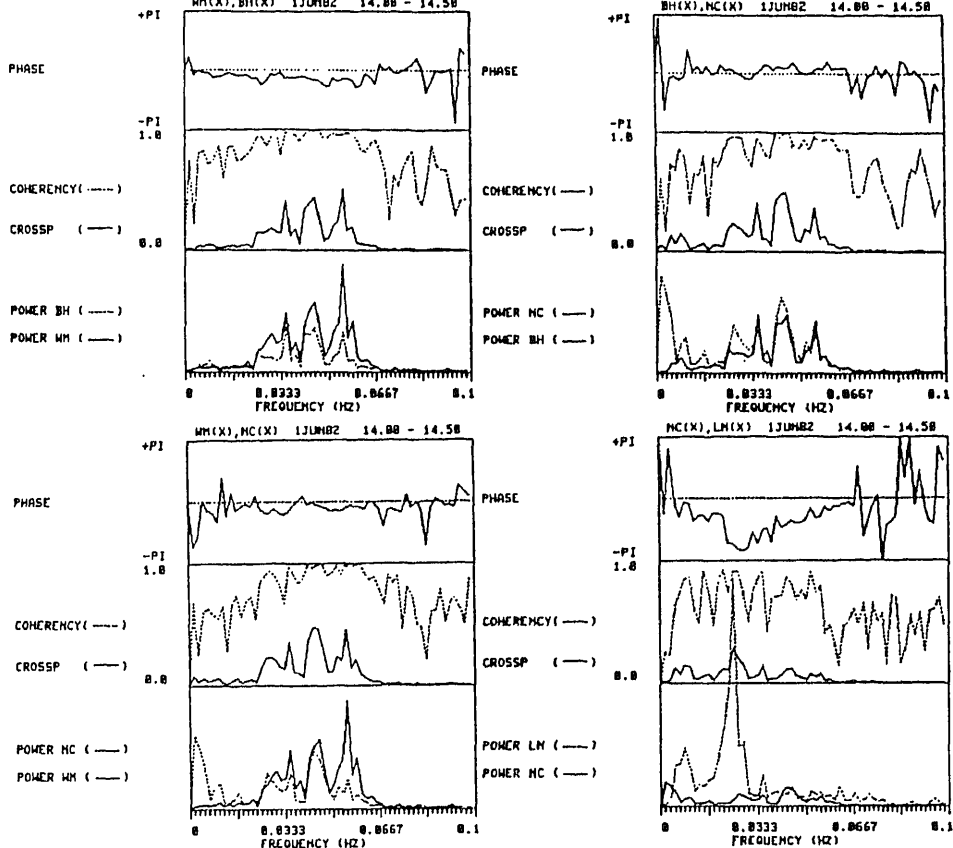


Figure 16a. Cross-spectral analysis for June 1, 1982 between 1400–1450 hr AEST, X component data.

harmonics associated with toroidal field line resonance and eigen frequencies calculated by Cummings *et al* (1969). An example of harmonic structure observed in the present data is seen in the pulsation spectra of June 1 between 1400–1450 hr AEST (figure 16). Harmonic structure is easily seen in the X components of the azimuthal pairs of stations WM, BH and NC where the spectral peaks at 35, 45 and 55 mHz are separated by a constant frequency difference of 10 mHz. From the present results it appears that the same multiple bands are seen at longitudinal stations and on both components but are more apparent on the X component. Frequencies may be different between components which may be expected for resonant waves since the poloidal (radial) and toroidal (azimuthal) modes have different eigen periods (Jacobs 1970). The observation of harmonic structure with $\Delta f = 10$ mHz can be explained by a standing Alfvén wave consisting of many discrete harmonic frequencies (Takahashi and McPherron 1982). They have observed the harmonic events with $\Delta f = 14$ mHz prenoon and 10 mHz afternoon in the azimuthal (east-west) component. The present observations are best

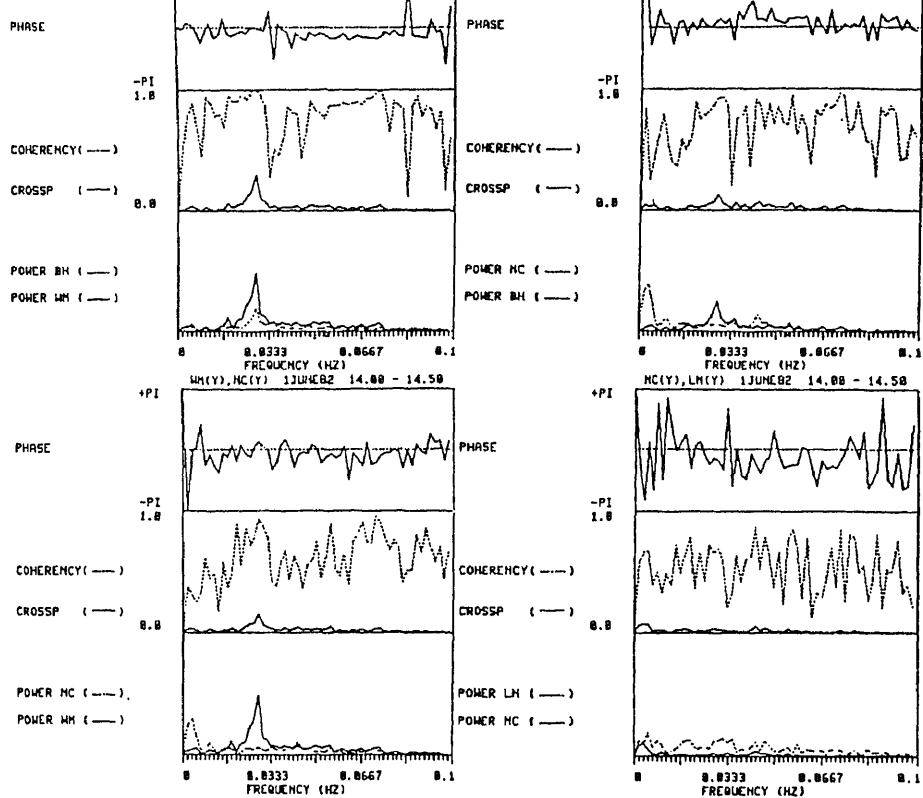


Figure 16b. Cross-spectral analysis for June 1, 1982 between 1400-1450 hr AEST, Y component data.

seen in the north-south components. This may be explained by taking into account 90° rotation of the polarization major axis azimuth by the ionosphere which occurs when signals are transmitted to the ground (Hughes and Southwood 1976). Tonegawa *et al* (1983) have also reported harmonic structure in Pc3 frequency range (20-80 mHz) on the ground at high latitudes.

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Appendix

Grey scale digital (GSD) dynamic power spectral plotting

The power spectral density was calculated using FFT methods. The distribution of power spectral density is scanned to determine the minimum and maximum power for scanning of the upper (black) and lower (white) limits for the grey scale plot. The dc spectral component is set to zero. The maximum and minimum power values are used for linearly scaling the power density to a level from which a half-tone grey scale picture can be plotted.

Unlike the usual printer where lights and lenses are used to perform the half-toning process in an analog fashion, the Grey Scale software accepts the scaled pixels and processes them digitally using a 32 nib (dot) half-toning screen to produce a raster line with 33 levels of grey shading (0–32 nibs on/off). Horizontal and vertical replication is used to obtain the grey scale plot of the required size. The density of nibs on the printer is currently 132/inch horizontal by 72/inch vertical. A replication ratio of 2:1 is used to overcome the difference in horizontal and vertical nib densities.

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Sub-seasonal scale fluctuations of the ITCZ over the Indo-Pacific region during the summer monsoon season. Part I: Features over the Indian region

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Abstract. Monex-79 and ISMEX-73 data have been analysed to study the sub-seasonal scale fluctuation of near equatorial oceanic intertropical convergence zone (ITCZ) over the North Indian ocean during the summer monsoon of 1979 and 1973. The oceanic ITCZ is characterised by a narrow shear zone between the equatorial westerlies and the tropical easterlies, associated with organised convective clouds. Synoptic analysis presented in this paper shows the steady northward propagation of the oceanic ITCZ from its near equatorial position ($5-10^{\circ}\text{N}$) to the continental position ($20-25^{\circ}\text{N}$) during the onset and mid-season revivals of monsoon after breaks. The northward propagation is initiated by the strengthening of the equatorial westerlies which result in the intensification of the shear zone and the embedded disturbances. The establishment of the northward propagating mode near normal monsoon trough position over the continent characterises the active phase of monsoon. As the monsoon cycles from active to weak/break phase, the monsoon trough (continental ITCZ) dissipates near the foothills of the Himalayas and the oceanic ITCZ gets emphasised once again near the equatorial region. The major phase changes in the ITCZ occur at an interval of about 30–50 days which dominantly control the intra-seasonal fluctuation of the Indian summer monsoon. The paper also discusses the characteristic features of the oceanic ITCZ during different phases of the monsoon.

Keywords. Monex-79; ISMEX-73; intertropical convergence zone; maximum cloud zone; sub-seasonal scale fluctuations; preonset; onset and mid-season revival of monsoon.

1. Introduction

During the northern hemisphere spring season (May), a quasi-stationary zone of convergence (ITCZ) between the equatorial westerlies and the tropical easterlies, with maximum cyclonic vorticity, exists in the lower troposphere over the equatorial Northern Indian ocean. In the satellite cloud photographs an east-west oriented belt of a maximum cloud zone (MCZ) also exists near the equatorial region which almost coincides with the quasi-stationary position of the ITCZ during the spring season. To the north of the MCZ, over the plains of India (where the summer heat-trough is in the process of getting established) the skies are mostly clear. Questions have been raised in the GARP document (WMO 1976) as to (i) what happens to this spring season near equatorial ITCZ with the onset and advance of summer monsoon over India and (ii) whether it loses its identity in the equatorial position during the established phase of the monsoon under the influence of the continental monsoon trough?

One of our objectives in this paper is to provide an answer to the afore-mentioned questions through the examination of synoptic weather and satellite picture data over the Indo-Pacific region. Simultaneously we also propose to look into the activity and

the characteristics of the near-equatorial ITCZ over the region of study. Our aim is also to provide some understanding of the linkage between epochs of intensification/weakening of the ITCZ over the two oceanic regions in the evolution of the monsoon system especially the low frequency intra-seasonal ones. The study is primarily based mainly on the Summer Monex data for the season of 1979, which had the best data coverage so far achieved for the regions. It also examines the data of the season of 1973, year of another monsoon experiment, viz., Indo-Soviet Monsoon Experiment (ISMEX-73) in order to substantiate some of the results obtained from the study of the Summer Monex data.

Part I of the paper discusses the above mentioned questions for the Indian region (equator to 30°N and 60° – 100°E). Part II of the paper besides studying the evolution and northward migration of the ITCZ over the Northern West Pacific region (equator to 30°N and 110° – 140°E) also discusses the interactions between the two regions on synoptic and sub-seasonal scales.

Section 2 deals with the methodology adopted for the data analysis. Section 3 discusses the sequence of major synoptic events, which occurred during different phases of the monsoon over India for the two seasons under study and §4 explores answers to the aforementioned questions. The characteristics of the oceanic ITCZ and its vertical structure at the stages of pre-onset, onset and mid-season revival of the monsoon are discussed in §5. The results are summarised in §6.

2. Data and methodology adopted

Daily aerological data of the www stations and the data obtained from special Summer Monex research ships during Monex-79 and ISMEX-73 are used in this study. Most of the research flights during the Monex-79 were undertaken between 05 and 10 z . The research ships and the land stations data used are corresponding to 06 and 12 z respectively. Satellite mosaics by us weather satellites are utilised to delineate the daily latitudinal position of the MCZ. As we are concerned with the description of large scale characteristics of the ITCZ, we believe that (3 to 6 hr) non-uniformity in the time of data coverage may not seriously hamper our results. The data have been analysed basically in three ways:

- (i) analyses of meridional sections/time sections, of the observed data.
- (ii) analyses of the wind, temperature and moisture fields at 850, 700 and 500 mb and other relevant parameters obtained at 2° latitude-longitude grid intervals.
- (iii) Daily latitudinal position of the MCZ and the trough at 700 mb (representative of location of ITCZ) at 70° and 90°E corresponding to the North Indian Ocean, and at 120° and 140°E corresponding to the monsoonal belt of the West Pacific Ocean.

The study mostly refers to 700 mb, as this level is considered to best reflect the low and middle level features of the ITCZ and monsoon activity. The grid points covered by the longitudes 66 to 80°E , 80 – 96°E and 110 – 140°E between equator and 20°N are referred to in the paper as pertaining to the Arabian sea, Bay of Bengal and the West Pacific Ocean respectively.

3. Discussion of major synoptic epochs over the Indian region

The summer monsoon season evolves through different phases, viz., onset and northward advance of monsoon, established, and withdrawal of the monsoon. The pre-onset phase begins from early May. The advance of the monsoon is reckoned with the first northward migration of the equatorial westerlies. The established monsoon phase, which is usually the July–August period, is characterised by the active-break cycle during which the rainfall fluctuates, from ample over most of India (active) to deficient over the central parts of the country (break). Activity of the monsoon on a day to day basis is modulated by the synoptic scale transient disturbances. In the following sub-sections we discuss some of these aspects briefly for the two seasons under study.

3.1 Season of 1979

3.1a Pre-onset phase: In the beginning of May 1979, ITCZ was quite active in the equatorial Bay of Bengal and a cyclonic storm formed on 7 May near 7°N and 88.5°E which moved northwest and struck the coast on 12 May. A temporary surge in the lower tropospheric equatorial westerlies occurred under the influence of this development during 7 May to 17 May. The ITCZ activity reverted to the equatorial region after the storm had weakened. Successive passage of active western disturbances across Northern India and the associated equatorwards penetration of the sub-tropical westerlies (Sikka 1981) during the second half of May inhibited the formation and intensification of the seasonal trough along the Gangetic plains. Figures 1a and b depict the average anomaly of the daily sea level pressure over India for the period 26 May to 5 June and the pressure distribution for the period. Figure 1c shows the 700 mb wind flow on a typical day (31 May) during the period. Prominent features in the second half of May, 1979, were the highly above-normal pressure (≥ 6 mb) over the region of seasonal low pressure and over Northwestern and Central India and the existence of a marked high pressure cell at sea level over the Arabian sea. The shear zone at 700 mb level between equatorial westerlies and subtropical easterlies over Arabian sea was along $5-7^{\circ}\text{N}$. This was also associated with a zonally oriented band of maximum cloud (MCZ) along about 4°N (figure 1d). Cloud clusters occasionally formed in the MCZ and moved in a general westerly direction. This MCZ could be identified with the position of the ITCZ during the pre-onset phase. During this period the Gangetic plain and Central India were mostly cloud free except during the passage of a western disturbance.

3.1b Onset and advance phases: (i) *Synoptic aspects*—Figures 2a and b show the average anomaly of daily sea level pressures over India and the average sea level pressure distribution for the period 6 to 8 June, and figure 2c depicts the wind flow at 700 mb for 7 June. From the end of May to the beginning of the 2nd week of June lowering of pressure by about 6 to 8 mb over Central and Northern India and increase in the pressure gradient between the equatorial region and Central India are noticeable from the data presented in figures 1a and b and figures 2a and b. Similarly the shear zone over the SE Arabian sea (figure 2c) and the associated MCZ (figure 2d) are also well marked than the same features shown in figures 1c and d. Figure 3 shows the meridional time section at 700 mb for wind, temperature and relative humidity during

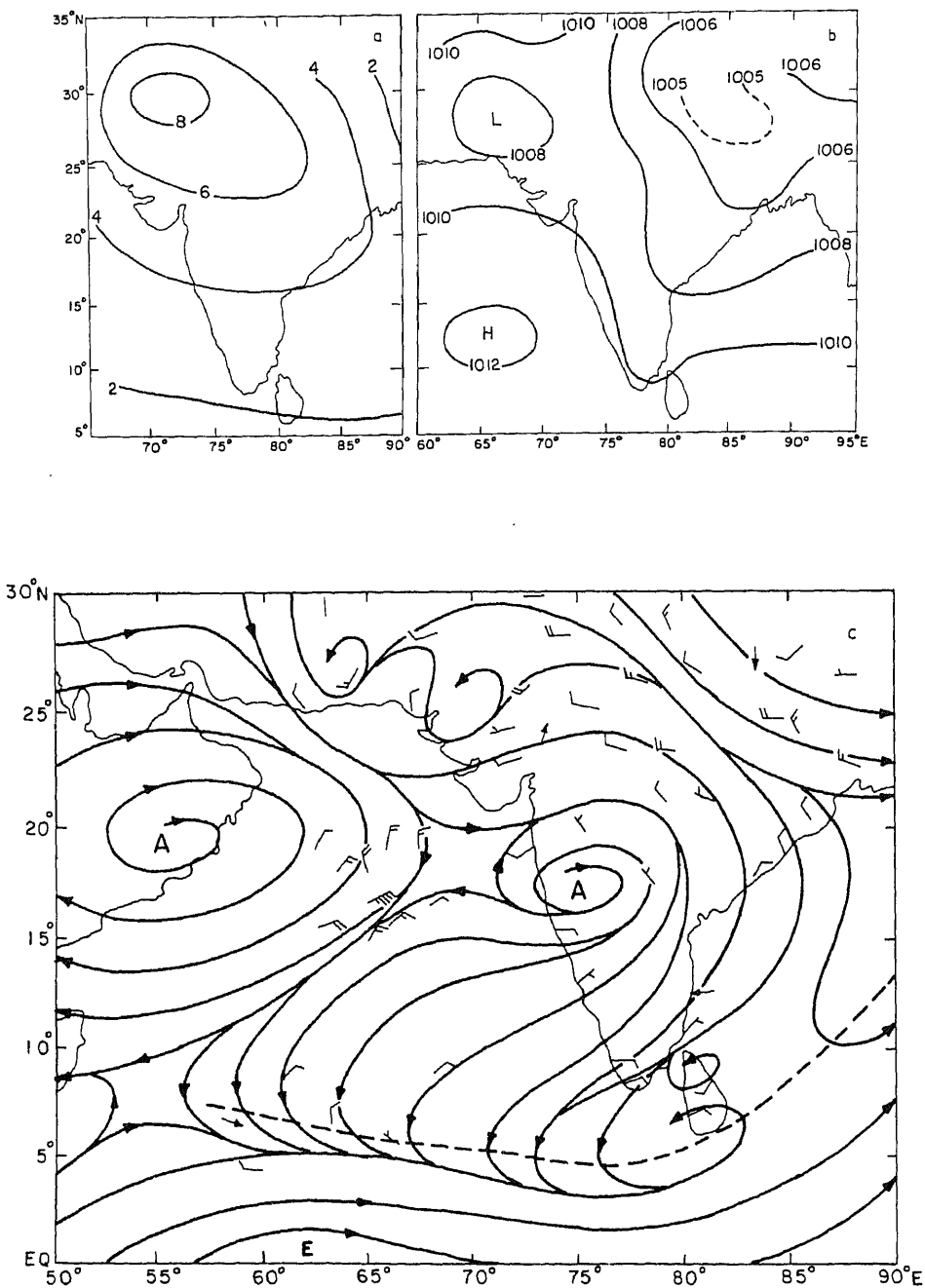


Figure 1. a-c. For captions, see p. 51

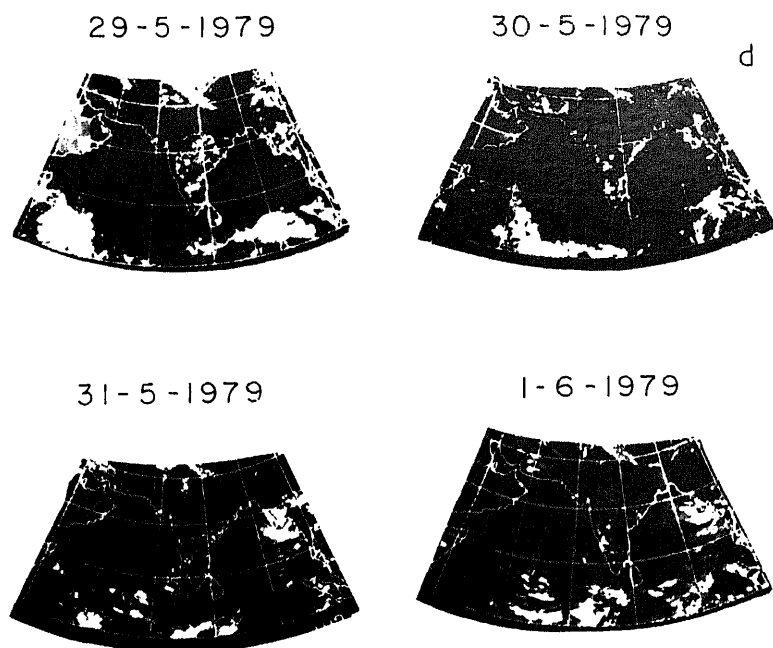


Figure 1a. Daily sea level pressure anomaly averaged for the period 26 May to 5 June 1979. **b.** Sea level pressure distribution averaged for the same period. **c.** Wind flow at 700 mb for 31 May 1979, 00 GMT. **d.** Satellite cloud imageries (visible) for the period 29 May to 1 June, 1979.

4 to 20 June. It is seen that prior to the onset of the monsoon, the shear zone fluctuated along about 6°N upto 9 June. The figure also shows that the equatorial westerlies were more moist and were somewhat cooler than the air of continental origin to the north. The wind-shifts over the location of the Russian ships between 4–5 June and 8–9 June suggested the passage of cyclonic disturbances within the ITCZ (Bhide *et al* 1982). The latter circulation intensified on 14 June while it moved northward. About the time of its formation and its intensification, cross-equatorial wind surges occurred over the West Arabian sea on 8 June and 11 June and the equatorial westerlies abruptly strengthened over the Arabian sea during 12–14 June. Formation of the monsoon onset vortex has been discussed in detail by Krishnamurty *et al* (1981). The meridional section (figure 3) shows the northward progression of the shear zone and the belt of stronger westerlies from its pre-onset phase position over the Southern Arabian sea on 9 June to about 20°N on 20 June.

Figure 4 which is a longitudinal time section along $20\text{--}25^{\circ}\text{N}$ for the pressure anomaly for the period 15 June to 7 July shows markedly above-normal pressure over the eastern part of the section in the middle of June followed by a period of negative pressure anomaly during 24 June to 3 July. It was in the latter period that the monsoon advanced over Central and Northeast India due to the passage of over-lapping monsoon disturbances. The rainfall distribution on 12 July when the monsoon has covered the entire country, is shown in figure 5.

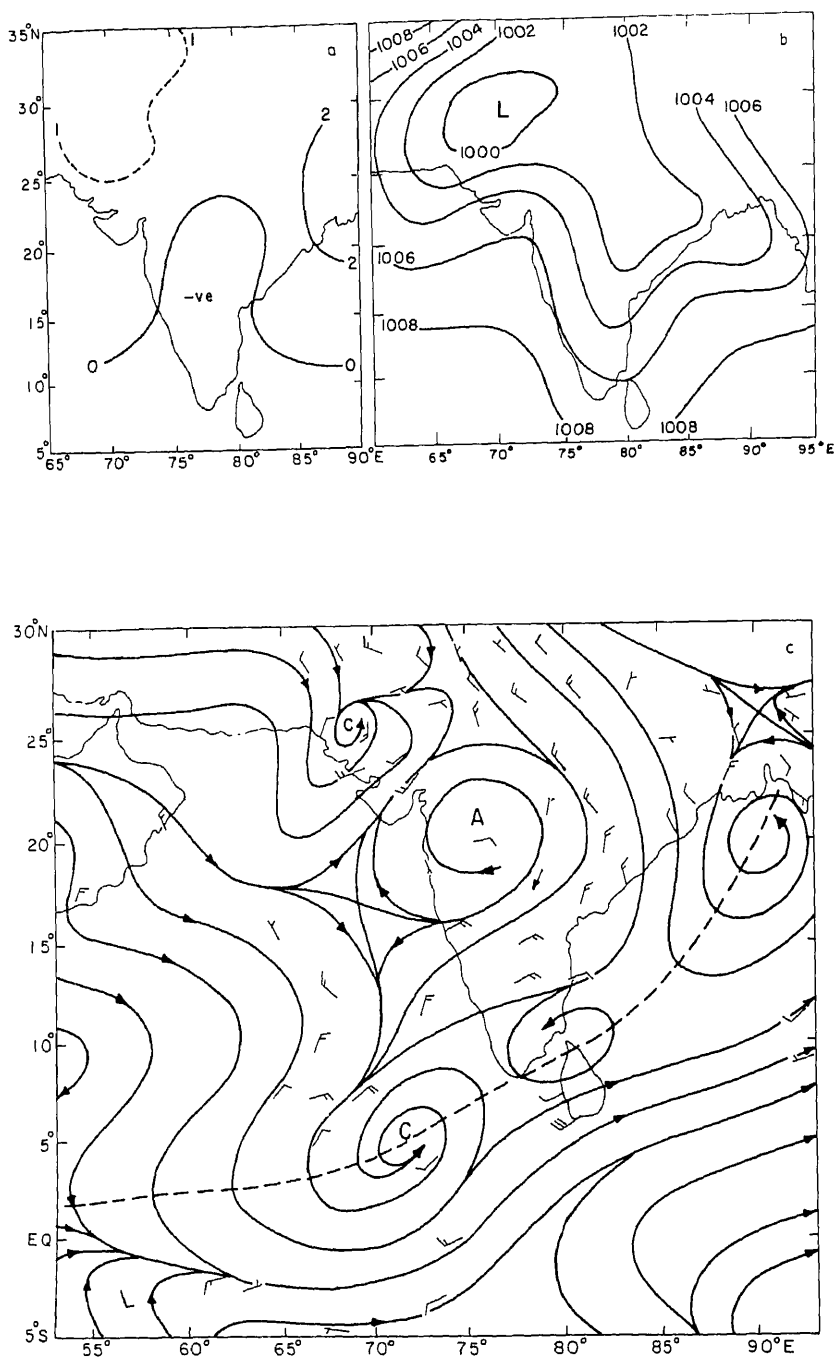


Figure 2. a-c. For captions see p. 53

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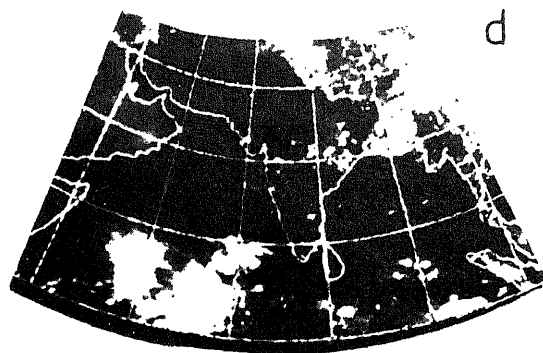
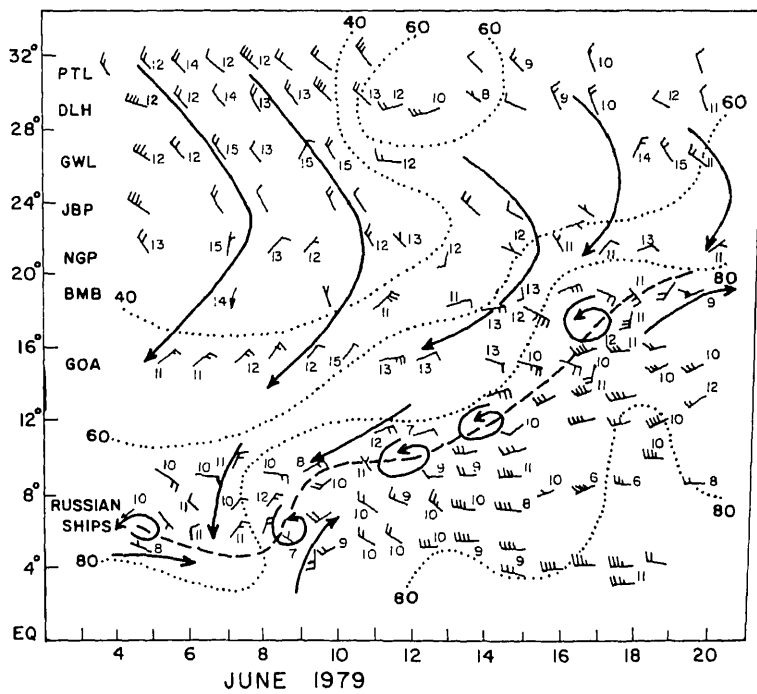


Figure 2a. Daily sea level pressure anomaly averaged for the period 6 to 8 June 1979. **b.** Sea level pressure distribution averaged for the same period. **c.** Wind flow at 700 mb for 7 June 1979, 00 GMT. **d.** Satellite cloud imagery (visible) for 8 June 1979.



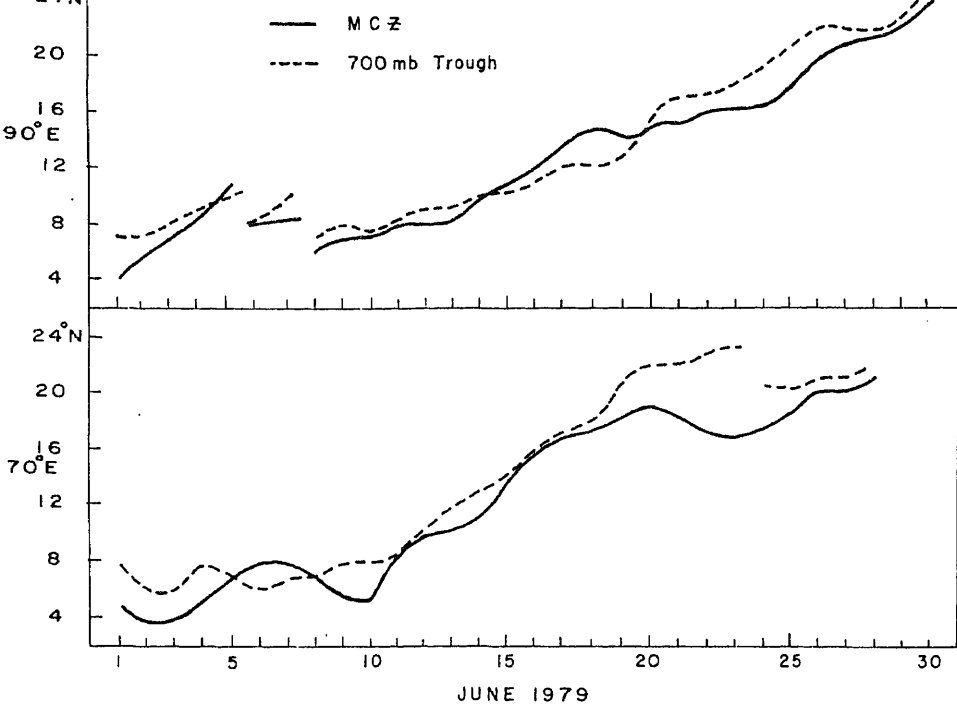


Figure 6. Daily latitudinal position of axis of maximum cloud zone (MCZ) and 700 mb trough line along 70°E and 90°E for June 1979.

during 1 to 5 May and shows a band of well organised convective clouds within the MCZ extending from the Southwest Bay off the Southeast Sri Lanka coast (8°N) to the east central Bay and the adjoining North Andaman sea off the Burma coast. A west/north-westwards movement of this cloud band is also seen in the figure. The MCZ and the 700 mb trough fluctuated between 8 and 12°N and 8 and 15°N respectively during 1 to 10 May along the 90°E longitude (figure 10). Synoptic scale disturbances formed within the ITCZ during the period, moved west/north-westwards across the Bay of Bengal causing a northward movement of the MCZ and the 700 mb trough. Thus the first organised northward movement of the MCZ in the Bay of Bengal sector can be considered to take place during the first fortnight of May. During the second fortnight of May, pressure over most of India was above normal till 27 May and below normal during the last few days of the month.

3.2b Onset and advance phase: (i) *Synoptic aspects*—Figure 11 shows the meridional-time section at 700 mb of wind, temperature and relative humidity during 21 May to 11 June in the Arabian sea sector (along 55–75°E). The figure shows that the shear-zone associated with the ITCZ fluctuated between 7 and 9°N during 22 to 29 May with cyclonic systems embedded within it during 22–23, 24–25 and 28–29 May. The ITCZ weakened on 29 May and it lay close to the equator till the beginning of June. Thus, the first spurt of ITCZ activity in the Southeast Arabian sea, which began on 22 May was rather a short-lived one (about a week).

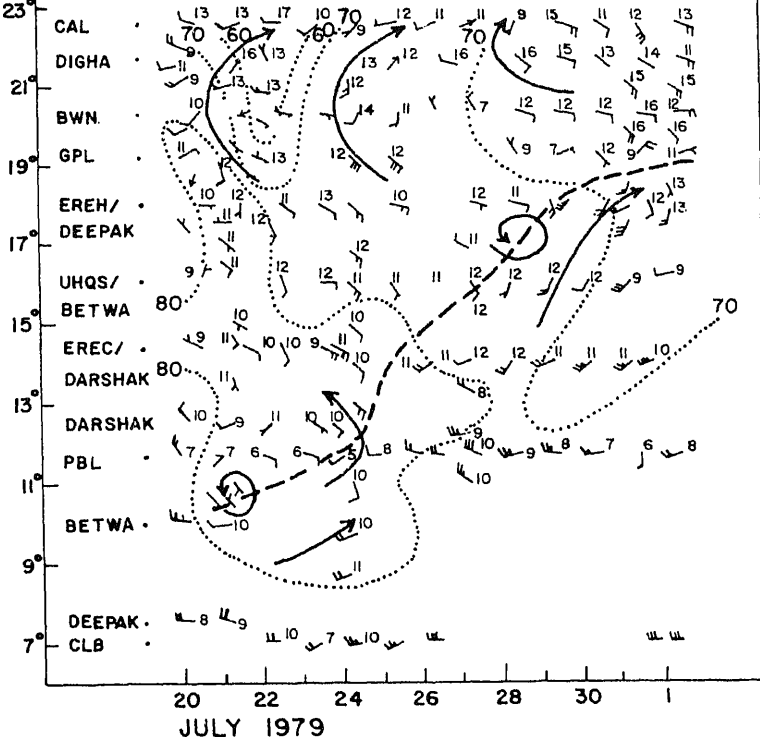


Figure 7. Meridional-time section at 700 mb of wind, temperature and relative humidity along the belt 85–90°E during 20 July to 1 August 1979.

Sea-level pressure over Northwest India and adjacent Pakistan was above normal during 30 May to 3 June and became below normal on 4–5 June. The equatorial convergence zone became active once again on 4 June. A cyclonic circulation formed within it on 4–5 June and the associated shear-zone was located near 10°N. Equatorial westerlies abruptly intensified to 30–35 m/sec on 6 June. As the system began to move northwards, strong (cold and moist) cross equatorial flow rushed towards the Central and East Arabian sea and led to the intensification of the cyclonic system to a depression on 6 June and cyclonic storm on 8 June. The ITCZ progressively moved northward along with the storm bringing the equatorial westerlies to about 20°N by 9 June.

The above description brings out that the onset process along the west coast in 1973 occurred twice—the first one—a temporary one during 23–27 May and the second—the regular one between 4–9 June.

(ii) *Northward progression of MCZ*—Figure 12 shows that at the time of temporary onset of monsoon over Kerala (23–27 May), the 700 mb trough and axis of MCZ fluctuated between 6 and 10°N, over the Arabian sea (70°E) and the Bay of Bengal (90°E). With the regular onset and advance of monsoon, the 700 mb trough and the

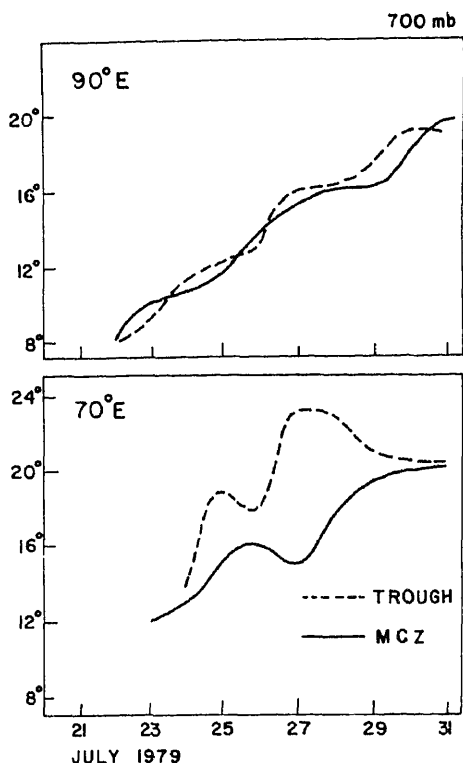


Figure 8. Daily latitudinal position of axis of MCZ and 700 mb trough line along 70°E and 90°E for 21–31 July 1979.

axis of MCZ marched progressively northward from 9–10°N on 4 June to 20°N on 9 June along both the longitudes.

There was a lull in the further advance of monsoon after 13 June. The seasonal trough moved rapidly northwards, near the foothills of the Himalayas and a lull prevailed in the monsoon-activity during the second half of June. The ITCZ remained weak as no perturbation formed within it and both the Arabian sea and the Bay of Bengal were devoid of any organised convective cloudiness. A series of western disturbances moved across Northwest India during this period.

Figure 13 shows the daily position of the 700 mb trough and the axis of MCZ along 70°E and 90°E (top part) and a meridional time section (wind, temperature and relative humidity) at 700 mb along 55–75°E (Arabian sea) and along 80–93°E (Bay of Bengal for the period 28 June to 10 July). A cyclonic circulation which formed on 28–29 June reactivated the ITCZ near 18°N. This system as well as a trough off the west coast of India during 27 June to 7 July, revived the monsoon activity and led to its further northward advance. The monsoon covered the entire country by 6 July, under the influence of two overlapping depressions which affected the country during 6 to 9 July.

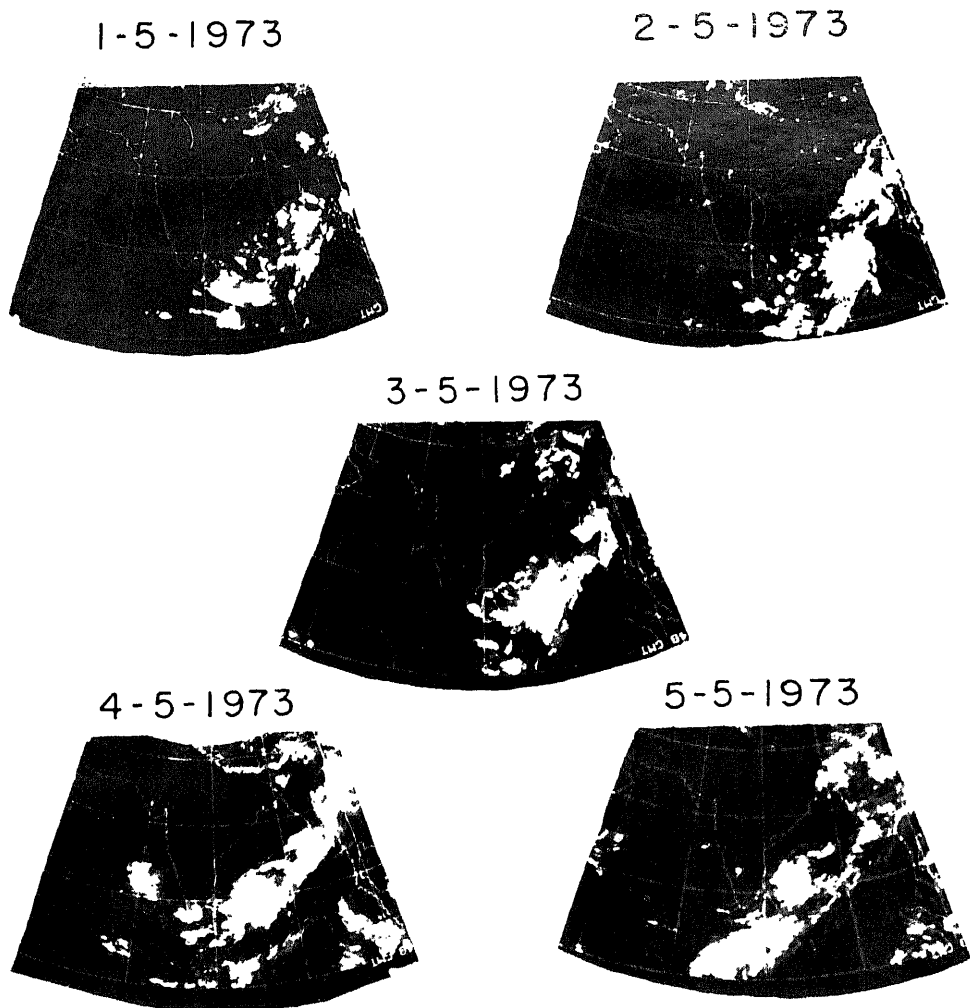


Figure 9. Satellite cloud imageries (visible) for the period 1 to 5 May 1973.

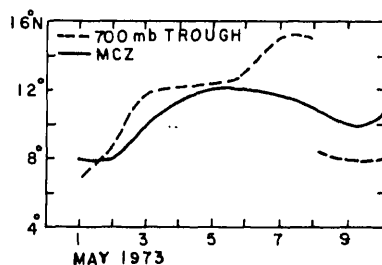


Figure 10. Daily latitudinal position of axis of MCZ and 700 mb trough line along 90°E for 1-10 May 1973.

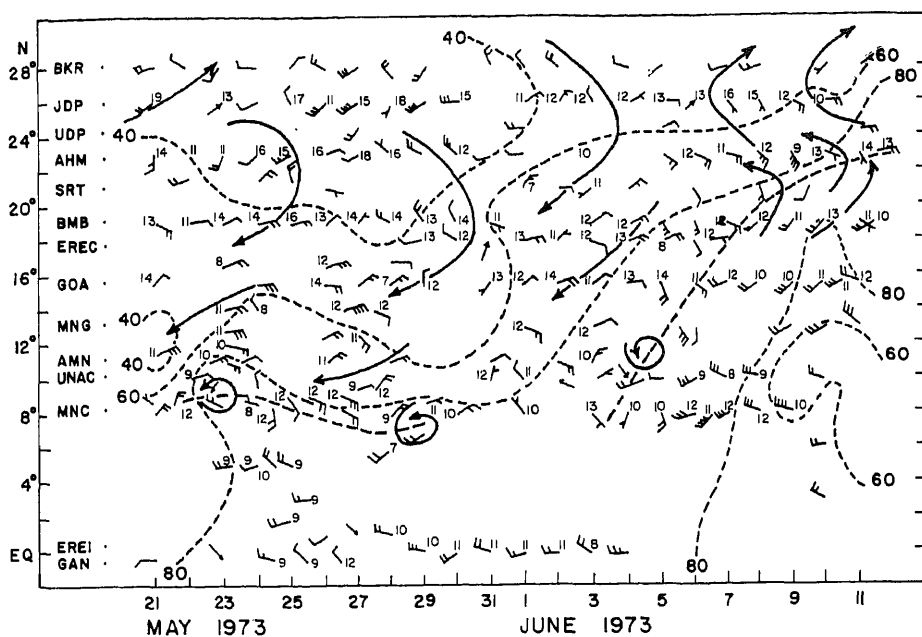


Figure 11. Meridional-time section at 700 mb of wind temperature and relative humidity along the belt 55–75°E during 21 May to 11 June 1973.

and the MCZ along 70°E and 90°E from 14–16°N on 28 June to 20–22°N on 8–10 July. Thereafter, during 12 to 22 July movement of two strong systems, a deep depression and a severe cyclonic storm, from the Bay of Bengal to the central parts of India kept the monsoon active over most of the country.

3.2c Revival of the monsoon after a 'break' in the established phase: Typical 'break' monsoon patterns of pressure and winds prevailed over major parts of the country on 24 July and lasted till 1 August. The oceanic ITCZ appeared near 8–10°N on 2 August. This is seen in figure 14. The figure also shows that the ITCZ intensified and moved progressively northwards, reached 20°N on 7 August and was even further north of 25°N on 9 August. This was rather a very rapid northward progression which is also reflected in the northward movement of the axis of MCZ. The oceanic ITCZ again became active near 10°N on 8 August and started its northward progression. This is also noticed from the northward movement of the MCZ from 7°N on 7 August to 15°N on 13 August.

The sequences of events described above clearly show that during the period 2–13 August, two successive meridional progressions of the oceanic ITCZ took place from the near-equatorial region. These events led to the revival or re-establishment of the monsoon activity over most parts of the country. In several respects the revival or re-establishment phase of the monsoon after a major break was similar to the multiple sequences of events which happened at the time of onset.

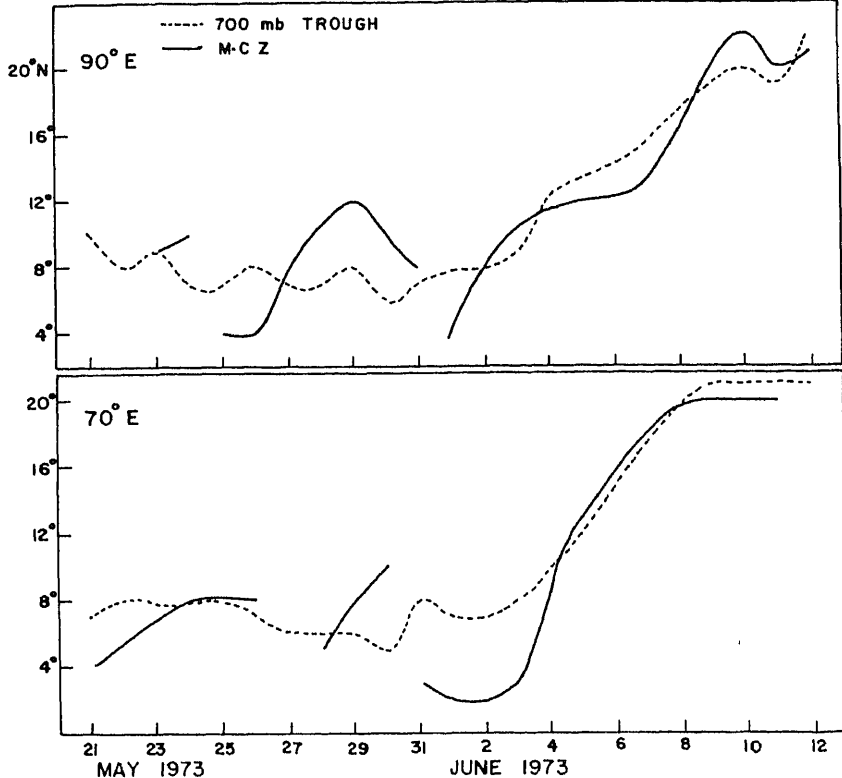


Figure 12. Daily latitudinal position of axis of MCZ and 700 mb trough line along 70 and 90°E for 21 May to 11 June 1973.

the Bay of Bengal and over the land-locked monsoon trough which moved in a westerly or northwesterly direction. These systems maintained active monsoon conditions over the country for about a month.

4. Discussion of results for the Indian monsoon region

Based on the above sequential description of synoptic events during the two typical monsoon seasons, the onset, advance and active-break-active cycles of the Indian monsoon may be understood in the following way. The location of the near equatorial oceanic ITCZ during the spring season is such that the heated continental region lies to its north. However, in the early spring season the intensity of the heat source is not yet sufficiently developed to significantly influence the ITCZ and hence it is located in a quasi-stationary position over the warm equatorial waters. As the heating of the continental region and the overlying airmass progresses the seasonal, regional and planetary scale pressure systems of the summer monsoon begin to intensify. A stage is reached when the ITCZ is observed to respond to the intense heat source. This process is aided by the latent heat released within the organised convection associated with the oceanic ITCZ and over the adjoining maritime continental region of Indonesia as well as

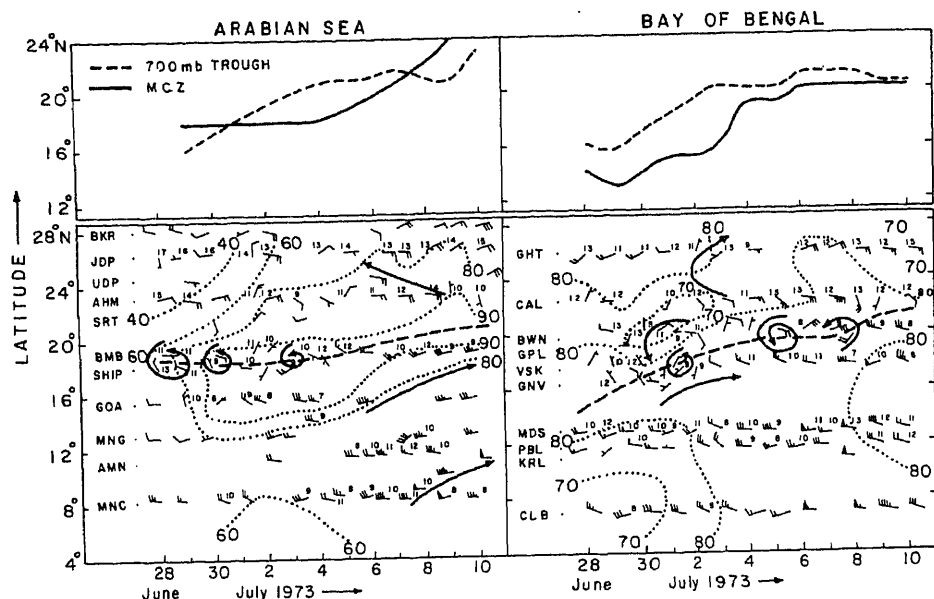


Figure 13. Meridional-time section at 700 mb of wind, temperature and relative humidity along the belt 55–75°E (Arabian sea) and 80–95°E (Bay of Bengal) and daily latitudinal position of axis of MCZ and 700 mb trough (top part) along the longitude 70°E (Arabian sea) and 90°E (Bay of Bengal) for the period 28 June to 10 July 1973.

the Tibetan Plateau (Northeast India). Krishnamurty and Ramanathan (1982) have quantitatively discussed the role of different heating processes during the onset phase of the monsoon.

The synoptic events also suggest that under the large scale response of the continental thermal low and dynamical influence of transient perturbations forming within the ITCZ, strong cross-equatorial monsoonal flow is established over the northern equatorial Indian ocean at the time of the monsoon onset. Further intensification of the transient disturbances of the ITCZ follows and the entire system progressively moves northwards towards the seasonal heat trough which is by then well established over North India.

Further progress in the advance of the monsoon over other parts of India can be viewed as the replacement of the continental heat trough by the continental monsoon trough (continental ITCZ). This is so because before the onset of monsoon upto 20°N, dry convective processes are dominant within the continental heat trough with the northward advance of the monsoon regional continental heat trough undergoes a transition to monsoon trough resulting in organised moist convection over the continental region of India.

As the monsoon trough moves further northward close to the foot of the Himalayas, 'break' or 'weak' monsoon conditions set in over the central parts of India with above normal pressure prevailing over the same region. During the transition from active to

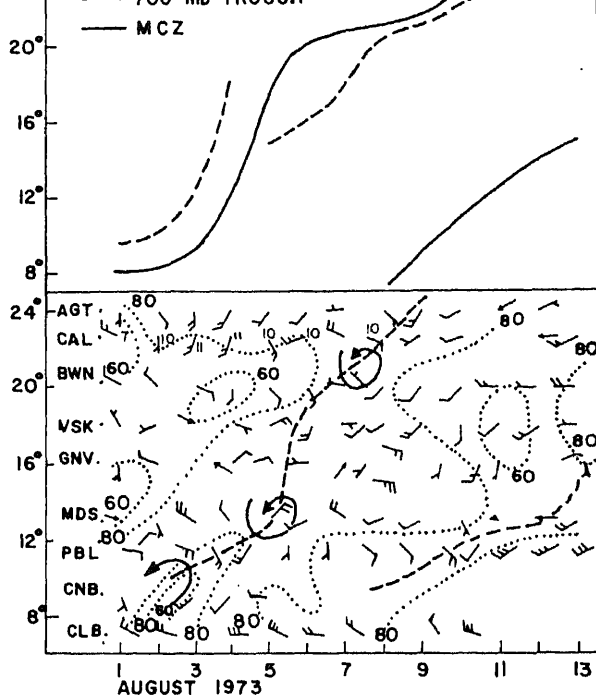


Figure 14. Meridional-time section at 700 mb of wind, temperature and relative humidity along the belt 80–95°E and daily latitudinal position of axis of MCZ and 700 mb trough (top part) for the period 1–13 August 1973.

establish the whole system once again from the equatorial region to its near normal position ($\sim 20^\circ\text{N}$) within a period of 7–10 days.

The existence and westward movement of the near equatorial cyclonic disturbances at 700 mb during the 'break' period was first identified by Koteswaram (1950). Mukherjee and Natarajan (1968) also documented such sequences. Table 1 summarises the sequence of events which occurred in the seasons of 1979 and 1973. It shows the existence of 30–40 day periodicity between the epochs of intensification of the oceanic ITCZ. The active phase of the monsoon commences when the ITCZ is established over 10°N after its northward propagation from the near equatorial region. The active phase is maintained by overlapping formations of disturbances in the monsoon trough one which may last for about two weeks, with some minor oscillation in the position of the monsoon trough and also the waxing and waning of its intensity. During this enveloping period of disturbances, good rainfall activity is maintained over India. After this the monsoon trough shifts to the foothills in a matter of 3–4 days and the oceanic ITCZ begins to emphasise itself once again. If this happens in the middle of the monsoon season, the continental heat source once again becomes prominent and the oceanic ITCZ responds to it. When this happens in the middle of September, the autumn transition in the northern latitude is underway and hence this leads to the withdrawal of the monsoon from Northwestern India. As such the next northward excursion of the equatorial ITCZ is then limited to the latitudes of about 15°N .

Table 1. Dates and periodicity of epochs of strengthening of oceanic ITCZ and its northward progression (with speed of movement) from the near equatorial position to its near normal position ($\approx 20^\circ\text{N}$) and the commencement of 'break' in monsoon.

Epochs	Dates (with latitudinal position) on which		Time of reaching 20°N position and latitudinal distance traversed	Speed of northward movement of ITCZ (latitudes per day)	Break monsoon commenced
	ITCZ became active near the equatorial region	ITCZ reached its near normal position ($\sim 20^\circ\text{N}$)			
I	11 June (8°N)	19 June (20°N)	9 days (12° latitude)	1.3°	20 July
1979 II	24 July (12°N)	31 July (19°N)	7 days (7° latitude)	1.0°	19 August
Periodicity (days)	44	43	—	—	31
I	23 May (8°N)	8 June (20°N)	16 days (12° latitude)	0.7°	14 June
1973 II	29 June (14°N)	8 July (20°N)	9 days (6° latitude)	0.7°	22 July
Periodicity (days)	38	31	—	—	39
III	2 August (11°N)	8 August (21°N)	5 days (10° latitude)	2.0°	—
Periodicity (days)	35	32	—	—	—

Sikka and Gadgil (1980) have demonstrated the meridional propagation of cloudiness from near equatorial region to the monsoon trough region on the scale of about 30–40 day during the summer monsoon season. Yasunari (1979, 1980, 1981) has also emphasised the 30–40 day mode in the monsoon cloudiness. Krishnamurty and Subramaniam (1982) using filtered zonal wind data at 850 mb have further revealed same quasi-periodicity in the zonal winds dominated by meridional propagation of trough ridge sequences from the equatorial region to the foothills of the Himalayas. Murakami *et al* (1983) also used filtered data to emphasise the low frequency oscillations in several features of the regional monsoon system. The detailed synoptic evidence presented in the present study confirms that the intra-seasonal fluctuations of the monsoon system is dominated by 30–40 day mode.

temporary onset of the monsoon. We propose to document the characteristics of the near equatorial shear zone and also those of the superimposed synoptic scale disturbances during different phases of the monsoon for the two seasons. The data is presented in two ways, viz, (i) longitudinally averaged (based on a 2° grid interval) characteristics for 700 mb over the belt 66 to 96°E and (ii) average relative vorticity and divergence for the active regime of the large scale ITCZ to represent the disturbed part of the ITCZ. Two sets of sequences are studied for each of the years corresponding to (i) the onset phase and (ii) the revival after a 'break' in the monsoon in the established phase. As aircraft reconnaissance and drop-sonde data were available for the sequences in the season of 1979, typical vertical structures of the ITCZ are also studied for these sequences along suitable meridians for which the data were maximum.

5.1 Large scale characteristics

5.1a *Pre-onset and onset phases of monsoon:* Figures 15 and 16 show the meridional profiles of the longitudinally averaged temperature, relative humidity, zonal wind and relative vorticity at 700 mb for three typical days in these phases for the two seasons. The profile of 7 and 10 June 1979 and 3 and 4 June 1973 define conditions during the pre-onset and just before the onset of monsoon. The profiles for 14 June 1979 and 7

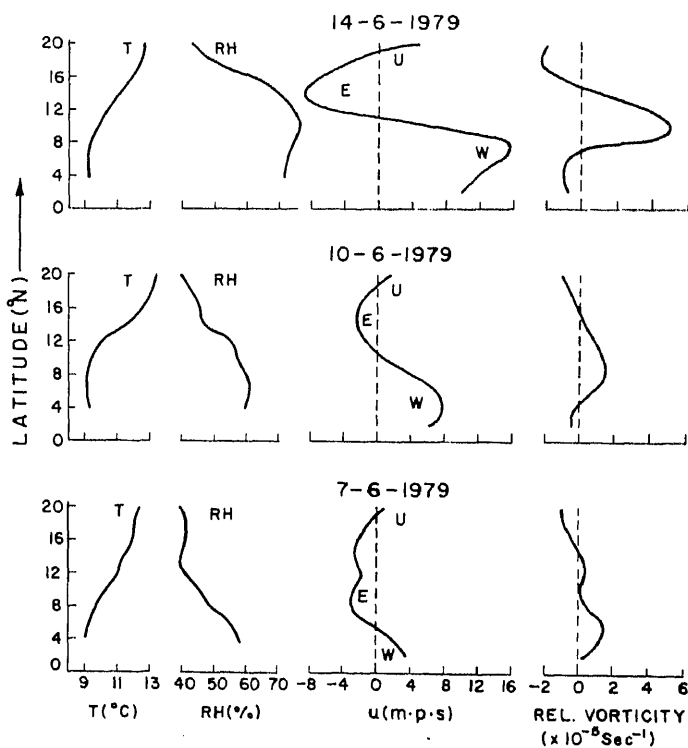


Figure 15. Meridional profile of characteristic parameters (temperature, relative humidity, zonal wind and relative vorticity) at 700 mb during the onset phase of season 1979.

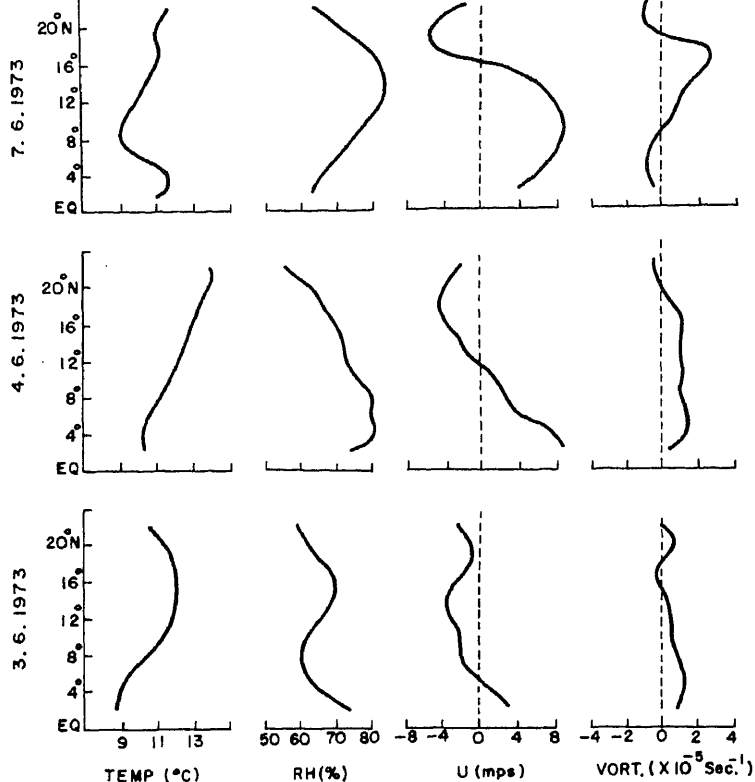


Figure 16. Meridional profile of characteristic parameters (temperature, relative humidity, zonal wind and relative vorticity) at 700 mb during the onset phase of season of 1973.

June 1973 define the conditions when the monsoon onset vortex had formed. The following features are observed:

- (i) The axis of the ITCZ progressively shifted northwards.
- (ii) Air temperature increases from the equator towards 20°N. The axis of the shear zone lies in the region of colder temperatures but to the north of it marked increase in temperature is observed.
- (iii) Equatorial westerlies are more moist. As the shear zone intensifies, a nose (maximum) in the relative humidity profile develops at the axis of the shear zone. The gradient in the relative humidity profile to the north of the shear zone also increases with the intensification of the shear zone.
- (iv) Progressive increase in the strength of the equatorial westerlies occurred between the pre-onset stage and the intensifying stage and the westerlies also extended northward.
- (v) Relative vorticity is generally positive from 4 to 15°N. The profile has a maximum which almost coincides with the axis of the shear zone and the magnitude of the maximum increases from about 1.5 units ($\times 10^{-5} \text{ sec}^{-1}$) on 7 June to 5.0 units on 14 June 1979 and from 1 to 3 units during 3 to 7 June 1973.
- (vi) The weather observed during the research flight on 14 June 1979 across the shear

zone showed deep convective clouds with moderate turbulence while crossing the shear zone. The rainfall over Kerala increased as the shear zone approached and lay near this area. Rainfall of 5 to 15 cm in 24 hrs was reported over Kerala on 12 and 14 June.

5.1b Revival of monsoon after a 'break' monsoon in the established phase: Figures 17 and 18 show the meridional profile of different parameters (longitudinally averaged) at 700 mb for three typical days representative of (i) early formative stage of the shears zone (21 July 1979 and 2 August 1973), (ii) well developed stage of the shear zone (24 July 1979 and 5 August 1973) and (iii) northward migrated shear zone located near 15°N—the transition regime between the near equatorial location and the normal monsoon trough zone (26 July 1979 and 6 August 1973). The following features are observed.

(i) Temperature increases from south to north. However, the axis of the shear zone lies in the region of weak temperature gradient.

(ii) The maximum moisture zone lies about 2° latitude to the south of the shear zone. Relative humidity increases from the formative to the intensified stage of the shear

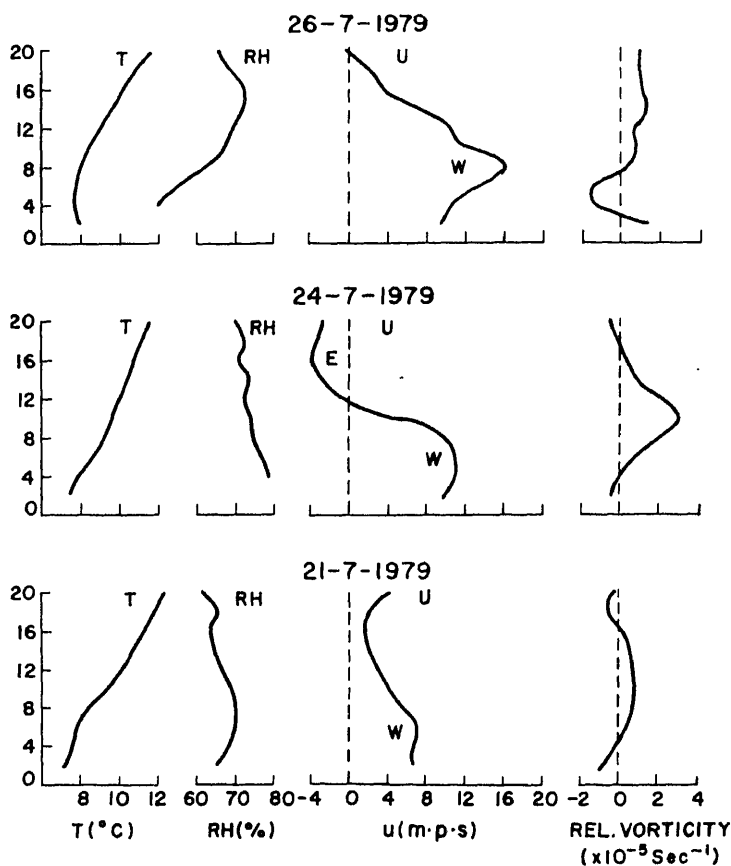


Figure 17. Meridional profile of characteristic parameters at 700 mb during the revival of

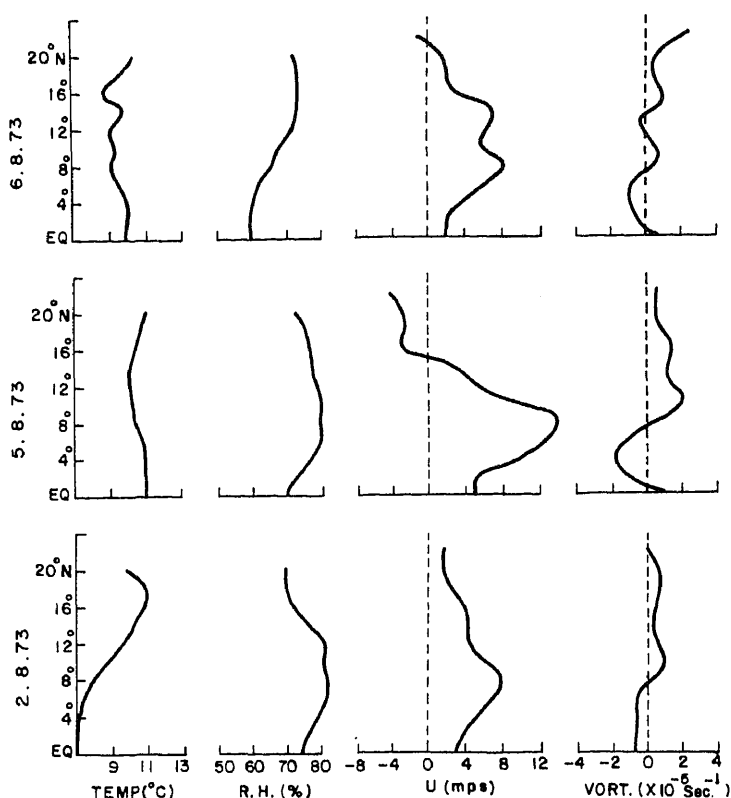


Figure 18. Meridional profile of characteristic parameters at 700 mb during the revival of monsoon after a break in the established phase of the season of 1973.

zone. On the third day, relative humidity has a maximum along 14°N in comparison to the maximum between $4\text{--}8^{\circ}\text{N}$ seen on the other two days.

(iii) A region of weak cyclonic vorticity maximum of 0.5 units is observed between $6\text{--}12^{\circ}\text{N}$ on the day of formation of the shear zone which intensified to about 2.5 units and became sharp along 12°N with the intensification of the shear zone. On the last stage, the region of cyclonic vorticity of about 1.0 units is observed between $9\text{--}20^{\circ}\text{N}$ indicating the broad and diffuse nature of the longitudinally averaged vorticity profile.

5.2 Disturbed state relative vorticity and divergence

The disturbed state represents the average for the section of the large scale ITCZ covering the longitudes of the active zone of the disturbance. This has been studied with reference to relative vorticity and divergence for the 1979 cases only where ample data were available. The longitudinal belt chosen for averaging purposes is $66\text{--}72^{\circ}\text{E}$ for the pre-onset and onset phases and $82\text{--}88^{\circ}\text{E}$ for the revival phase.

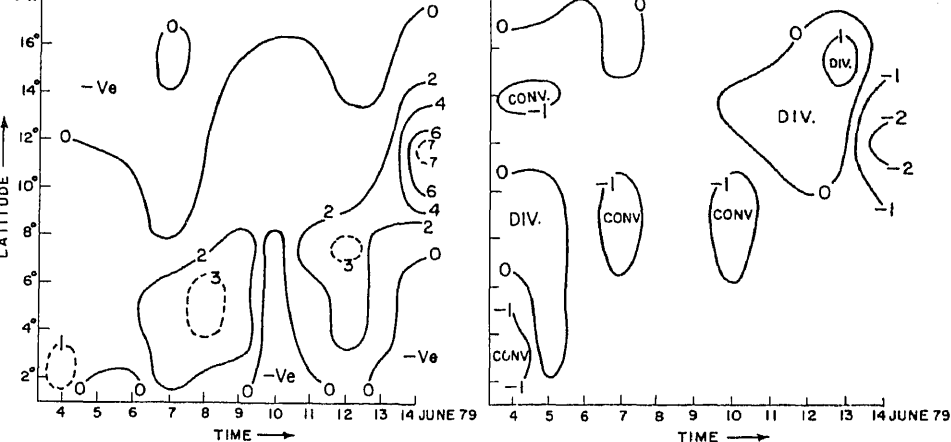


Figure 19. Meridional-time section of **a.** relative vorticity and **b.** divergence at 700 mb during 4–14 June 1979.

on 8 June represent the disturbed phase on the pre-onset conditions (Bhide *et al* 1982). The features on 14 June represent the highly disturbed phase of the monsoon onset when the northward progression had begun.

Note that the relative vorticity maximum has shifted from 2°N on 4 June to 12°N on 14 June, the shift being rather rapid between 12 and 14 June. The maximum has also intensified to a value of 7.5 units on 14 June. Convergence of magnitude $\geq 1 \times 10^{-5} \text{ sec}^{-1}$ is observed in the vicinity (within about 2° latitude) of the location of the shear zone.

(ii) *Revival phase*—Figures 20a and b show the typical disturbed state parameters for the revival phase of the oceanic ITCZ between 20 and 26 July 1979.

3.3 Vertical structure of the ITCZ

Figure 21a shows the distribution of wind, temperature and relative humidity across different latitudes along approximately 65°E for the pre-onset conditions of the ITCZ typified by the data of 7 June 1979. The same parameters along about 70°E are shown in figure 21b for 14 June 1979, typical of the monsoon onset conditions of the ITCZ. On 7 June the ITCZ extends at best upto 700 mb with a marked southward tilt as seen from its position along 12°N at 850 mb and about 4°N at 700 mb. On 14 June the ITCZ extended upto 400 mb without any significant slope between 850 and 400 mb. The horizontal shear across the axis also considerably increased as the equatorial westerlies strengthened to the south. The moist layer ($\geq 80\%$ relative humidity) also considerably deepened from 850 mb on 7 June to at least 400 mb on 14 June. There is also appreciable fall in temperature (3 to 4°C) north of 10°N at 850 and 700 mb and increase of about 2°C at 400 mb from the pre-onset (7 June) to the onset conditions (14

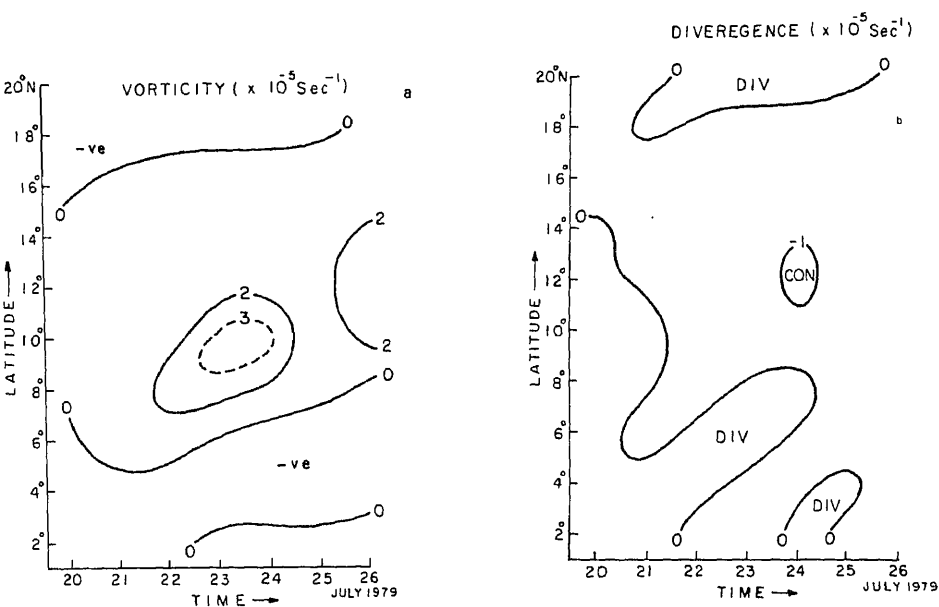
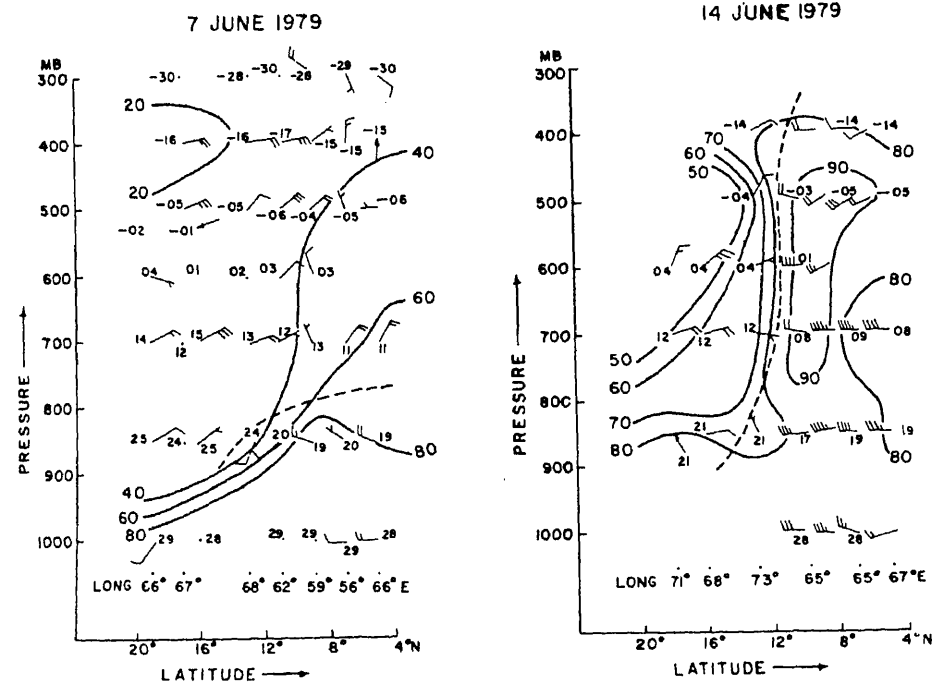


Figure 20. Meridional-time section of a. relative vorticity and b. divergence at 700 mb during 20-26 July 1979.



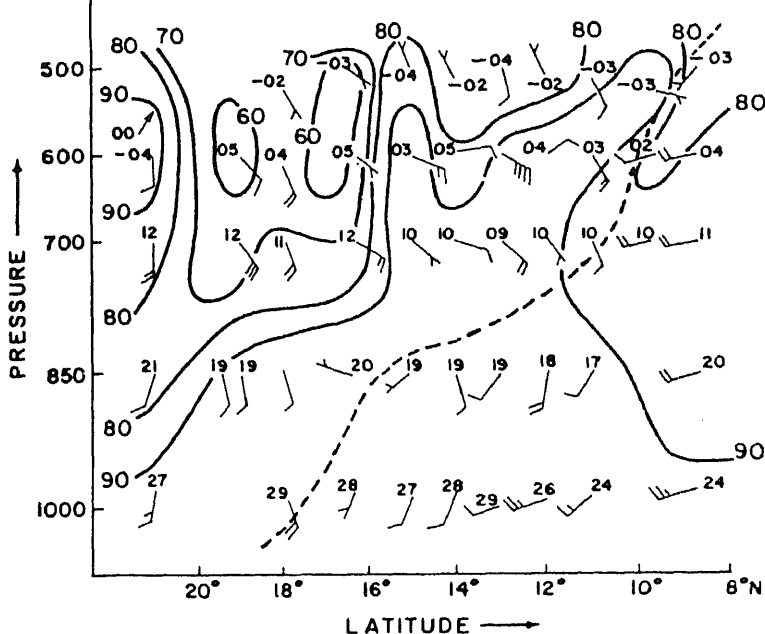


Figure 22. Meridional section in the vertical of wind, temperature and relative humidity (approximately along 70°E) based on data collected during a research flight on 24 July 1979.

June). These temperature changes are indicative of the large scale convective activity as the system intensified.

The vertical structure of the well developed near equatorial oceanic ITCZ in the Bay of Bengal during the revival phase of the monsoon for 24 July 1979 is shown in figure 22. The shear zone has a very large vertical slope between 1000 and 700 mb. Strong westerly flow is to the south of 12°N, 10°N and 8°N at 1000, 700 and 600 mb respectively. The temperature gradient is small across the shear zone at 700 to 400 mb. A deep moist layer (relative humidity, $RH \geq 80\%$), extending upto 500 mb, prevails in the vicinity of the shear zone between 9 and 16°N. The second maximum in the RH, which is observed near 21°N is perhaps due to the day-time cumulus activity over the marshy continental region adjoining the Northern Bay of Bengal.

6. Summary

Based on the synoptic analysis of the weather events for the two typical summer monsoon seasons of 1979 (Monex) and 1973 (ISMEX) chiefly at 700 mb and the satellite imagery, intra-seasonal fluctuating regimes of different phases of the summer monsoon system over the Indian region are described. The following aspects have been brought out by the study.

(i) The analysis has confirmed that the major phase changes in the sub-seasonal scale

are characterised by two preferred positions of the ITCZ, viz, (a) near equatorial one ($5-10^{\circ}\text{N}$), in the beginning of the season and also soon after the 'break' monsoon conditions, and (b) continental one ($20-25^{\circ}\text{N}$) during the established phase of the monsoon. The transition between the near equatorial oceanic position to the continental position is brought about during the onset and mid-season revival of the monsoon by northward propagation of the near equatorial oceanic ITCZ to take the position normally occupied by the monsoon trough.

(ii) The analysis also confirmed that the major phase changes in the ITCZ occur at an interval of about 30–50 days in the monsoon season.

(iii) The northward migration is initiated by the strengthening of the equatorial westerlies which result in the intensification of the shear zone.

(iv) The active phase of the monsoon in the middle of the season which lasts for 15–20 days is enveloped by the formation of overlapping disturbances (low pressure areas, depressions, mid-tropospheric cyclonic systems) in the vicinity of the regional monsoon trough extending from 70°E to 95°E over the monsoon trough region. The movement of the trough from 25°N to the foothills is accomplished rather rapidly (about 2–3 days).

(v) The 'break' monsoon phase in which the monsoon trough moves close to the foothills of the Himalayas and the near equatorial trough gets emphasised, lasts for about 5–10 days. The changeover from the ITCZ from about 15°N to 20°N is again accomplished somewhat rapidly (2–3 days).

(vi) The large scale oceanic ITCZ is characterised by: (a) cyclonic vorticity over the entire belt at the beginning of the onset phase. As the equatorial westerlies strengthen and the disturbance intensifies a region of anticyclonic vorticity is noticed to the south of the axis of the maximum wind. The cyclonic vorticity over the perturbed part of the ITCZ at the intensifying stage, is about 3 to 4 times the large scale magnitude. The cyclonic vorticity also extends in the vertical up to about 400 mb at the intensifying stage; (b) deep moist layer extending upto even 400 mb prevails in the vicinity of the disturbed portion of ITCZ; (c) temperature is colder in the layer 850 to 700 mb in the vicinity of ITCZ in comparison to the environment and it increases to the north of the axis of the ITCZ.

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Quasi-specular sea scatter with modified reflection coefficient

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Abstract. The paper deals with the relationship between the sea surface wind speed (U) and the backscattering coefficient (σ^0) for microwave frequencies and quasi specular range of angles. Valenzuela's effective reflection coefficient based on the theory of scattering of electromagnetic waves from rough dielectric surfaces has been introduced in the expression of backscattering coefficient and the final result compared with the σ^0 - U relationship derived from SEASAT scatterometer and concurrent sea truth data.

Keywords. Scattering; SEASAT; sea surface wind speed; quasi specular reflection.

1. Introduction

Satellite scatterometry has made significant advancement in the last two decades. Scatterometers onboard the satellites SKYLAB and SEASAT have mapped the oceanic wind on a global scale. Determination of the oceanic wind vector by scatterometers is possible because the backscattering coefficient at microwave frequencies is correlated to the ocean surface wind vector. This is explained by the mechanism of Bragg scattering for large incidence angles and quasi-specular reflection near normal incidence (Moore and Fung 1979). Normal incidence scattering coefficient measurements by altimeters have also been used to retrieve surface wind speed (Brown *et al* 1981).

Current scattering theories (Fung 1981) have explained the trends in the ocean backscatter measurements to a large extent. But considerable refinements and modifications are required before they can be used for definite remote sensing applications with confidence. In the present paper is given a modified expression of backscattering coefficient from rough ocean surface as a function of surface wind speed for quasi-specular range of angles. The modifications are made by using Valenzuela's expression (1970, 1978) for effective reflection coefficient for rough surfaces in the expression for backscattering coefficient in quasi-specular scattering (Barrick 1974). The angular dependence of the coefficients of the wind-backscatter relationship are generated as analytical functions.

2. Quasi-specular sea scatter

Based on the specular point model, which involves derivation of the average number of specular points for a two-dimensionally rough surface and the Gaussian curvature of these points. Barrick (1974) has developed a simple expression for the backscattering

coefficient in the quasi-specular range of incidence angles (i.e., incidence angles upto 20°):

$$\sigma^\circ = \pi \sec^4 \theta P(-\tan \theta, 0) |R(0)|^2, \quad (1)$$

where,

θ = angle of incidence from the vertical,

$P\left(\frac{\partial Z}{\partial x}, \frac{\partial Z}{\partial y}\right)$ = joint probability density function of the surface slopes,

Z = surface height above the mean (xy) plane, and

$R(0)$ = Fresnel reflection coefficient of the surface at normal incidence.

It has been assumed that the xz plane is the incident plane.

It is reasonable to interpret (1) as the scattered signal, being merely proportional to the probability of the surface having regions tilted at slope $\tan \theta$ which make them normal to the incident ray. For an isotropic rough surface of Gaussian statistics, (1) becomes

$$\sigma^\circ = (1/S^2) |R(0)|^2 \sec^4 \theta \exp(-\tan^2 \theta / S^2) \quad (2)$$

where S^2 is the total variance of slopes.

3. Effective reflection coefficient

Barrick (1974) used the Fresnel reflection coefficient at normal incidence in the expression for σ° (2). However, the reflection properties of the surface can be significantly modified by the amplitude of the roughness present. Hence, for the reflection coefficient of a wind driven rough surface, an effective reflection coefficient should be used. Using Rice's theory for forward scattering of electromagnetic waves from a dielectric rough surface (Rice 1951), Valenzuela (1970, 1978) has derived the following expression for effective reflection coefficients:

$$R_\perp \text{ effective} = R_\perp + \frac{1}{2} \frac{\beta^2 \gamma (\epsilon - 1)}{(\gamma + \eta)^2} I_\perp \quad (3)$$

$$R_\parallel \text{ effective} = R_\parallel - \frac{1}{2} \frac{\beta^2 \gamma (\epsilon - 1) \epsilon}{(\epsilon \gamma + \eta)^2} I_\parallel \quad (4)$$

where

$$R_\perp = \frac{\gamma - \eta}{\gamma + \eta}; \quad R_\parallel = \frac{\epsilon \gamma - \eta}{\epsilon \gamma + \eta};$$

$$I_\perp = \int_{-\infty}^{\infty} dp \int_{-\infty}^{\infty} dq \left[\eta + \frac{\beta(\epsilon - 1)}{d(p, q)} \left(q^2 / D_{pq} - 1 \right) \right] W(p - \alpha \beta, q);$$

$$I_\parallel = \int_{-\infty}^{\infty} dp \int_{-\infty}^{\infty} dq \left\{ -\frac{\alpha}{\beta} [p\eta - \alpha c(p, q)] + \eta - \frac{\beta \alpha^2 \epsilon b(p, q)}{D_{pq}} \right. \\ \left. + \frac{p\beta \alpha (\epsilon - 1) \eta b(p, q)}{D_{pq}} - \frac{p\beta \alpha \eta}{D_{pq}} - \frac{\beta(\epsilon - 1) \eta^2}{D_{pq}} \left(1 - \frac{p^2}{D_{pq}} \right) \right\} W(p - \alpha \beta, q);$$

$$\eta = (\varepsilon - \alpha^2)^{1/2}; \quad \gamma = \cos \theta; \quad \alpha = \sin \theta;$$

$$b(p, q) = (\beta^2 - p^2 - q^2)^{1/2}; \quad c(p, q) = (\beta^2 \varepsilon - p^2 - q^2)^{1/2};$$

$$d(p, q) = b(p, q) + c(p, q); \quad D_{pq} = b(p, q) c(p, q) + p^2 + q^2$$

Hence, for computations of the backscattering coefficient, the following expression has been used here:

$$\sigma^\circ(\theta) = \frac{|R_{\text{effective}}(0)|^2}{S^2} \sec^4 \theta \exp\left(-\frac{\tan^2 \theta}{S^2}\right) \quad (5)$$

It is realized that the choice of the sea spectrum (which is undergoing continuous refinement in recent times) will influence the final results. In this paper, the following specific form of the spectrum $W(K)$ is selected:

$$W(K) = \begin{cases} CK^4/(\kappa^2 + K^2)^4, & K \leq K_c \\ 0, & K > K_c \end{cases} \quad (6)$$

where K is the wavenumber. Equation (6) represents a polynomial approximation of the so-called Pierson Moskowitz spectrum for the steady state response of the ocean surface to a surface wind speed U and has been used earlier by Brown (1978) for studying scattering by composite random surfaces. The constant κ is given by $(335.2U^4)$, K in (cm^{-1}) and U in m/sec, the spectral constant C was taken to be 0.0046 and cut off wavenumber K_c to be 12 cm^{-1} .

The spectrum was selected merely for the sake of an example and a realistic spectrum should be used for exact evaluation.

4. Comparison with SEASAT scatterometer results

To evaluate the performance of the original Barrick's expression (2), and the expression with the modified reflection coefficient (5), backscattering coefficient values were computed for the SEASAT-A scatterometer (SASS) system parameters (Boggs 1982). SEASAT provided an opportunity to collect a large amount of calibrated scattering coefficient data. Britt and Schroeder (1984) developed a functional data base by merging large numbers of SASS measurements (35073 horizontal and 70744 vertical polarization measurement samples) with high quality surface truth wind data. The data base is represented by a multivariate regression equation by relating wind speed, relative wind direction and incidence angle, to backscattering coefficient:

$$\sigma^\circ(\text{db}) = \sum_{i=1}^{12} b_i x^i y^i z^i \quad (7)$$

where $x = \sin \theta$, $y = \log(U)$, $z = \cos \phi$

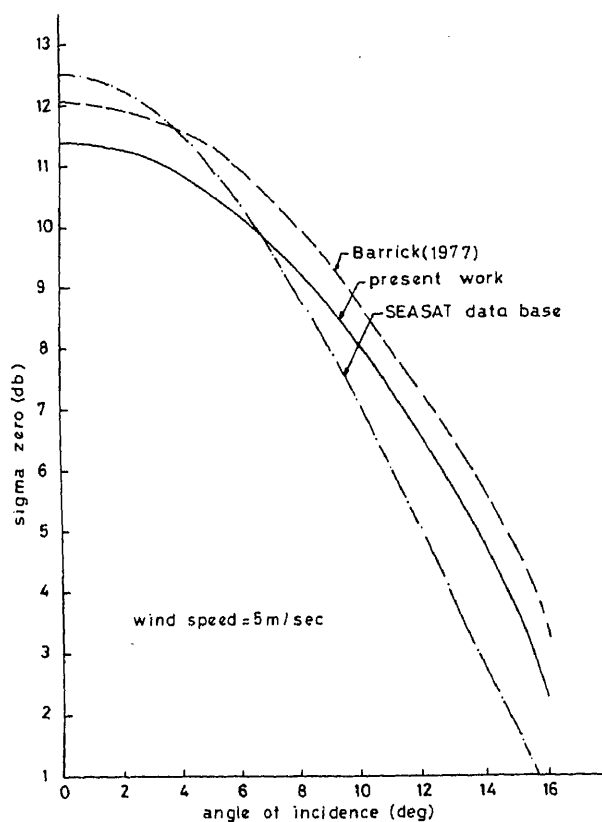
θ is the incidence angle and ϕ the relative wind direction.

The regression coefficients are given in table 1.

In figures 1-3 are illustrated the variations in the values of scattering coefficient over the angular range of 0 to 16° for the upwind case for wind speed values of 5, 15 and 20 m/sec respectively. It should be noted that the effect of polarization and wind direction are insignificant in the quasi-specular range of angles. In all the cases, it is seen that Barrick's original expression (2) is closer to the SEASAT regression model curve for

Table 1. Regression coefficients

i	f	g	h	b_i	
				H-pol	V-pol
1	2	0	0	-295.0534	-313.9546
2	3	0	0	267.0684	300.6422
3	4	0	0	000.0000	000.0000
4	0	1	0	-004.4289	-004.4752
5	2	1	0	93.9786	102.2082
6	3	1	0	000.0000	000.0000
7	4	1	0	-109.9966	-126.5153
8	2	1	1	000.0000	000.0000
9	2	1	2	7.7536	13.0239
10	2	1	3	2.8942	0.9506
11	2	1	4	-002.0757	-005.3607
12	0	0	0	15.6307	15.9729

Figure 1. Angular variations of σ^0 for wind speed of 5 m/sec.

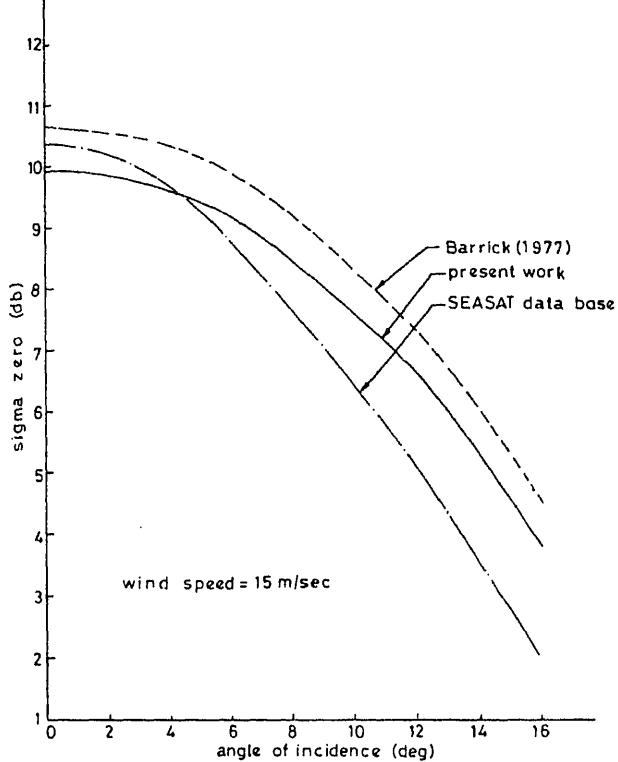


Figure 2. Angular variations of σ^0 for wind speed of 15 m/sec.

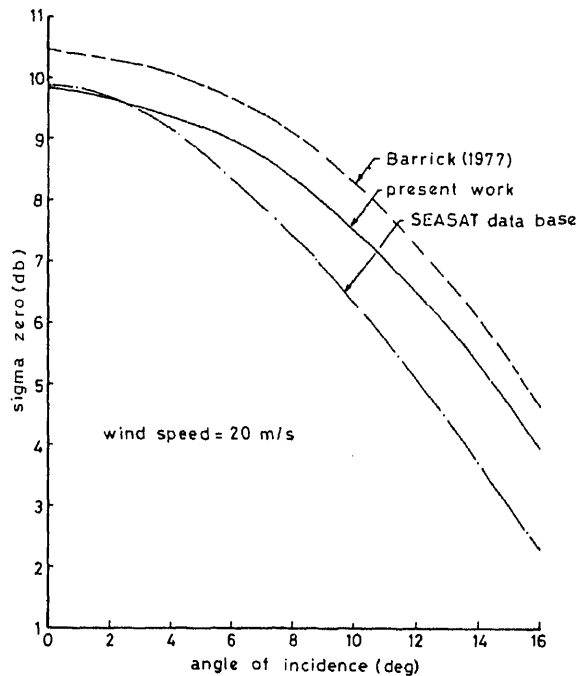


Figure 3. Angular variations of σ^0 for wind speed of 20 m/sec.

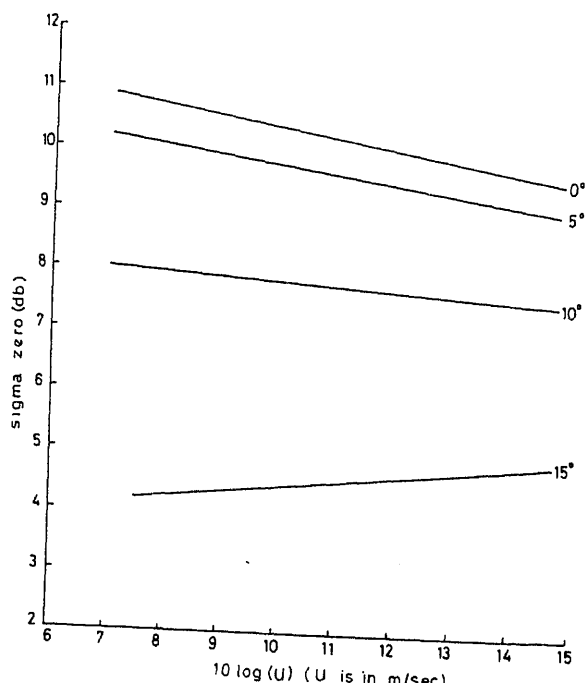
angles upto about 7 degrees, while beyond that the modified expression provides more accurate results.

5. Power law relationship

A simple power law relationship of the type $\sigma^\circ = A(\theta) \cdot U^{B(\theta)}$ is found to be a fairly good representation of the relationship between surface wind speed and the backscattering coefficient. The coefficients A and B would depend on the frequency, polarization and angle of observation. It is possible to generate analytical functions for A and B through the method of numerical curve fitting. For the case of quasi-specular sea scatter as given by Barrick (1974), the angle dependent functions $A(\theta)$ and $B(\theta)$ were generated for the X band (Sarkar and Bhaduri 1984). Following similar procedures, functions $A(\theta)$ and $B(\theta)$ have been generated for the modified backscattering coefficient, given by (5). In figure 4 is shown the power law relationship between σ° and U on a logarithmic scale for different angles of incidence. In figure 5 is illustrated the angular dependence of A and B for sass frequency. The following analytical functions have been obtained for $A(\theta)$ and $B(\theta)$:

$$A(\theta) = \sum_{i=0}^4 (A_i \theta^i), \quad (8)$$

$$B(\theta) = \sum_{i=0}^4 (B_i \theta^i), \quad (9)$$



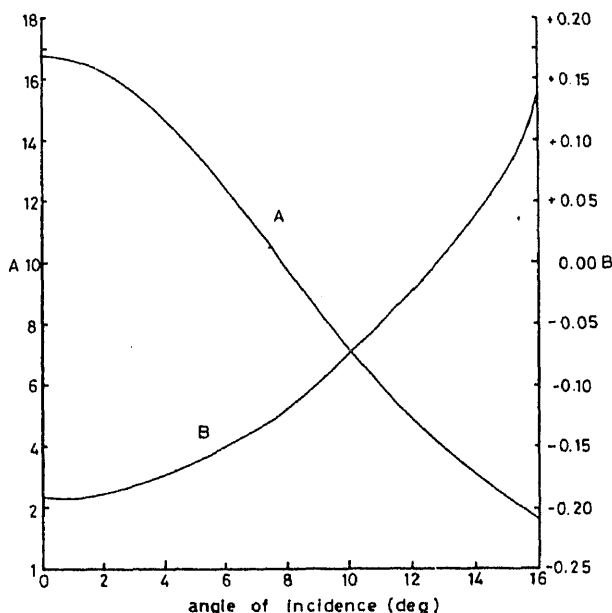


Figure 5. Angular variations of the coefficients A and B .

Table 2. Coefficients A_i and B_i .

i	0	1	2	3	4
A	16.5795	0.1964	-0.2146	0.01178	-0.1704E-03
B	-0.1932	-0.1019E-02	3.77379	-0.163244	2.97941

where θ is in degrees in (8) and in radians in (9). The coefficients A and B are given in table 2. Fourth degree polynomials are found to give excellent fits with RMS errors of only 0.65 % and 0.067 % for $A(\theta)$ and $B(\theta)$ respectively.

6. Results and discussion

Barrick (1974) found a fairly good agreement of measured scattering coefficient values at normal incidence with those obtained through his quasi-specular model. However, at angles away from normal, considerable disagreement is noticed (Valenzuela 1978). This is also seen in figures 1–3. Possible remedies lie in using mean square slopes of large waves (i.e. for the case where gravity-capillary waves have been suppressed) and modified reflection coefficient (as it has been done in this paper) in the model for the quasi-specular sea scatter. Figures 1–3 indeed clearly illustrate the improvement brought about by the modified reflection coefficient over the angles away from normal. Modifications due to the spray and foam can also be introduced if an appropriate

evolve realistic functional forms of probability density functions of ocean slope and elevation, which have been elusive owing to the nonlinear nature of ocean waves. In figures 1–3 one finds an improvement of about 1 dB over the quasi-specular model. Effectively this will bring about improvement of the order of 15% in retrieved wind speed.

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The effect of filtering on drift and anisotropy parameters determined by full correlation analysis

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Abstract. Ionospheric drift records obtained by the closely spaced receiver technique at Udaipur are subjected to low and high pass filtering with varying cutoff frequencies and the effects of filtering on various drift and anisotropy parameters as determined by full correlation analysis are studied. Limits in which filtering could be employed are suggested on the basis of results obtained.

Keywords. Ionospheric drift; full correlation analysis; spaced receiver method; filtering.

1. Introduction

Various methods have been developed for evaluating the drift parameters from spaced receiver records. The three methods used are (i) The similar fade method (Mitra 1949), which gives the apparent drift velocity V_a , and most commonly used earlier; (ii) The full correlation analysis (Briggs *et al* 1950; Phillips and Spencer 1955; Fooks 1965) mostly used nowadays that gives the apparent drift velocity, the true drift velocity V , characteristic velocity V_c and various other parameters defining the characteristic ellipse representing the average size of the ground diffraction pattern namely the semi-minor axis b , the axial ratio r and the orientation of the major axis ψ ; and (iii) cross spectral analysis, developed by Jones and Maude (1965) which gives the apparent drift velocity at different fading frequencies. The cross spectral method was developed to see if the diffraction pattern has wave motion. However, there are several possibilities which could result in dispersion in velocity like variation of velocity in the record or the presence of vertical gradients in velocity.

A different approach for the study of the variation of drift velocity with fading frequency was introduced by Sprenger and Schminder (1969). In this method the fading records are first filtered and then subjected to full correlation analysis. In principle one can thus study the apparent and the true drift velocity as well as other anisotropy parameters of the ground pattern as a function of the fading frequency by varying the cutoff frequency of the filter. Sprenger and Schminder (1969) reported an increase of the true drift velocity with the increase of the cutoff frequency of the high pass filter while the apparent drift velocity remained independent. Chandra and Briggs (1978) investigated theoretically the effect of low pass filtering assuming a Gaussian form of correlation functions and showed that the true drift velocity decreased with low pass filtering and therefore this need not be taken as evidence for dispersion. The filtering effect was seen only when $V_c/\pi f_0 \xi_1 \gg 1$ (ξ_1 is the scale size and f_0 the filter cutoff

frequency) and for typical ionospheric conditions filter effects will be noted when cutoff frequency is less than about 1 Hz. The apparent drift velocity, however, remained independent of filtering. In real cases of dispersion the correlation functions would be asymmetric and would show change in the apparent drift velocity with filtering. The theoretical investigations were compared and confirmed with experimental observations conducted at Buckland Park using the partial reflections. Thus it is obvious that filtering followed by correlation analysis cannot be used to determine true drift velocity at different fading frequencies.

Recently Chandra (1985) has studied theoretically the effect of low pass filtering on drift and anisotropy parameters and showed that while the true drift velocity V decreases with filtering, the characteristic velocity V_c decreases with filtering unless $V \gg V_c$ when V_c shows an increase; resulting in an increase of the ratio V_c/V with filtering since the decrease in V is steeper than the decrease in V_c . The size (semi-minor axis) of the diffraction pattern was also shown to increase with filtering while the axial ratio would remain independent for near isotropic patterns but change for elongated patterns such as observed near the magnetic equator. The effects due to high pass filtering would be just the opposite as high pass filtering would result in narrowing down the correlation functions. The effect of filtering was shown, however, only when $V_c/\pi f_0 \xi_1 \gg 1$ which worked out to be about 1 Hz for typical ionospheric conditions. A few samples of observational data recorded earlier at an equatorial station, Tiruchirapalli, confirmed these theoretical results.

In the present investigations the effect of low and high pass filtering is studied on a large number of spaced fading records obtained at a low latitude station Udaipur (Geog. lat. 24°N). The primary aim of the study is to confirm the theoretical findings of Chandra (1985) and also to find out the limits in which filtering can be done without affecting the drift results.

2. Observations and analysis

About 80 records obtained at Udaipur during the year 1981–82 have been analysed in this study. The experimental arrangement used has been described earlier (Rastogi *et al* 1978). The amplitude data has been digitized manually and fed to a computer for full correlation analysis. Record lengths between 1–2 minutes have been used for analysis with a sampling interval of 0.25 seconds. Filters of running mean type have been used with width varying between 0.50 to 10 sec. The cutoff frequency is determined as $f_c = 1/\tau_f$, where τ_f is the width of the filter. Hence in the present investigations cutoff frequency varies between 0.1 Hz to 2 Hz.

3. Results

The average variation of the various parameters computed from the full correlation analysis are plotted as a function of the cutoff frequency of the filter and shown in figures 1–5. The results for both low pass and high pass filters are shown in the same figure and the unfiltered values are also marked.

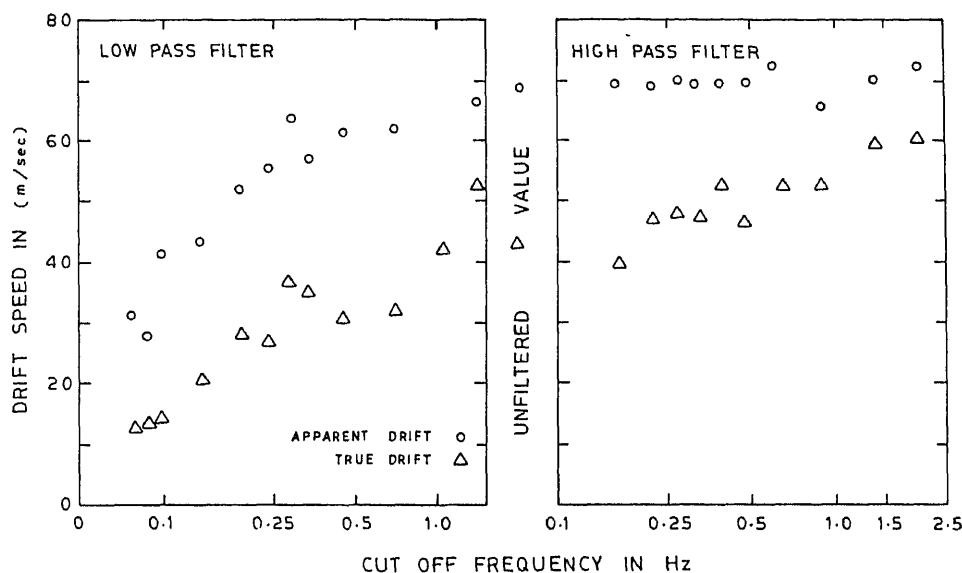


Figure 1. The variation of the apparent drift velocity V_a and the true drift velocity V_t , determined by full correlation analysis, as a function of the cutoff frequency of the filter shown both for low pass and high pass filtering.

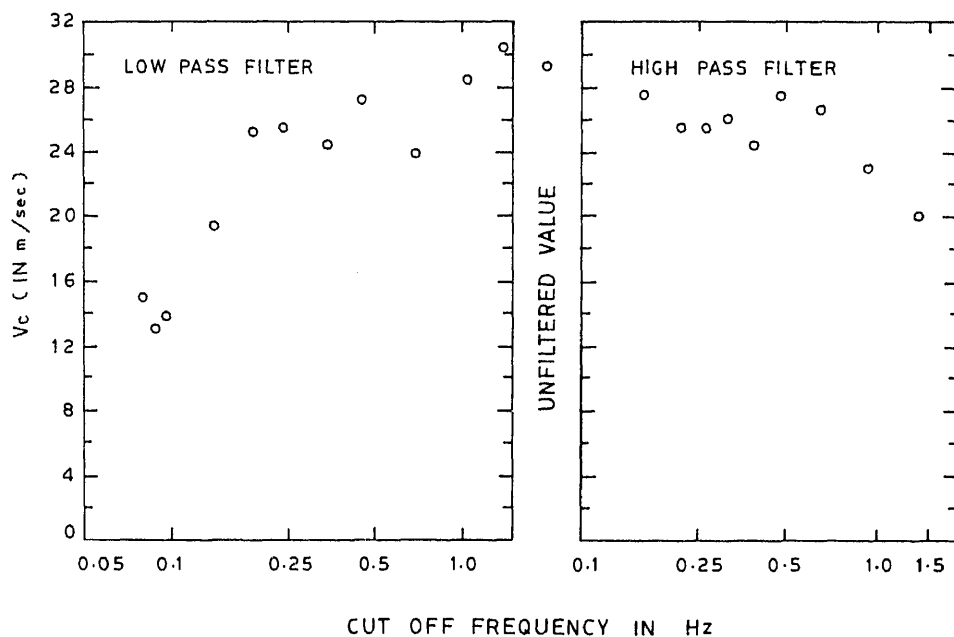


Figure 2. The variation of the characteristic velocity V_c as a function of the cutoff frequency

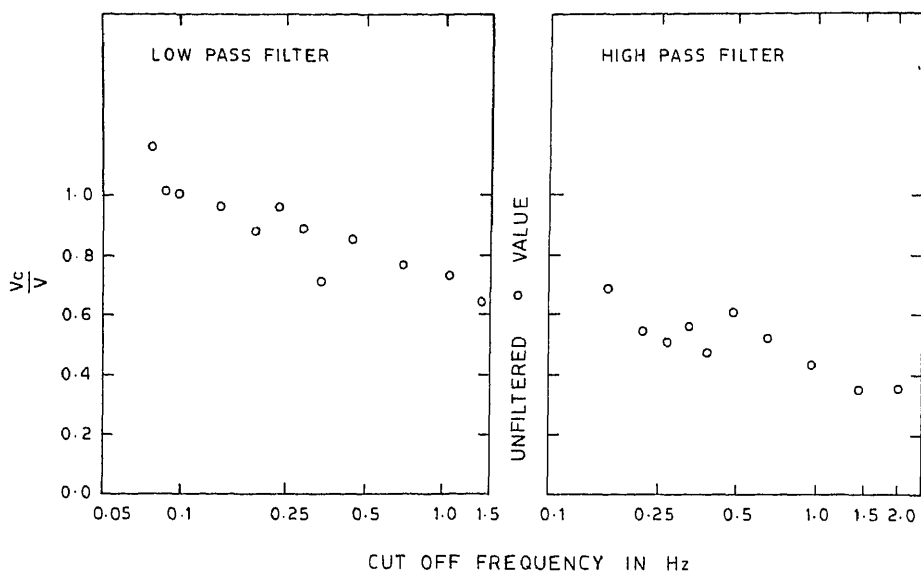


Figure 3. The variation of the ratio V_c/V as a function of the cutoff frequency of the filter shown both for low pass and high pass filtering.

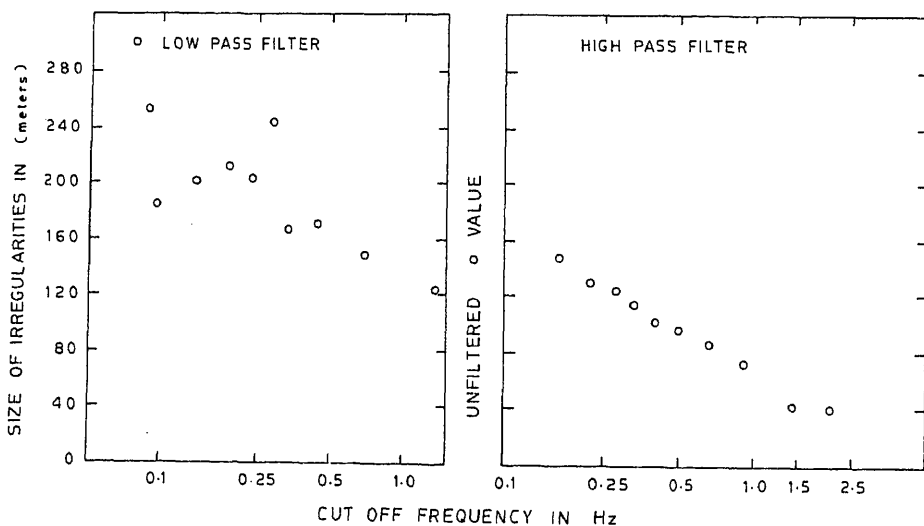


Figure 4. The variation of the scale size (b) of the irregularities determined by full

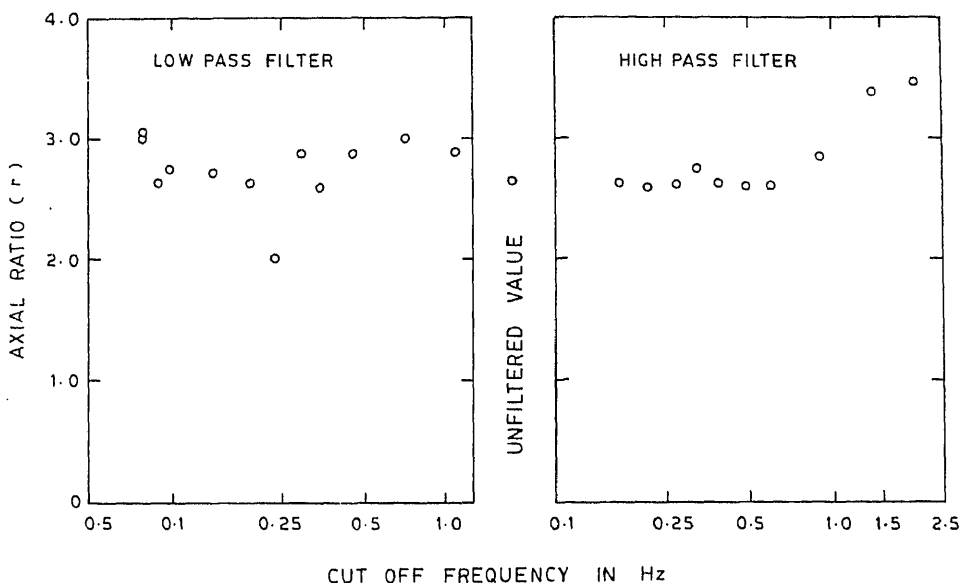


Figure 5. The variation of the axial ratio of the irregularities determined by correlation analysis as a function of the cutoff frequency shown both for low and high pass filtering.

The average variation of the apparent drift speed V_a with cutoff frequency is shown in figure 1. It is seen that V_a remains independent of the high pass filtering but decreases when low pass filtering is used. This decrease suggests the presence of dispersion. The variation of the true drift speed with filtering is also shown in the same figure. It is found that the true drift speed V increases with high pass filtering and decreases with low pass filtering. The unfiltered value of 43 m/s decreases to about 13 m/s with low pass filtering and approaches V_a with high pass filtering. These results are similar to the results obtained earlier from experimental data (Sprenger and Schminder 1969; Chandra and Briggs 1978) and are also in agreement with the theoretical results of Chandra and Briggs (1978).

The average variation of the characteristic velocity V_c with cutoff frequency is shown in figure 2. The value of V_c does not show any significant variation with high pass filtering but decreases with low pass filtering, the decrease being from 29 m/s to about 13 m/s. The average variation of the ratio V_c/V shown in figure 3 indicates decrease of the ratio V_c/V with high pass filtering and increase with low pass filtering. The unfiltered value of about 0.65 increases to more than 1.0 with low pass filtering and decreases to less than 0.4 with high pass filtering. Thus, while the effect of filtering (decrease) on parameters V_c and V is similar, the magnitude of effect is more pronounced for V than for V_c . The unfiltered value of V_c/V being about 0.6, no increase in V_c with filtering is expected on theoretical grounds.

The effect of filtering on the semi-minor axis (b) of the characteristic ellipse is shown in figure 4. The scale size, b , decreases with high pass filtering and increases with low pass filtering. The average value of the scale size which is about 140 m increases to about 300 m with low pass filtering and decreases to a low value of about 40 m with high pass

in figure 3. There is no significant variation in the value of axial ratio with filtering, the value lying between 2.5 to 3.0 on most cases.

4. Discussion

The effect of low pass filtering on the apparent and true drift velocity determined by full correlation analysis has been studied theoretically by Chandra and Briggs (1978). On theoretical grounds it is expected that the true drift velocity would decrease with low pass filtering while apparent drift velocity remains independent. Chandra (1985) has studied theoretically the effect of low pass filtering on characteristic velocity V_c , the ratio V_c/V , scale size b as well as axial ratio r . According to Chandra (1985) there would be a decrease in the characteristic velocity V_c with low pass filtering unless $V \gg V_c$, when there is an increase in V_c , and the ratio V_c/V would always increase. The scale size would increase with low pass filtering. The limiting value of the increase given by $[(V^2 + V_c^2)/V_c^2]^{1/2}$. The results in the present investigations are in conformity with these theoretical investigations. Theoretical investigations by Chandra (1985) have also shown that for isotropic irregularities the axial ratio would remain independent of the filtering. Since the axial ratio at Udaipur is about 2.5, one would not expect any significant effect of the filtering. This result is also observed in the experiment results presented here. For a low latitude station Udaipur, the limits in which filtering could be employed comes out to be about $\frac{1}{2}$ Hz (cutoff frequency).

Cross spectral analysis has been applied to a large number of F -region records at Udaipur in an earlier study (Sardesai *et al* 1983) and it was noted that about 21 % cases showed velocity increasing with fading frequency (positive dispersion). No dispersion in velocity was noted in about 18 % cases and nearly 46 % cases showed velocity varying randomly with fading frequency. Similar patterns were also found for both E and F region observations at an equatorial station Tiruchirapalli (Vyas *et al* 1980). Positive dispersion has been thought to arise due to the variation of velocity within the record or due to the presence of vertical gradients in velocity (Felgate and Golley 1971; Huang and Petterson 1970; Chandra 1979). A comparison of the dispersion results by cross spectral and filtering method showed similar variation of velocity with the fading frequency (Vyas and Chandra 1985). The eighty records studied here were also analysed by the cross spectrum method. Positive dispersion was noted on about 22 % of records while 8 % of the records showed negative dispersion. Approximately 25 % of the records did not show any dispersion and the rest of the records showed no consistent variation of the velocity with fading frequency. The variation of the drift and anisotropy parameters as a function of the filter cutoff frequency was then studied separately for the four categories of dispersion as seen by the cross spectral method. Except for the apparent drift velocity the result for other parameters were similar to those shown in figures 1–5 which are averages of the data. The apparent drift velocity, for the positive dispersion group, increased with the cutoff frequency of the high pass filters and decreased with the decrease of the cutoff frequency of low pass filters (increasing filtering). For the group showing negative dispersion the results were just the opposite i.e. the apparent drift velocity showing an increase with low pass filtering and a decrease with cutoff frequency of high pass filters. For the groups showing 'no dispersion' and 'random dispersion' the effect of filtering on the apparent drift velocity were similar. Hence the filter method can only study the variation of the apparent drift velocity with fading

frequency. The variations in other parameters arise by the nature of the analysis itself and have no physical significance. The cross spectral method has limitations in that it can determine only the apparent drift velocity and due to this reason it appeared earlier that the filter method could be used to study various drift and anisotropy parameters at different fading frequencies. However, both the theoretical work of Chandra (1985) as well as the present experimental investigations suggest that the results obtained by the filter method may lead to wrong interpretation if the limitations of the method are not taken into account. In actual experimental data some sort of filtering is always present, imposed by the finite length of data and the sampling interval. In addition, detrending is necessary sometime. However, when using filters for detrending, care must be taken that the filter cutoff frequency chosen satisfies the conditions that parameters to be studied do not change appreciably. This is given by the condition $V_e/\pi f_0 \xi_1 \gg 1$ and for the ionospheric fading data for a low latitude station like Udaipur is about $\frac{1}{2}$ Hz.

Acknowledgements

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Geochemistry of coexisting hornblende and biotite from the Ambalavayal granite, Kerala

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Abstract. Major and trace element geochemistry of coexisting hornblendes and biotites from the Ambalavayal granite, northern Kerala, are presented. The hornblendes correspond to edenitic composition, whereas the biotites correspond to annite. The hornblendes typically show high Al_2O_3 contents (9.69–11.89 %) comparable with those from anorogenic granites. The biotites are characteristically low Mg-type, similar to those reported from alkaline rocks. The distribution coefficients calculated for all the major and trace elements are presented and an evaluation of the nature of variation indicate near-chemical equilibrium conditions during the crystallization of the two minerals. The hornblende-biotite tie lines in the Fe^{3+} – Fe^{2+} –Mg compositional triangle, lie parallel to those of buffered biotites, indicating crystallization in an environment closed to oxygen and well above the Ni–NiO buffer. It is inferred that the $f\text{H}_2\text{O}$ increased towards the residual stage and $f\text{O}_2$ values were high, in the range of 10^{-15} bars.

Keywords. Granite; hornblende; biotite; geochemistry; distribution coefficients; $f\text{H}_2\text{O}$; $f\text{O}_2$; Ambalavayal.

1. Introduction

The 595 million year (m.y.) old granite pluton of Ambalavayal in Wynad district of northern Kerala, emplaced within Precambrian biotite gneisses, is a typical member of the Late Precambrian Early Paleozoic granitic intrusives in the south-western Indian shield (Nair *et al* 1982; Santosh *et al* 1985). The granite is spatially related to the Moyar fault zone and the associated pegmatites and quartz veins show disseminated molybdenite mineralization, marking an event of taphrogenic metallogeny (Santosh and Nair 1983). Geochemical characters denote an alkaline nature and tectonic aspects suggest a probable anorogenic affiliation, like the other plutons in the region. This paper presents the major and trace element geochemistry of hornblende and biotite, the principal mafic minerals in the granite, based on which the conditions of crystallization are deciphered.

The granite is pink, medium grained and composed of interlocking quartz and alkali feldspar. Plagioclase of albitic composition occurs in subordinate amounts. The hornblende (constituting 3.74–5.63 volume %) and biotite (3.88–5.23 volume %) occur as subhedral laths with textural relations generally suggesting their formation towards the later stages of crystallization history as they commonly form interstitial laths between grains of plagioclase and perthite. Biotite shows a pleochroic scheme of $X < Y = Z$, where X = straw yellow and $Y = Z$ = dark greyish brown, whereas the pleochroism of hornblende is $X < Y \leq Z$ with X = yellowish green, Y = green and Z = deep bluish green. The accessories include zircon, apatite, epidote, monazite and sphene. Minor reibeckite is also noted, showing deep blue colour, anomalous extinction and $Z \wedge c$ varying between 3–9°. The opaque is mainly magnetite.

Hornblende and biotite fractions for the present study were obtained by mineral separation involving a combination of repeated heavy liquid and electromagnetic techniques. The final purity of the samples were checked by hand picking under a stereo-microscope and was found to be about 99%.

The major elements were analysed by conventional wet chemical techniques and trace elements by atomic absorption spectrophotometer (Perkin-Elmer-4000), calibrated using internal standards.

2. Geochemistry of biotite

The chemical analyses of biotite samples from Ambalavayal and their structural formulae (based on 22 atoms, anhydrous) are presented in table 1. Si cations range from 5.570 to 5.923 per formula unit compared to 6.000 for ideal phlogopite-annite series.

Table 1. Chemical analyses and structural formulae of biotite from Ambalavayal.

	A/BT/1	A/BT/2	A/BT/3	A/BT/4	A/BT/5	A/BT/6
<i>Chemical analyses</i>						
SiO ₂	35.01	36.10	36.28	37.10	38.18	37.51
Al ₂ O ₃	13.77	13.26	14.62	12.75	14.07	12.82
TiO ₂	2.80	2.88	2.23	2.60	1.96	2.14
Fe ₂ O ₃	5.02	5.44	5.21	8.39	7.46	6.47
FeO	29.21	28.10	26.28	26.05	21.37	25.86
MnO	0.33	0.42	0.46	0.18	0.53	0.34
MgO	0.96	1.12	1.42	0.65	2.02	0.84
CaO	1.57	1.84	1.13	1.12	1.48	1.12
Na ₂ O	0.80	0.76	0.74	0.65	0.58	0.63
K ₂ O	8.51	7.89	9.12	8.10	8.98	8.42
P ₂ O ₅	0.43	0.52	0.59	0.57	0.54	0.58
Moisture	0.44	0.66	0.25	0.18	0.28	0.18
LOI	0.81	0.98	0.73	1.68	0.56	1.64
Total	99.66	99.97	99.06	100.02	98.01	98.55
<i>Structural formulae*</i>						
Si	5.570	5.697	5.732	5.830	5.923	5.955
Al ^{iv}	2.430	2.303	2.268	2.170	2.077	2.045
Al ^{vi}	0.153	0.164	0.455	0.192	0.496	0.356
Ti	0.336	0.343	0.265	0.307	0.229	0.256
Fe ³⁺	0.601	0.647	0.619	0.991	0.871	0.773
Fe ²⁺	3.886	3.709	3.472	3.423	2.772	3.434
Mn	0.045	0.056	0.062	0.024	0.070	0.046
Mg	0.228	0.264	0.334	0.150	0.467	0.198
Ca	0.268	0.311	0.191	0.189	0.246	0.191
Na	0.246	0.234	0.036	0.198	0.176	0.194
K	1.726	1.592	0.010	1.624	1.776	1.731

There is sufficient Al in the sample to fill the tetrahedral sites not occupied by Si. The octahedral sites are occupied by 4.905 to 5.249 Y group cations compared with 6.000 in the trioctahedral mica. The sum of the inter-layer X group cations (K, Na, Ca) ranges from 2.011 to 2.240, compared with the ideal 2.000 cations per formula unit. Plots of Fe/Fe + Mg ratios of biotite against Al atoms/22 oxygens define a field close to the annite-phlogopite join (figure 1), with Al atoms indicating composition close to that of annite (2.2–2.8).

The most striking feature of the biotites is the very low magnesium content of the octahedral layers (0.150–0.467). Figure 2 depicts a comparison of the composition of these biotites with those delineated for biotites from elsewhere. The Ambalavayal biotites lie outside Foster's (1960) field, away from the Mg apex and are comparable

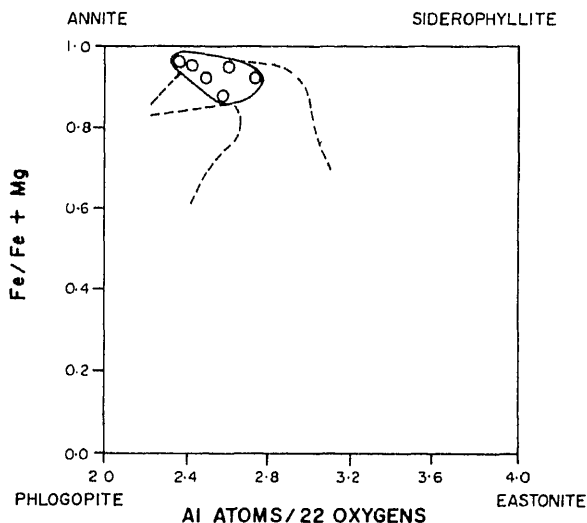


Figure 1. Composition of Ambalavayal biotite in terms of Fe/Fe + Mg vs Al atoms/22 oxygens. The broken lines represent the field of biotites as given by Anderson (1980).

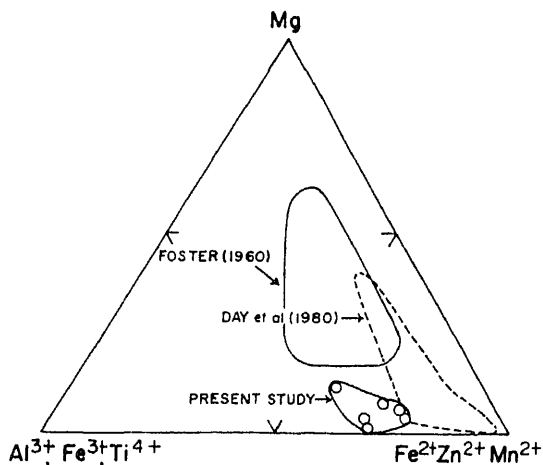


Figure 2. Composition of Ambalavayal biotites compared with those from elsewhere (note the low Mg-content).

with low Mg biotites from alkaline environments reported by Day *et al.* (1980). The biotites show low sodium content in the alkali site varying from 0.036 to 0.246. These characters are comparable with those for biotites from alkaline rocks (*cf.* Czamanske *et al.* 1977).

3. Geochemistry of hornblende

Chemical analyses and structural formulae (based on 23 oxygen atoms, anhydrous) of hornblende separates from Ambalavayal granite are presented in table 2. The analyses and structural formulae overall satisfy the requirements for 'superior analyses' and indicate an edenitic composition (Leake 1978). Si cations range from 6.306 to 6.920 per formula unit with sufficient Al in all the samples to fill the tetrahedral sites not occupied

Table 2. Chemical analyses and structural formulae of hornblende from Ambalavayal.

	A/HB/1	A/HB/2	A/HB/3	A/HB/4	A/HB/5	A/HB/6
<i>Chemical analyses</i>						
SiO ₂	39.83	40.23	39.45	39.83	41.21	39.78
Al ₂ O ₃	10.20	10.67	11.89	10.71	10.71	9.69
TiO ₂	1.42	1.41	1.35	1.53	1.58	1.53
FeO ₃	5.58	5.11	4.96	6.80	6.56	6.68
FeO	20.20	20.42	20.61	22.85	19.80	23.40
MnO	0.53	0.64	0.61	0.55	0.42	0.56
MgO	2.72	4.01	3.86	2.56	2.56	2.56
CaO	10.53	9.22	8.78	9.41	7.51	10.30
Na ₂ O	1.87	1.79	1.94	2.15	2.01	2.13
K ₂ O	2.41	1.98	2.21	1.78	2.48	1.78
P ₂ O ₅	0.31	0.28	0.12	0.22	0.16	0.15
Moisture	2.21	1.02	1.97	1.08	1.85	1.97
LOI	1.95	1.95	2.22	0.97	2.56	2.65
Total	99.76	98.53	99.97	100.44	99.41	98.41
<i>Structural formulae*</i>						
Si	6.947	6.443	6.337	6.306	6.716	6.371
Al ^{iv}	1.053	1.557	1.663	1.694	1.784	1.629
Al ^{vi}	0.894	0.442	0.526	0.306	0.774	0.154
Ti	0.175	0.170	0.163	0.182	0.191	0.135
Fe ³⁺	0.673	0.616	0.601	0.811	0.805	0.809
Fe ²⁺	2.734	2.735	2.769	3.025	2.700	3.137
Mn	0.073	0.086	0.083	0.074	0.058	0.077
Mg	0.656	0.957	0.924	0.604	0.622	0.606
Ca	1.826	1.582	1.512	1.596	1.322	1.771
K	0.588	0.556	0.604	0.660	0.634	0.654
Na	0.498	0.404	0.454	0.360	0.514	0.366
Sum Y	5.205	5.019	5.066	5.002	5.150	4.918
Sum W + X	2.912	2.542	2.570	2.616	2.472	2.791
Mg/Mg + Fe ²⁺	0.194	0.259	0.250	0.166	0.187	0.162

*Based on 23 oxygens per formula unit; LOI—loss on ignition

by Si. The total number of Y group cations range from 4.891 to 5.205 per formula unit compared with 5.0 in the ideal hornblende formula.

A notable aspect is the high Al-content of the hornblende ($\text{Al}_2\text{O}_3 = 9.69 - 11.89\%$), which compares with the characteristic high Al_2O_3 values of hornblendes from anorogenic granites (9–11 weight %) and contrast with the low Al values for hornblendes from calc-alkaline plutons associated with orogenic belts (Anderson 1980). In all samples except one, the sum of W and X group cations (Ca, Na, K) exceed the theoretical maximum of 2.000 cations per formula unit in the M_4 site and suggests partial occupation of the A sites. The $\text{Mg}/\text{Mg} + \text{Fe}^{2+}$ values are fairly uniform, ranging between 0.162 and 0.259.

4. Distribution of major elements

For coexisting minerals α and β , the distribution coefficient of element A is defined as:

$$K_{D(A)}^{\alpha-\beta} = \frac{X_A^\alpha}{1 - X_A^\alpha} \frac{1 - X_A^\beta}{X_A^\beta}$$

where X_A is the atomic ratio of element A to the sum of major elements occupying the same site or sites in phases α and β . If the elements considered behave as ideal mixtures and reach chemical equilibrium under uniform P - T conditions, then the distribution coefficient, $K_{D(A)}$ is constant (cf. Kretz 1959, 1960; Stephenson 1977). The distribution coefficient, $K_{D(\text{Fe}^{2+} - \text{Mg})}^{B-H}$ was calculated using the atomic ratio, $\text{Fe}^{2+}/(\text{Fe}^{2+} + \text{Mg})$. $K_{D(\text{Al}^{\text{iv}})}^{B-H}$ was calculated using the atomic ratio, $\text{Al}^{\text{iv}}/(\text{Al}^{\text{iv}} + \text{Si})$. For the remaining distribution coefficients, the atomic ratio of the element considered is based on the total number of octahedral cations, e.g., for $K_{D(\text{Ca})}^{B-H}$, $X_{\text{Ca}} = \text{Ca}/\Sigma$ octahedral cations. The calculated atomic ratios of elements to the sum of major elements occupying the

Table 3. Atomic ratio of elements to the sum of major elements occupying the respective sites in hornblende and biotite from Ambalavayal.

	$X_{\text{Al}^{\text{iv}}}^H$	$X_{\text{Al}^{\text{vi}}}^H$	X_{Ti}^H	$X_{\text{Fe}^{2+}}^H$	$X_{\text{Fe}^{3+}}^H$	X_{Mn}^H	X_{Mg}^H	X_{Ca}^H
<i>Hornblende</i>								
A/HB/1	0.132	0.172	0.034	0.129	0.525	0.014	0.126	0.670
A/HB/2	0.195	0.088	0.034	0.123	0.545	0.017	0.191	0.622
A/HB/3	0.209	0.104	0.032	0.119	0.547	0.017	0.182	0.588
A/HB/4	0.212	0.061	0.036	0.162	0.605	0.015	0.121	0.610
A/HB/5	0.223	0.150	0.037	0.156	0.524	0.011	0.121	0.535
A/HB/6	0.204	0.031	0.028	0.165	0.638	0.016	0.123	0.635
	$X_{\text{Al}^{\text{iv}}}^B$	$X_{\text{Al}^{\text{vi}}}^B$	X_{Ti}^B	$X_{\text{Fe}^{2+}}^B$	$X_{\text{Fe}^{3+}}^B$	X_{Mn}^B	X_{Mg}^B	X_{Ca}^B

Biotite

A/BT/1	0.304	0.029	0.114	0.115	0.740	0.009	0.043	0.120
A/BT/2	0.288	0.032	0.066	0.125	0.716	0.011	0.051	0.146
A/BT/3	0.284	0.087	0.051	0.119	0.667	0.012	0.064	0.093
A/BT/4	0.271	0.038	0.060	0.195	0.673	0.005	0.030	0.094
A/BT/5	0.260	0.101	0.047	0.178	0.565	0.014	0.095	0.112
A/BT/6	0.255	0.070	0.051	0.152	0.554	0.009	0.030	0.091

respective sites in coexisting hornblendes and biotites from Ambalavayal are given in table 3 and the distribution coefficients are given in table 4. The variations in K_D values of each element are shown in figure 3, where the linear to curvilinear trends depict near-chemical equilibrium conditions of crystallization of the two minerals.

Among the tetrahedrally coordinated cations, Al and Si show variable distribution with $K_{D(Al^{iv})}^{B-H}$ ranging from 1.255 to 2.303. The distribution of Al^{iv} and Si between coexisting hornblende and biotite is shown to be non-ideal with their K_D values systematically related to the Al^{iv} contents of hornblende (Gorbatshev 1970). The linear distribution pattern of $X_{Al^{iv}}^H$ and the curvilinear trend of $X_{Al^{iv}}^B$ and $K_{D(Al^{iv})}^{B-H}$ depicts that the Al-Si distribution coefficient is related to compositional variables (Stephenson 1977). A check on the trend defined by Al^{iv} and Si between the coexisting minerals as per the equation of Gorbatshev (1970) indicate near-chemical equilibrium conditions.

Among the octahedral cations (table 4), $K_{D(Fe^{2+})}^{B-H}$ ranges from 1.063 to 1.410 and $K_{D(Mg)}^{B-H}$ ranges from 0.248 to 0.785. $K_{D(Fe^{2+}-Mg)}^{B-H}$ relationship was also checked and found to be uniform, consistent with the observations of other workers (*cf.* Saxena 1966; Leelanandam 1970). The variation trends of Al^{vi} in biotite and Al^{vi} , Fe^{3+} , Ca, Mn and Ti in hornblende are rather ambiguous and the trend lines drawn are only overall estimates.

The higher charged octahedral cations like Fe^{3+} , Al^{vi} and Ti show overall uniform distribution. The increase in $K_{D(Mn)}^{B-H}$ values against increasing Mn content in biotite (figure 4a) and their negative correlation in hornblende are consistent with similar observations of Saxena (1966) and Kretz (1959). The non-ideal distribution of Ti and Al^{vi} between hornblende and biotite also correlates with their observations with the feature attributed to be a result of variation in Al^{iv} , Al^{vi} , Fe^{3+} and alkalis in hornblende (Kretz 1960). Fe^{2+} and Ca in biotite show good fit against their respective K_D values, whereas in hornblende, the Ca distribution is not strictly regular.

5. Distribution of trace elements

The trace element geochemistry of hornblendes and biotites from Ambalavayal is presented in table 5 and the distribution coefficients are given in table 6. The pattern of distribution of elements between the coexisting minerals are shown in figure 4, where the K_D values are plotted against the respective $K_{D(Mg-Fe)}^{B-H}$ value. Systematic relationship

Table 4. Distribution coefficients for coexisting hornblende and biotites from Ambalavayal.

K^{B-H}	$D(Al^{iv})$	$D(Al^{vi})$	$D(Fe^{3+})$	$D(Fe^{2+})$	$D(Ti)$	$D(Mn)$	$D(Mg)$	$D(Ca)$
1	2.303	0.169	0.891	1.410	3.353	1.614	0.341	0.179
2	1.477	0.364	1.016	1.314	1.941	0.647	0.267	0.235
3	1.359	0.837	1.000	1.219	1.594	0.727	0.352	0.158
4	1.278	0.623	1.203	1.112	1.607	0.313	0.248	0.154
5	1.666	0.672	...	...	...	...	...	...

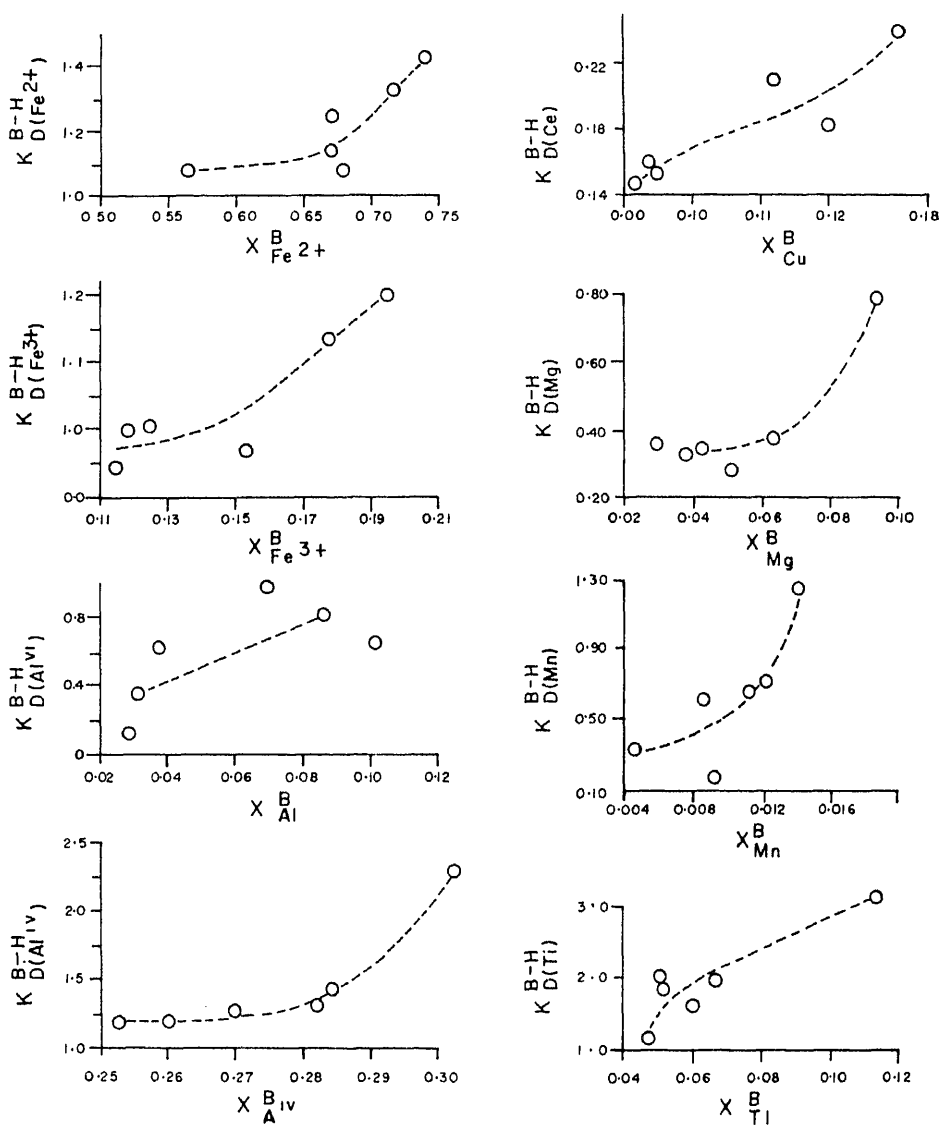


Figure 3. Variations of K_D values of major elements between coexisting biotite and hornblende from Ambalavayal.

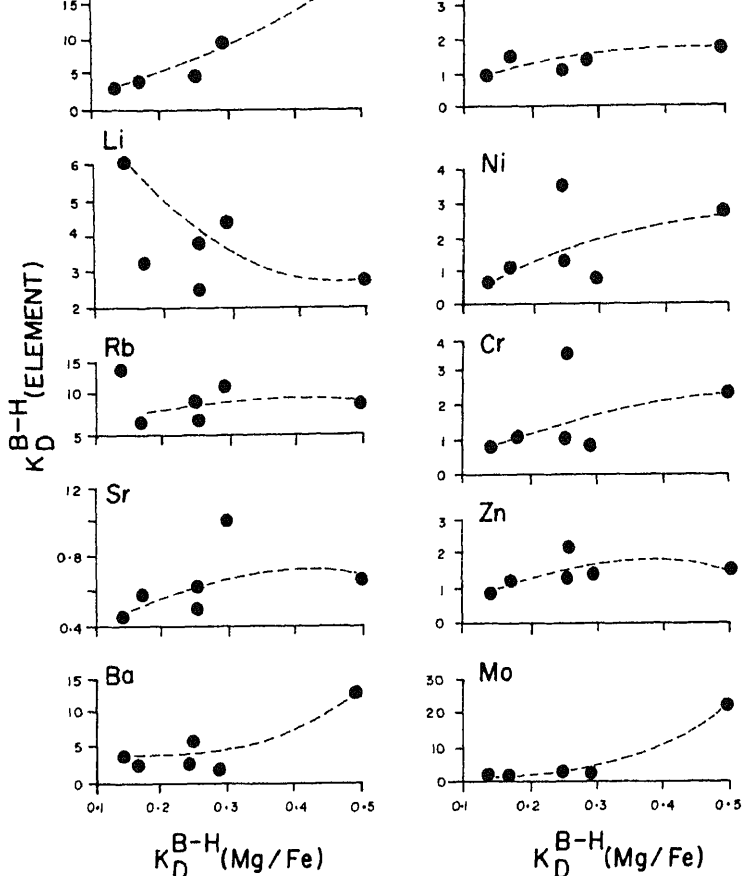


Figure 4. Variations of trace element K_D values between coexisting hornblende and biotite from Ambalavayal.

is observed in the distribution of Ba, Cu, Mo and Co, whereas Sr, Li, Zn, Cr, Ni and Rb show an overall correlation, suggesting the maintenance of near-chemical equilibrium during the crystallization of the minerals. The increase in Li and the decrease in other trace elements like Cu, Ni, Co, Zn, Sr and Cr with decreasing Mg/Fe values correspond with the crystal-chemical characters.

6. Intensive parameters

As shown by Wones and Eugster (1965), the dominant factors that influence the conditions of crystallization of biotite are temperature and fO_2 . Calculations based on the composition of biotite will hence give the minimum values of fO_2 . The most important elements to be considered are Fe^{2+} and Mg as the variations of these elements are largely temperature dependent. Also, Fe^{3+} and Fe^{2+} give an indication of

Table 5. Trace element analyses of hornblendes and biotites from Ambalavayal.

	Ba	Bi	Co	Cr	Cu	Li	Mo	Ni	Pb	Rb	Sr	V	Zn	Zr
<i>Hornblende</i>														
A/HB/1	72	50	136	122	2	89	24	86	40	62	54	120	1272	180
A/HB/2	64	58	128	128	4	88	22	98	34	56	66	130	1310	170
A/HB/3	76	48	132	116	6	92	38	80	48	72	48	110	1344	280
A/HB/4	22	74	148	128	4	42	24	104	60	24	27	100	1210	260
A/HB/5	30	60	146	116	2	72	2	122	46	50	39	140	1360	160
A/HB/6	60	78	153	124	4	48	14	102	38	30	15	100	1284	460
Mean	54	61	141	122	4	72	21	99	44	49	42	117	1297	252
<i>Biotite</i>														
A/BT/1	426	170	529	426	50	350	44	310	108	582	33	60	2600	300
A/BT/2	176	78	175	152	18	284	16	105	50	315	38	10	1530	100
A/BT/3	224	141	144	118	32	216	28	98	84	462	24	52	1840	220
A/BT/4	88	64	154	98	14	260	24	84	38	342	12	20	1142	400
A/BT/5	418	86	235	268	42	204	46	320	96	468	26	24	1860	280
A/BT/6	84	152	220	86	39	212	26	78	34	320	16	58	1780	360
Mean	236	115	243	191	33	254	31	166	68	415	25	37	1792	277

Table 6. Distribution coefficients of elements between coexisting biotite and hornblende from Ambalavayal.

K_D^{B-H}	Mg/Fe	Ba	Bi	Co	Cr	Cu	Li	Mo	Ni	Pb	Rb	Sr	V	Zn
1	0.25	5.9	3.4	3.9	3.5	2.5	3.9	1.8	3.6	2.7	9.4	0.6	0.5	2.1
2	0.17	2.3	1.3	1.4	1.2	4.5	3.2	0.7	1.1	1.5	3.2	0.6	0.1	1.2
3	0.25	2.9	2.9	1.1	1.0	5.4	2.4	0.7	1.2	1.8	2.4	0.5	0.5	1.4
4	0.14	4.0	0.9	1.0	0.8	3.5	6.2	1	0.8	0.6	6.2	0.4	0.2	0.9
5	0.50	13.9	1.4	1.6	2.3	21	2.8	23	2.6	2.1	2.8	0.7	0.2	1.4
6	0.29	1.4	2.0	1.4	0.7	9.8	4.4	1.9	0.8	0.9	4.4	1.1	0.6	1.4
Mean	0.32	5.2	2.0	1.7	1.6	11.5	3.8	4.9	1.7	1.6	4.7	0.7	0.4	1.4

the degree of oxidation. Composition of the Ambalavayal biotite when related to Wones and Eugster's (1965) experimental system suggest that oxygen fugacities during biotite crystallization were slightly higher than that defined by the Ni-NiO buffer and that the granite magma was buffered with respect to oxygen by oxides existing within the magma (Dodge *et al* 1969; Dodge and Ross 1971). If buffering continued during the crystallization of both biotite and hornblende, the tie lines between coexisting biotite and hornblende tend to be parallel to Wones and Eugster's estimated trends of buffered biotites. This is the case with Ambalavayal, where tie lines between coexisting biotites and hornblendes lie almost parallel to the trends of buffered biotite in the com-

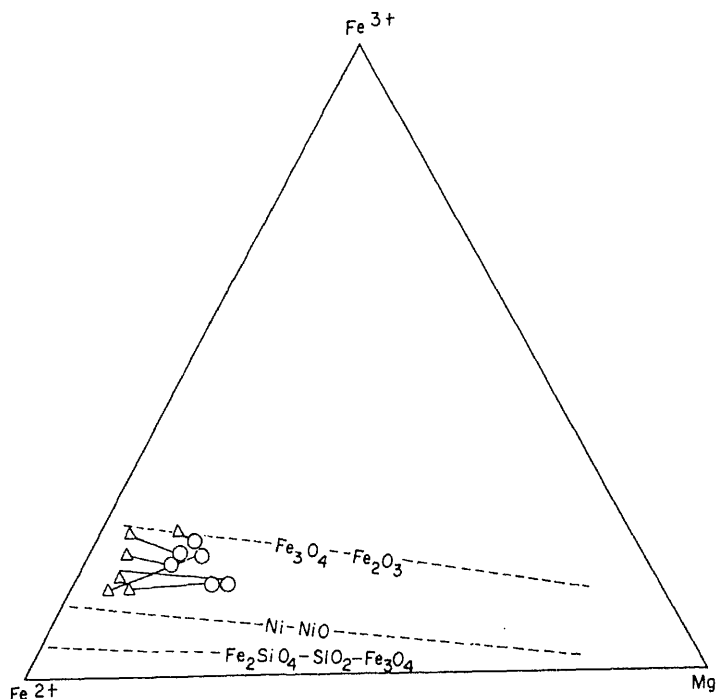


Figure 5. Fe^{3+} - Fe^{2+} -Mg plots of coexisting biotite (circles) and hornblende (triangles) from Ambalavayal. The buffer limits are after Wones and Eugster (1965).

plutonic rocks commonly formed in an environment closed to oxygen with oxygen fugacities determined by buffer reactions (Wones and Eugster 1965).

Qualitative aspects of intensive variables such as T , P , $f\text{H}_2\text{O}$ and $f\text{O}_2$ can be deduced from the sequence of crystallization of constituent minerals. The sequence of crystallization at T - $f\text{O}_2$ constrained to the Ni-NiO buffer for a rock of adamellitic composition was determined by Maaloe and Wyllie (1975). Their data show that hydrous minerals like biotite crystallize early under wet conditions and late under dry conditions. Hornblende and biotite are texturally late crystallized minerals in the Ambalavayal granite. This, together with the presence of pegmatites and aplites in the granite suggest an increase in $f\text{H}_2\text{O}$ to a probable saturation towards the final stages of crystallization.

Ishihara (1977) noted that Fe-Ti oxides indicate the relative levels of oxygen fugacity. His 'magnetite-series' refers to granites with high total Fe-Ti oxides with higher proportion of magnetite to ilmenite. The high $\text{Fe}^{3+}/\text{Fe}^{2+}$ ratios of coexisting biotites and hornblendes indicate higher oxygen fugacities during their crystallization ($f\text{O}_2 > \text{Ni-NiO}$). An estimate of $f\text{O}_2$ is depicted in figure 6 with the aid of an independent geothermometer obtained from coexisting feldspar pairs. Santosh (1986) estimated crystallization temperatures of 704–740°C for the coexisting alkali feldspar-plagioclase pairs from Ambalavayal, based on the mole % Ab content following Brown and Parsons' (1981) method. For granitic plutons, the lower limit of temperature is their wet

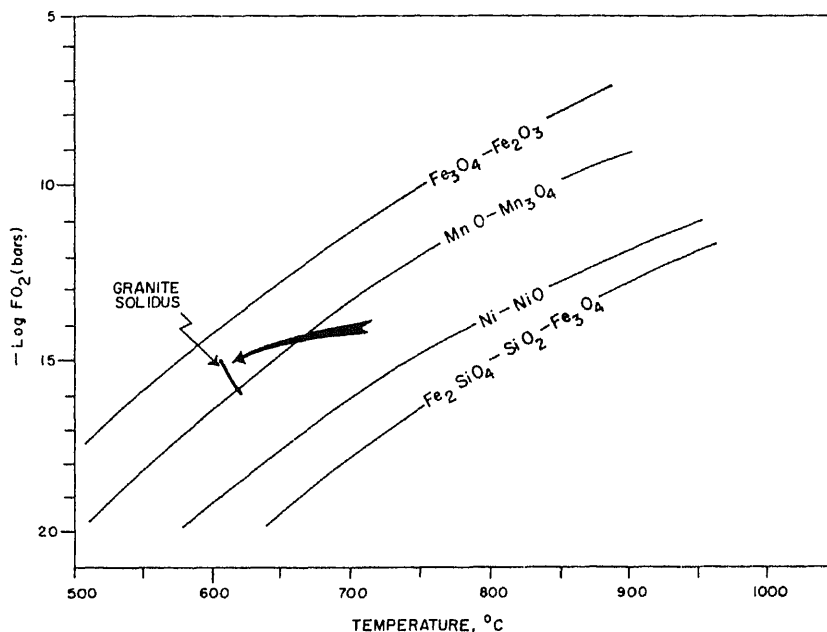


Figure 6. Log fO_2 vs temperature diagram with buffer limits, after Wones and Eugster (1965), showing the stability field of biotites from Ambalavayal. The upper limit of solidus temperature has been estimated from independent two-feldspar geothermometer (see text for further discussion).

equilibrium with biotite can be plotted. Thus, the stability fields depicted in figure 6 are derived from the two-feldspar geothermometer, which defines a window in P - T - fO_2 space, denoting high fO_2 conditions of crystallization of about 10^{-15} bars. The result is consistent with the qualitative estimate that the magnetite series granites crystallize at higher levels of fO_2 , well above the Ni-NiO buffer.

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Radioelement geochemistry of alkali granites of the Kerala region, south-west India

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Abstract. Th, U and K abundances in four alkali granites of the Kerala region, south-west India, are presented. The plutons show high radioelement levels, correlatable with those of alkali granites in other regions. The nature of variation is consistent with the correlation of Th and U with accessory phases like sphene, zircon, allanite, apatite and monazite. A geochronologic correlation is also observed between the alkali granites and the Th-bearing beach placers of the region. The petrogenetic features of the alkali plutons, their taphrogenic association, Pan-African affiliation and high Th/U levels suggest that the alkali plutons are favourable locales for radioelement exploration.

Keywords. Radioelement geochemistry; alkali granites; geochronologic correlation; radioelement exploration.

1. Introduction

The Kerala region forms a significant portion of the Indian shield and the south-western part of the continental margin. The region comprises largely Precambrian crystallines including charnockites, khondalites and migmatitic gneisses. Recent studies reveal the occurrences of a number of granite and syenite intrusives representing a Late Precambrian-Early Paleozoic magmatic regime (Santosh and Nair 1983a; Santosh and Thara 1985; Nair and Santosh 1984). The spatial relationship of the intrusives with regional fault-lineaments suggests a taphrogenic association (Santosh and Nair 1983b; Nair *et al* 1983). Among the intrusives are a group of granites which show alkaline character and unique petrochemical features (see for example, Nair and Santosh 1984). Even though general geochemical studies of these alkali granites have been attempted, their radioelement abundance has not been reported.

The present paper documents the abundances of Th, U and K in the alkali granites.

2. The alkali granites

The alkali granites of the region are represented by E-W elongated plutons near Chengannoor in Alleppey, Ambalavayal in Wynad, Munnar in Kottayam and Peralimala in Cannanore districts. These intrusives are spatially related to major fault-lineaments of the region (figure 1). All of them are emplaced within Precambrian basement rocks (figure 2), with which they maintain sharp contacts. Each pluton is exposed over an area of 25–50 km². Their general geologic characters, including age, are summarised in table 1.

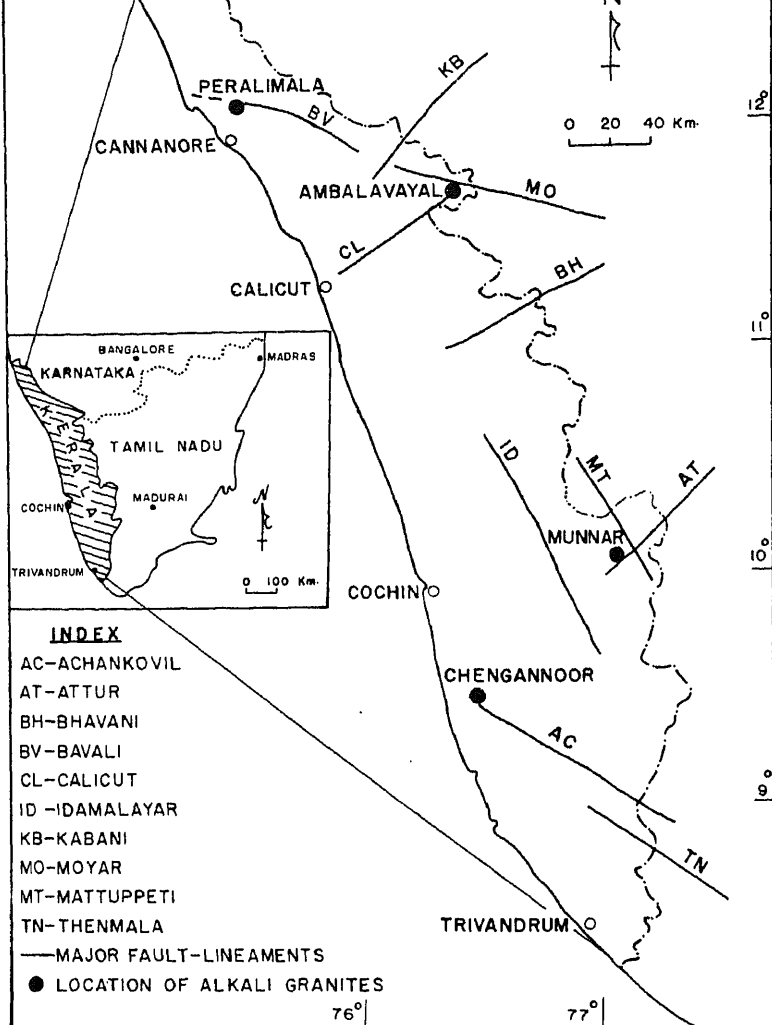


Figure 1. Lineament map of the Kerala region showing the location of the alkali granites.

quartz and feldspar. In thin sections, they show a general hypidiomorphic granular texture with alkali feldspar as the dominant constituent. The perthitic phase in the K-feldspar grains range from micro and string perthite in Ambalavayal, Munnar and Peralimala to coarse mesoperthite in Chengannoor. Plagioclase is subordinate and shows a range in composition of $Ab_{95}An_5$ in Ambalavayal to $Ab_{85}An_{15}$ in Chengannoor. It generally occurs as minor laths showing lamellar twinning. Two-feldspar geothermometry based on the Ab-component in coexisting alkali feldspar-plagioclase pairs give an estimate of 704–740°C for the Ambalavayal granite (Santosh 1985a). The modal $Q-A-P$ proportions of the granite correspond to a range between quartz alkali feldspar granite and normal granite as per Streckeisen's scheme (1976).

The major mafic mineral in all the granites, except Munnar, is greenish pleochroic

Table 1. General geologic and petrologic data for the alkali granites of Kerala.

Granite	Radiometric age (10 ⁶ yr)	Country rock	Associated fault-lineament	Average modal composition				
				Quartz	Alkaline feldspar	Plag.	Hb/ Biot.	Others
Chengannoor	550 ^a	Charnockite	Achankovil	24.31	61.30	7.00	2.52	4.87
Ambalavayal	595 ^b	Biotite gneiss	Moyar and Calicut	31.67	27.10	30.74	4.89	5.60
Munnar	740 ^c	Migmatitic gneiss	Attur	21.53	60.32	12.06	3.29	2.80
Peralimala	n.d.	Migmatitic gneiss	Bavali	25.95	57.90	7.29	6.88	1.98

^aafter Soman *et al* (1983); ^bafter Santosh *et al* (1985); ^cafter Odom (1982); n.d.— not determined; Plag.— plagioclase; Hb.— hornblende; Biot.— biotite.

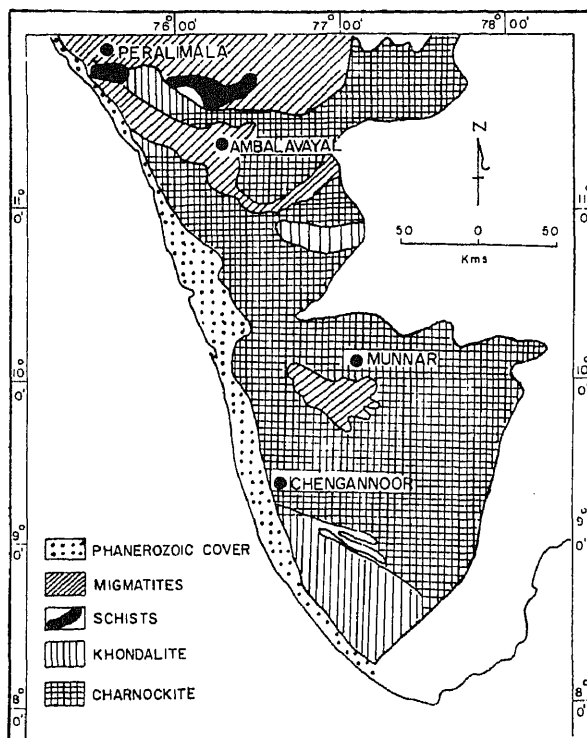


Figure 2. Generalised geological map of the Kerala region showing the location of the alkali granites.

hornblende. In Munnar, biotite forms the dominant mafic constituent, whereas biotite is found in subordinate amounts in the others. Sphene and apatite are ubiquitous accessories in all the granites. Other accessories include zircon, epidote, allanite, calcite and Fe-Ti oxides, Euhedral grains of monazite occur as minor inclusions in biotite and

Table 2. Average major element contents of the alkali granites.

Oxide	Chengannoor	Ambalavayal	Munnar	Peralimala
SiO ₂	69.39	74.51	71.74	64.53
Al ₂ O ₃	15.65	13.67	13.84	18.0
TiO ₂	0.38	0.16	0.24	0.08
Fe ₂ O ₃	2.68	0.94	1.68	0.93
FeO	1.05	1.87	0.84	0.65
MnO	0.03	0.02	0.02	0.05
MgO	1.62	0.42	0.66	0.62
CaO	0.53	1.44	1.70	1.48
Na ₂ O	3.48	3.01	2.56	3.11
K ₂ O	4.19	3.64	5.66	9.68
P ₂ O ₅	0.08	0.01	0.02	0.05

Data sources: Chengannoor, after Santosh and Nair (1983a); Ambalavayal, after Nair *et al* (1982); Munnar, after Nair *et al* (1983) and

granites. Zircon, monazite and allanite grains frequently exhibit metamict texture.

The average major element compositions of the alkali granites are given in table 2. Moderate variation in SiO_2 and Al_2O_3 levels, sympathetic variation of alkalis and alumina, higher alkali content and pronounced depletion of FeO' , MgO , TiO_2 and P_2O_5 with increasing differentiation are typical geochemical characters of these alkali granites. In the $\log_{10}\text{K}_2\text{O}/\text{MgO}$ vs. SiO_2 diagram (Rogers and Greenberg 1981), the plots of the granites fall mainly in the field delineated for alkali granites. Petrochemical features suggest that the alkali granites were derived from partial melts generated at mantle depths as a probable response to crustal distension and mantle degassing prior to the rifting of the continent (Santosh and Nair 1983b; Nair and Santosh 1984).

3. Abundance of Th, U and K

Fresh representative samples of the alkali granites were collected, mainly from working quarries. Seven samples from Chengannoor, ten from Ambalavayal, five from Munnar and twelve from Peralimala were selected for the present study. The samples were crushed and pulverized to -200 ASTM mesh and were analysed for equivalent U and Th using a ECIL single channel analyser (sc 604 B) coupled to a NaI (Tl) detector and calibrated using natural standards. K was analysed by flame photometry.

The analytical results are given in table 3, along with Th/U ratios, $\text{Mg}/\text{Mg} + \text{Fe}'$ levels, SiO_2 values and mean and standard deviation for each pluton. Th, U and Th/U levels of the alkali granites are compared with those of similar granites from other regions in table 4.

3.1 Thorium

The mean thorium values of Chengannoor (41.4 ppm), Ambalavayal (21.6 ppm) and Munnar (23.8 ppm) are considerably higher than the average for granites (18 ppm; Rogers and Adams 1969). However, they are closely comparable with the generally high Th values exhibited by alkali granites of other regions as compiled in table 4. Rogers *et al* (1978) observe that Pan-African granites are Th-rich. In this regard, it is significant to mention that the beach placers in the western coastal tract of Kerala are rich in Th-bearing minerals, the probable provenances of which are the granite and syenite bodies as well as the rare metal and rare earth bearing pegmatites of the region (Santosh 1984; Soman *et al* 1983). This observation is substantiated by the accessory mineral assemblages in the plutons. Also, the ages reported for monazite (600×10^6 yr; Holmes 1955) and zircon (700×10^6 yr; Parthasarathy and Sankardas 1976) from the beach sands are correlatable with the ages of the plutons. Th shows inhomogeneous distribution in each pluton, with large standard deviation (SD) values. Thus, Th levels in Chengannoor show a range of 7–130 ppm (SD = 42.4), in Ambalavayal from 10 to 41 ppm (SD = 10.96), in Munnar from 6 to 42 ppm (SD = 17.01) and in Peralimala from 3 to 25 ppm (SD = 6.9).

3.2 Uranium

The mean uranium contents of Chengannoor (10.29 ppm), Ambalavayal (9.7 ppm), Munnar (8 ppm) and Peralimala (8.08 ppm) are higher than for usual granite (4 ppm,

Table 3. Radioelement abundance and other salient geochemical characters of the alkali granites of Kerala.

	Chengannoor										Mean	SD
	CR-1	CR-2	CR-4	CR-5	CR-6	CR-7	CR-8					
Th	7	11	26	20	130	53	41	42.40	41.14			
U	2	3	4	3	31	18	11	10.29	10.83			
Th/U	3.5	3.7	6.5	6.7	4.2	2.9	3.6	4.44	1.52			
K	3.65	2.64	3.41	2.71	2.71	5.02	3.58	3.39	0.84			
SiO ₂	66.13	71.21	70.89	68.42	69.80	68.17	73.20	69.69	2.33			
Mg/Mg + Fe ^t	0.31	0.28	0.35	0.25	0.27	0.22	0.23	0.27	0.05			
	Ambalavayal										Mean	SD
	AL-1	AL-2	AL-3	AL-6	AL-10	AL-14	AL-15	AL-23	AL-25	AL-35		
Th	15	11	16	10	19	33	41	36	16	19	21.6	10.96
U	8	8	8	4	4	24	6	15	6	14	9.7	6.25
Th/U	1.9	1.4	2	3.5	4.8	1.4	6.8	2.4	2.7	1.4	2.73	1.75
K	3.79	3.17	3.48	4.63	3.76	3.89	3.27	3.77	3.11	3.02	3.59	0.49
SiO ₂	72.46	73.88	73.17	74.03	73.58	72.87	74.67	74.52	71.30	73.86	73.43	1.02
Mg/Mg + Fe ^t	0.14	0.09	0.08	0.18	0.14	0.10	0.07	0.08	0.13	0.13	0.11	0.15

Peralimala

	PM-1	PM-2	PM-3	PM-4	PM-5	PM-6	PM-7	PM-8	PM-9	PM-10	PM-11	PM-13	Mean	SD
Th	6	9	13	25	9	7	21	5	5	4	7	3	9.5	6.9
U	5	15	3	8	13	7	2	7	10	10	7	10	8.8	3.8
Th/U	1.2	0.6	4.3	3.1	0.69	1.0	10.5	0.7	0.5	0.4	1.0	0.3	2.02	2.93
K	8.57	8.69	8.92	7.48	8.98	8.12	8.87	7.75	8.58	8.98	6.0	9.10	8.34	0.90
SiO ₂	63.71	64.55	64.29	63.96	66.10	63.48	65.32	63.09	63.30	66.24	63.46	66.86	64.53	1.29
Mg/Mg + Fe ⁺	0.24	0.23	0.22	0.28	0.15	0.25	0.11	0.27	0.26	0.17	0.31	0.13	0.22	0.06

Munnar

	MR-4	MR-28	MR-34	MR-35	MR-36	Mean	SD
Th	42	42	13	16	6	23.8	17.01
U	3	7	4	14	12	8.0	4.85
Th/U	14	6	3.3	1.1	0.5	4.98	5.49
K	4.79	3.83	5.19	5.27	4.05	4.61	0.64
SiO ₂	71.47	71.18	72.72	71.35	71.43	71.68	0.70
Mg/Mg + Fe ⁺	0.22	0.22	0.17	0.43	0.24	0.26	0.10

Major element data of Ambalavayal granite from Santosh (1983, unpublished data) and other granites from references listed under table 2. SD-standard deviation.

Table 4. Th, U and Th/U values of the alkali granites of Kerala region compared with those from other regions.

	CR	AL	MR	PM	P-2	P-7	S-6013	S-6015
Th	42.40	21.60	23.80	6.90	10.0	32.0	21.20	50.0
U	10.29	9.70	8.0	3.80	5.0	6.30	3.90	5.50
Th/U	4.44	2.73	4.98	2.93	2.0	5.08	5.44	9.09

	SA	HR	116A	34R/B	28B	NANTB
Th	13.0	14.0	12.0	23.0	9.0	27.0
U	4.0	4.0	3.0	3.0	2.0	4.0
Th/U	3.25	3.50	4.0	7.67	4.5	6.75

CR—Chengannoor; AL—Ambalavayal; MR—Munnar; PM—Peralimala; P-2 and P-7—Chamundi granite (Reddy *et al* 1983); S-6013 and S-6015—Arsikere granite (Reddy *et al* 1983); SA—average of alkali granites of Arabian Shield (Drysdall *et al* 1984); HR—alkali granites of central Hijaz region, Arabia (Jackson *et al* 1984); 116A, 34 R/B and 28 B—alkali granites of Labrador (Collerson 1982); NANTB—alkali granites of Lleyln Peninsula, North Wales (Croudace 1982).

Rogers and Adams 1969). Wilson and Akerblom (1982) observe that high U contents are characteristic of alkali granites. This is substantiated by the general correlation between U values of the alkali granites with similar granites from other regions (table 4). Uranium also shows inhomogeneous distribution like Th, yielding high sd values like 10.83 for Chengannoor and 6.25 for Ambalavayal. In Munnar and Peralimala, the inhomogeneity is less pronounced (sd = 4.85 and 3.80 respectively).

3.3 Th/U ratio

Even though Th and U show wide variation in their distribution (figure 3), the average Th/U ratios lie in a limited range of 1.52 to 5.49, comparable with similar values for alkali granites elsewhere (table 4). The highest variation is exhibited by Munnar with Th/U ranging from 0.5 to 14, whereas Chengannoor shows the least variation (sd = 1.52). In general, alkali granites show higher Th/U ratios as compared to normal granites, where the average is only 4 (Rogers and Adams 1969).

3.4 Potassium

The average K contents of Ambalavayal (3.59%), Munnar (4.6%) and Peralimala (8.34%) are higher than the Clarke of 3.6% for normal granites (Rogers and Adams 1969), but comparable with the high K-content of alkali granites in other regions (Rogers and Greenberg 1981). The K-rich nature is a common characteristic of the alkaline plutons in the Kerala region, which, together with the trace element characters

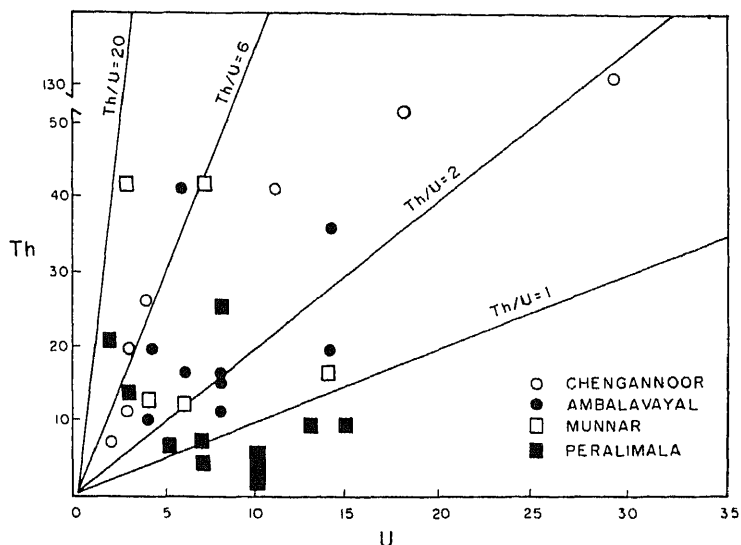


Figure 3. Th vs U plots.

or upper mantle source (*cf.* Nair and Santosh 1984; Santosh and Thara 1985). There is a consistent homogeneity in the distribution of K in individual plutons yielding low standard deviation values (0.49–0.90).

4. Discussion

Plots of Th, U and Th/U with respect to SiO_2 (figure 4) and K (figure 5) indicate a poor correlation. Studies by other workers (e.g., O'Connor *et al* 1982) have also shown weak correlation between U and Th with these major elements. However, plots against $\text{Mg}/\text{Mg} + \text{Fe}'$ indices (figure 6) exhibit definable trends of variation. Th and U tend to concentrate (except the Th depletion in Munnar) towards lower $\text{Mg}/\text{Mg} + \text{Fe}'$ values, consistent with an enrichment in the radioelement abundance with increasing fractionation. The trends are similar to those obtained by O'Connor *et al* (1981) for Carrigmore complex. O'Connor *et al* (1982) observe strong correlation of U and Th with FeO and MgO. The dependence in the distribution of radioelements with $\text{Mg}/\text{Mg} + \text{Fe}'$ relates to the higher modal content of hornblende, biotite and other accessories in these alkali granites and testifies to the significant role played by ferromagnesium minerals in the fractionation processes (*cf.* Nair *et al* 1983). Inter-element correlation and fission-track results presented by various workers (O'Connor *et al* 1981, 1982) indicate that the radioactive elements are mainly bound in restite accessory phases like zircon, sphene, apatite, allanite and monazite. The ubiquitous occurrence of these accessories in the alkali granites possibly account for the higher levels of Th and U. This is further established by the correlation between the averaged total of accessories and the average abundances of U and Th, signifying that the U and Th in these granites are mainly locked up in the accessory minerals. Higher Th-content as compared to U is

(1978) suggest that there is a preferential release of U relative to Th from the mantle into the derived melt during partial melting. U is released from the mantle not only into the magma but also directly into the volatile phases. Petrochemical and tectonic studies of the alkaline plutons suggest their derivation from partial melts generated in the mantle (Santosh and Thara 1985). An intrinsic association of volatiles with the petrogenesis of these plutons is also suggested (Nair and Santosh 1984). Fluid inclusion studies on the alkaline plutons (Santosh 1985b) and related ore mineralization (Santosh 1984) also indicate that volatiles played a significant role. A taphrogenic control over the generation and emplacement of the alkali granites with the fault-lineaments acting as conduits for volatile transfer from the upper mantle is also deduced (Santosh and Nair 1983b; Santosh 1985b) This together with the common occurrence of Th-bearing

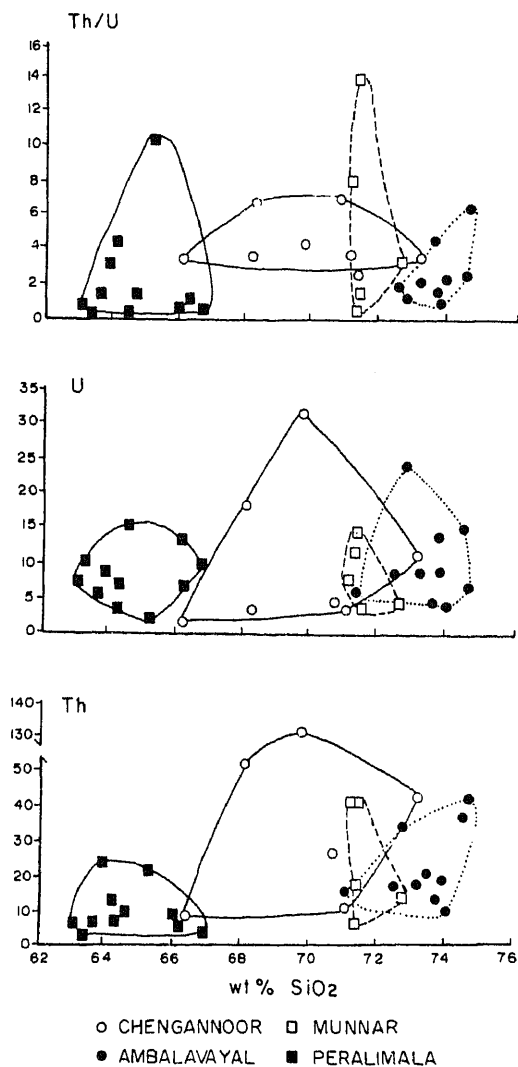


Figure 4. Th, U and Th/U vs SiO₂ plots of the alkali granites.

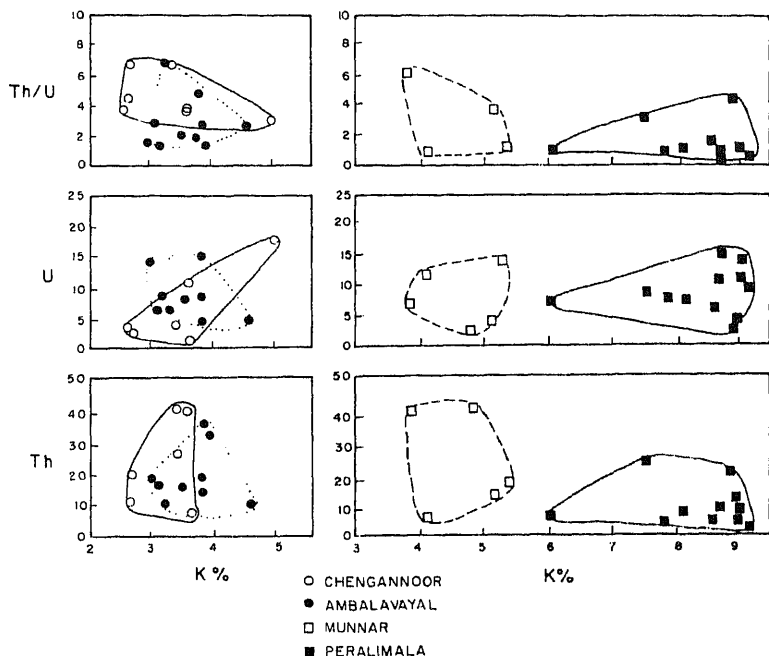


Figure 5. Th, U and Th/U vs K plots of the alkali granites.

minerals in the beach placers promise scope for detailed radioelement exploration in the alkali granites. Above all, their similarity to Pan-African alkali granites elsewhere is also significant as investigations have shown that Pan-African provinces are favourable tectonic belts for radioelement mineralization (Ragland and Rogers 1980).

5. Conclusion

The alkali granites of the Kerala region show higher Th, U and Th/U contents as compared to normal granites, but are closely comparable with similar alkaline plutons from adjacent terranes and elsewhere. The nature of distribution of radioelements testifies to their correlation with the ferromagnesium minerals and accessory phases. There is a general trend of concentration of Th and U and decrease in Th/U ratio with progressive fractionation. Their Pan-African tectonic setting, alkaline nature and higher Th/U levels, together with the findings of related rare metal pegmatites and the occurrence of Th-rich beach placers in the region are encouraging factors for detailed radioelement exploration in these alkali granites.

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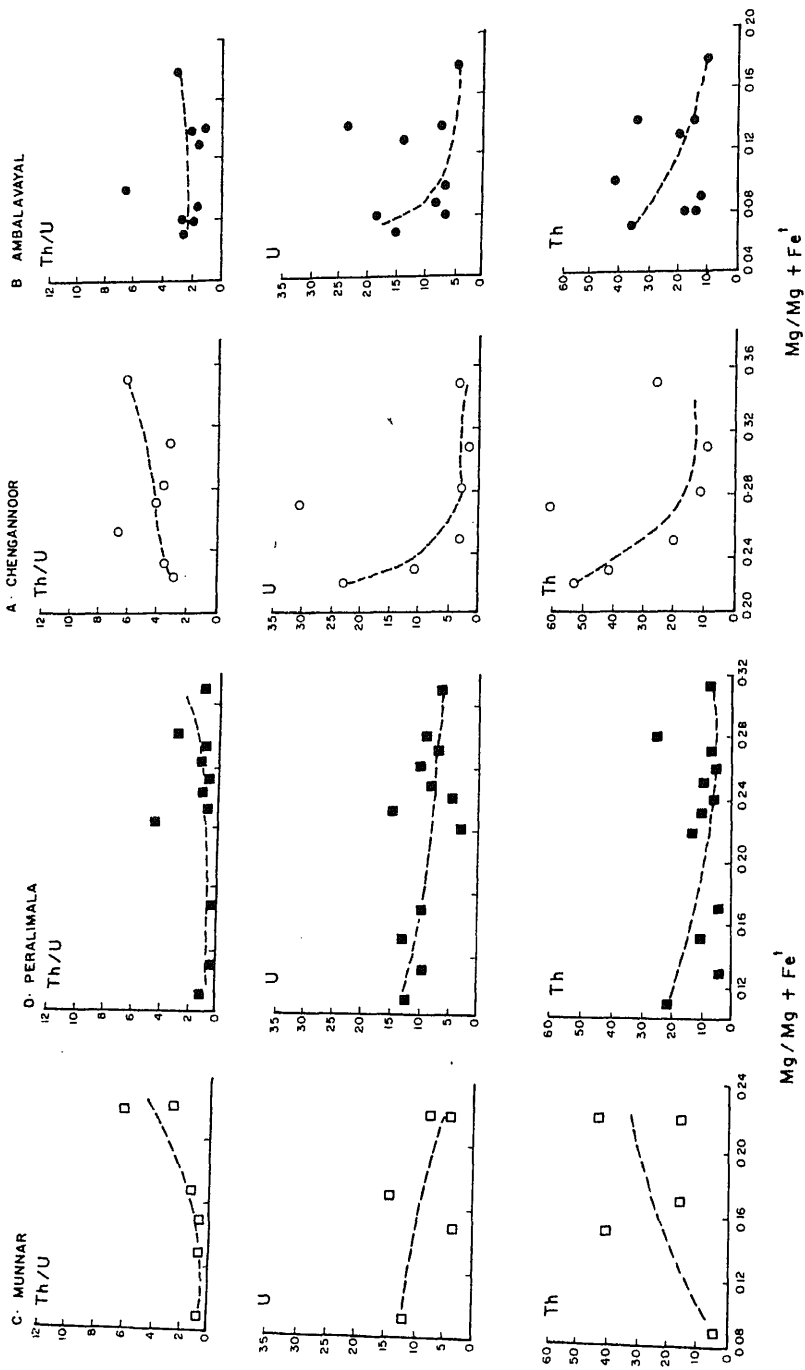


Figure 6. Th, U and Th/U vs $Mg/Mg + Fe'$ plots of the alkali granites. Note that the felsic end is towards the left (decreasing $Mg/Mg + Fe'$ levels).

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The petrology and geochemistry of massif anorthosites and gabbros of the Bavali fault zone, North Kerala

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Abstract. The massif anorthosites and gabbros occurring in the north Kerala region are localised along the Bavali fault. Several other bodies of granophyres and syenites also occupy this fault. All these bodies are considered to be a continuous graduating igneous series. This paper aims at delineating the field relations, petrochemistry and origin of massif anorthosites and gabbros occurring in the vicinity of the fault.

Anorthosites (An 70-55) are nonlayered and highly deformed with no igneous structure present. Gabbros show relict poikilitic texture with a mineralogy of plagioclase, clinopyroxene, hornblende and biotite. Geochemical study suggests dominantly calc-alkaline characteristics for both anorthosites and gabbros. However, their chemical variation is not transitional to indicate evolution by continuous differentiation from a single parent magma. The rock types seem to be chemically unrelated and appear to have evolved independently from discrete magma sources and the magma migration might have only been promoted by the Bavali lineament.

Keywords. Anorthosite; gabbro; Bavali fault; major element; trace element; magma source; differentiation.

1. Introduction

The massif type anorthosites are generally of Proterozoic age (Isachsen 1969; Bridgwater and Windley 1973). The classic examples are the Adirondacks of New York State and the Nain complex of Labrador. Several bodies of massif anorthosites are known from India (De Anirudda 1969). The nature of parental magma, tectonic milieu and restricted time of emplacement of anorthosites, have become important problems to petrologists interested in the study of Precambrian crustal evolution (Wiebe 1980).

In the north Kerala region widespread occurrence of massif anorthosite, gabbro, syenite and granophyre represent important magmatic activities. They are localised along a major fault, also known as the Bavali lineament. The Bavali lineament is a westward extension of the Moyar lineament. Regular alignment of the above igneous plutons along the Bavali lineament is suggested as being due to differentiation and emplacement from a single parent magma. The geochemistry of granophyre pluton (figure 1, area 1) indicates that it is a shallow level fractionate showing a range of composition from tonalite to granite (Sinha-Roy and Ravindra Kumar 1986). Syenite (figure 2, area 2) is ultrapotassic ($K_2O \approx 13\%$), probably generated from peridotitic parent magma at mantle depths (Ravindra Kumar and Sinha-Roy 1986). The geochemistry of massif anorthosites and gabbros were not known. In this paper the field relations and petrochemistry of anorthosites and gabbros are presented and the nature of these relations and the possible mode of origin has been discussed. The chemistry of the gabbros of Adakkatodu (figure 1, area 4) and Kartikulam (figure 1,

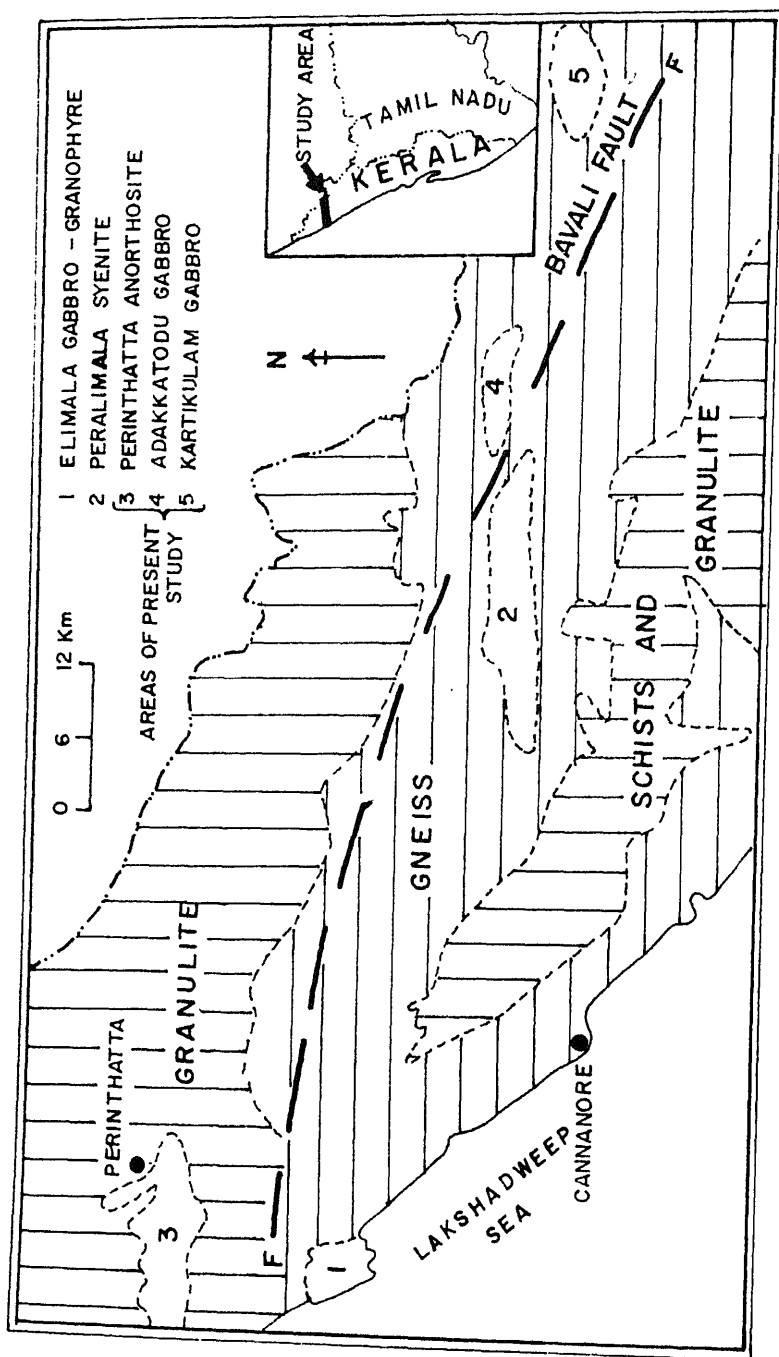


Figure 1. General geological map of the Wynad Schist belt showing the regional distribution of gabbro-anorthosite-syenite and granulite bodies (after Nair and Vidyadharan 1982). The inset shows the location of the study area.

area 5) have been treated together based on similarities in field relations and petrography.

2. Geological setting

The anorthosite and gabbro bodies are seen within the linear Wynad belt composed of rocks of the Precambrian age. The belt has all the characteristics of a Precambrian polymetamorphic and polydeformed terrain with rock types like metasedimentaries, charnockite-granulites and a variety of gneisses. The rocks can be divided into (i) the basement group composed of migmatitic banded hornblende-biotite gneiss, (ii) shelf facies sedimentary group with quartzite-pelite, (iii) mafic and ultramafic group comprising of amphibolites and ultramafites and, (iv) igneous plutons, represented by gabbro-anorthosite-syenite-granite indicating a post-kinematic alkaline and calc-alkaline magmatism. A complex of a minimum of three deformation phases have been suffered by these rock types. The mineral assemblages of the rock types reflect a varying greenschist facies to upper amphibolite facies and at places granulite facies metamorphism. This change is conspicuous from west to east of the belt.

Figure 1 gives a schematic representation of the rock types. The distribution of the gabbro-anorthosite-syenite and granophyre bodies along the fault indicates some probable control of magmatic activity in the restricted vicinity of the fault. The anorthosite body is seen to expose in rough elliptical outline to the northwest of the fault. Gabbros are exposed all along the fault, predominanting towards the east. The host rock gneisses include at places complexly deformed and metamorphosed schists. The map (figure 1) also shows the irregular outline of the anorthosite body within the granulites. Contact relations are not displayed. Extension of the anorthosites is difficult to mark due to the thick mantle of laterite. Vidyadharan *et al* (1977), who first identified the anorthosite body, have suggested by well section studies a total area coverage of more than 50 km² for anorthosites.

The anorthosites are nonlayered and highly deformed with very coarse grained plagioclase laths measuring from 2 to 5 cm. They are well exposed near Perinthatta and in adjoining areas. Recognisable primary igneous structures are not present. Plagioclase laths measuring upto 7 × 1.5 cm are noticed. A crude alignment of plagioclase laths in a general E-W direction is observed. Regional dips in the surrounding rocks are observed towards the anorthosite body suggesting that the anorthosite was emplaced into a synform structure (Vidyadharan *et al* 1977). Several bodies of hornblendites and gabbros occur within the anorthosite massif. Field relations between these rocks and anorthosites are not clear as contacts are not preserved. Small bodies of gabbroic anorthosites and gabbro occur away from the anorthosite massif. Vidyadharan *et al* (1977) recorded this as a case of zonation. Anorthosites at a few places show the presence of pseudotachylite veins. This probably suggests extreme grinding of the rock, later during deformation. In several places, tongues of anorthosites project into the granulites producing hybrid rocks in the immediate contact. Shearing is a common phenomenon in these contacts. Bands of pegmatite with predominant potassic feldspar crystals traverse the rock types in a random orientation.

The gabbro bodies occur roughly in an ellipsoidal shape, generally oriented in an

anorthosite, huge plutons are seen away from the anorthosite (figure 1), measuring over 50 km². Volume-wise gabbro dominates over anorthosite. To the west-southwest of the gabbro, a unique alkali syenite (K₂O ~ 13 %) (Ravindra Kumar and Sinha-Roy, 1986) is seen occupying an area of about 80 km².

Gabbros are mainly exposed in three places, at Perinthatta and Elimala adjacent to anorthosite and granophyres bodies, along the fault around Adakkatodu, and at Kartikulam near Manantoddy. The rock is medium to coarse grained. Foliation is not very prominent but mafics are aligned in an E-W direction. Deformation is very intense and gabbro bodies, specially close to the Bavali fault, have been extensively mylonitised. Pegmatite, quartz and pseudotachylyte veins traverse the rock generally along the weak fracture planes.

3. Petrography

Anorthosites are constituted by coarse crystals of plagioclase (An 70-55), minor clinopyroxene ($Z\Lambda C = 45^\circ$; $2Vr = 60^\circ$), minor orthopyroxene (enstatite) and carbonates (table 1). Plagioclases are well twinned and generally the twinning is very conspicuous to the unaided eye. The plagioclases are bent, fragmented and in a few places granulated. They are randomly arranged. In the marginal zones alteration of plagioclase and carbonates is seen. Augitic clinopyroxene ($Z\Lambda C = 45^\circ$; $2Vr = 60^\circ$) is dark green and feebly pleochroic from yellowish green to dark green, and biotite with shades of dark brown to yellow surround the clinopyroxene. Clinopyroxenes are also studded with magnetites. The petrographic study suggests that only plagioclase and pyroxene are primary, as evidenced by subophitic texture and zoning in feldspar, while hornblende, biotite and carbonates grew at the expenses of primary minerals.

The gabbro is characterised by the dominance of plagioclase (An 45) (~ 60 %), clinopyroxene ($Z\Lambda C = 38^\circ$; $2Vr = 62^\circ$) (30 %-20 %), hornblende (10-5 %), biotite (~ 2 %) with minor magnetite (< 5 %) and ilmenite (table 1). A large number of samples show relict poikilitic texture. With the increase in the modal presence of hornblende gabbro shifts towards diorite composition. A few samples show presence of garnet. Plagioclase (An 50-45) is well twinned. Augitic pyroxene occurs as subhedral grains surrounded by a rim of colourless to olive green hornblendes. Hornblendes are of secondary origin. Occasionally primary hornblendes are seen interlocked with pyroxene and may have a weak rim of secondary yellowish green amphibole and inclusions of plagioclase. Hornblende also occurs as interstitial to plagioclase laths with blebs of anhedral magnetites. In certain deformed varieties, recrystallised secondary quartz granules appear as fine ground mass and surround the plagioclase and other mafics (pyroxene/amphibole).

4. Geochemistry

The major and minor elements in samples from ten anorthosites, one pegmatite and ten gabbros were determined by conventional methods using USGS Sy1 and Sy2 as standards. The analytical error was found to be within the range of 3-5 %. Trace element determinations were performed on Perkin Elmer 5000 atomic absorption spectrophotometer.

Table 1. Average volumetric data of seven anorthosites and eight gabbros.

	Plagioclase	Quartz	Clinopyroxene	Hornblende	Biotite	Opaques
Anorthosite	96	—	2	1	1	1
Gabbro	62	3	18	10	2	3

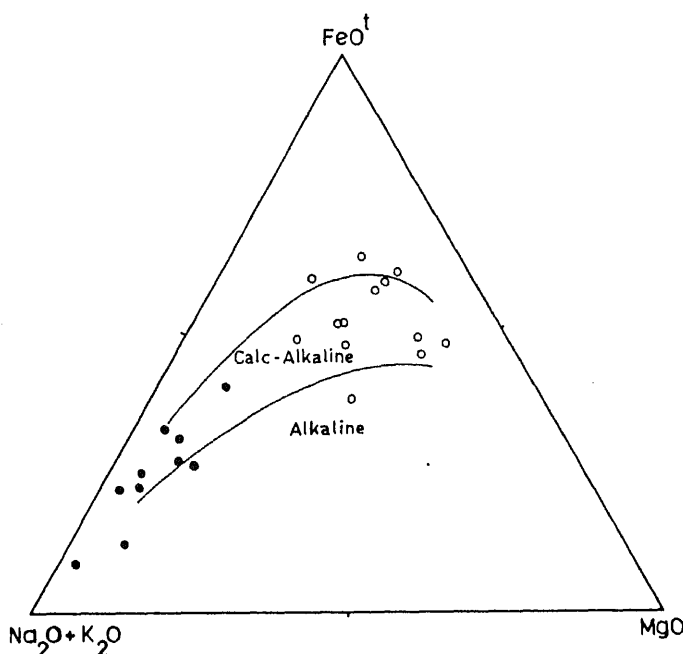


Figure 2. Triangular A-F-M projection for gabbros and anorthosites. Note the separate grouping of anorthosites and gabbros and their dominant calc-alkaline nature. Closed circles: anorthosite; open circles: gabbro.

mean major element abundances of anorthosites and gabbros are compared in table 4 with the average chemical data of other well known bodies.

The major element abundances in anorthosites and gabbros are comparable to the typical massif anorthosites (by $\text{CaO}/\text{Na}_2\text{O}$ and FeO/MgO ratios) and gabbros (cf. table 4). In anorthosites, SiO_2 varies from 51.39 % to 55.88 %, whereas in gabbros, it varies from 44.40 % to 55.39 %. Alumina has a narrow range between 23.23 % and 27.52 % in anorthosites but varies from 11.73 % to 18.35 % in gabbros. Total alkalis ($\text{Na}_2\text{O} + \text{K}_2\text{O}$) vary from 4.94 % to 6.04 % in anorthosites. An increase in total alkalis up to 12.56 % is discernible towards the pegmatitic phase. In gabbros the total alkali content varies from 1.66 % to 5.85 %. The variation of CaO (9.63 to 12.00 %) is subtle in anorthosites, whereas it varies from 6.05 to 12.10 % in gabbros.

The chemical composition of the rocks clearly tend to reflect the mineralogical composition. Norms indicate that anorthosites are slightly oversaturated. Few samples

Table 2. Chemical analyses of anorthositic (GAN 1-10) and pegmatite (GAN 11) of Perinthatta.

GAN	1	2	3	4	5	6	7	8	9	10	11
SiO ₂	54.72	55.88	54.48	52.74	51.39	54.47	53.47	52.69	52.25	52.06	64.67
TiO ₂	0.56	0.36	0.02	0.49	1.26	0.41	0.20	0.64	0.35	0.22	0.17
Al ₂ O ₃	23.23	23.56	23.84	23.23	24.97	26.00	27.52	25.49	26.51	26.00	18.34
Fe ₂ O ₃	1.05	0.72	1.04	0.86	2.96	1.24	0.12	1.90	0.57	0.68	1.16
FeO	1.63	1.36	1.47	1.67	1.43	0.75	0.68	1.16	0.93	0.61	0.18
MnO	0.04	0.03	0.02	0.05	0.04	0.02	0.01	0.03	0.02	0.01	0.02
CaO	10.30	9.93	9.40	12.00	10.30	9.63	10.30	10.08	12.32	10.30	1.12
MgO	0.50	0.42	0.82	1.17	1.12	0.48	0.64	0.48	0.22	1.79	0.48
Na ₂ O	4.80	5.40	5.30	5.50	4.30	5.24	4.70	4.77	4.41	5.23	1.74
K ₂ O	0.14	0.24	0.25	0.12	0.57	0.47	0.67	0.65	0.54	0.01	10.72
P ₂ O ₅	ND	ND	ND	ND	0.10	0.11	0.10	0.08	0.09	0.08	0.06
H ₂ O ⁺	0.23	0.31	0.29	0.19	0.29	0.17	0.12	0.29	0.21	0.20	0.06
H ₂ O ⁻	0.16	0.68	0.72	0.40	0.64	0.37	0.35	0.85	0.66	0.58	0.48
Total	97.43	98.89	97.65	98.48	99.45	99.36	99.08	99.11	99.08	99.37	99.21
Trace elements (ppm)											
Cr	78	160	116	140	94	108	86	81	82	84	75
Cu	46	69	11	14	21	23	9	8	9	31	31
Co	11	11	5	12	84	71	72	81	74	65	15
Pb	2	5	2	20	110	82	40	66	176	166	134
Ni	42	83	55	86	102	95	69	72	51	75	50
Zn	97	172	59	103	175	171	143	161	160	201	170
Mo	ND	ND	ND	Nf	Nf	12	Nf	Nf	12	4	12

V	75	25	Nf	35	8	2	2	4	4	2	6
Ba	190	29	154	23	170	178	186	266	188	266	940
Sr	ND	ND	ND	ND	243	225	217	233	265	250	170
Mg/Mg + Fe ²⁺	0.24	0.25	0.35	0.43	0.33	0.31	0.59	0.23	0.21	0.72	0.41
CaO/MgO	24.60	28.41	13.40	11.97	9.19	20.06	16.09	21.00	56.00	5.75	2.30
CaO/Al ₂ O ₃	0.58	0.55	0.52	0.66	0.41	0.37	0.37	0.39	0.46	0.39	0.06
Ca/Sr	—	—	—	—	302	302	339	309	332	294	47
K/Sr	—	—	—	—	19	17	26	23	17	27	523
K/Ba	6	69	13	43	28	22	30	20	24	25	95
Na/K	30.66	20.12	18.96	40.98	6.87	9.97	6.27	6.56	7.30	5.77	0.15
<i>CIPW Norms</i>											
qz	4.98	2.88	1.92	—	1.02	1.56	0.72	0.96	—	—	10.38
or	1.11	1.67	1.67	0.56	3.34	2.76	3.82	3.89	3.34	5.86	63.38
ab	40.35	45.58	44.54	40.74	37.20	44.02	39.82	40.35	37.20	36.29	14.62
an	41.42	39.29	40.58	38.36	46.70	46.15	50.32	46.15	50.86	45.84	5.50
ne	—	—	—	2.81	—	—	—	—	—	4.19	—
co	—	—	—	—	—	—	0.61	—	—	—	1.84
di	4.97	5.85	5.01	8.23	2.81	8.65	—	2.59	—	4.75	—
wo	—	1.28	—	4.06	—	—	—	0.23	4.29	—	—
hy	—	—	1.30	—	1.50	0.90	2.39	—	—	—	1.2
ol	—	—	—	—	—	—	—	—	0.88	2.08	—
mt	1.62	1.16	1.62	1.16	0.93	1.16	0.23	1.86	0.70	0.23	—
he	—	—	—	—	2.40	0.48	—	0.64	—	0.48	1.12
il	0.84	0.76	—	0.91	2.43	0.76	0.30	1.22	0.60	0.46	0.30
ap	—	—	—	—	0.34	0.34	0.34	—	0.34	—	—

Table 3. Chemical analyses of gabbros (GGB).

GGB	12	13	14	15	16	17	18	19	20	21
SiO ₂	51.52	52.81	49.34	55.39	47.89	47.11	52.41	48.25	46.57	44.40
TiO ₂	1.59	2.09	3.37	1.46	1.57	3.17	0.26	1.93	2.13	3.08
Al ₂ O ₃	18.35	11.73	14.27	16.31	14.78	16.82	15.29	15.29	15.80	16.31
Fe ₂ O ₃	4.90	7.00	6.22	3.41	4.55	5.00	3.74	3.85	2.83	7.49
FeO	4.79	7.64	6.98	4.26	6.83	8.58	4.40	8.33	2.26	6.98
MnO	0.14	0.23	0.18	0.10	0.15	0.16	0.10	0.15	0.14	0.19
CaO	7.84	7.39	6.72	6.05	11.20	7.39	10.08	12.10	10.98	9.63
MgO	4.93	6.72	5.82	6.05	8.16	3.36	6.72	3.84	8.32	6.08
Na ₂ O	3.82	1.78	3.62	3.88	2.84	3.50	1.62	2.18	2.27	3.30
K ₂ O	1.09	0.90	2.23	1.86	0.26	1.96	0.04	0.57	0.98	0.50
P ₂ O ₅	0.23	0.16	0.30	0.20	0.13	0.26	0.05	0.16	0.18	0.32
H ₂ O ⁺	0.15	0.18	0.13	0.21	0.17	0.14	0.17	0.16	0.18	0.28
H ₂ O ⁻	0.11	0.48	0.56	0.45	1.17	2.13	3.87	2.36	1.64	1.03
Total	99.46	99.41	99.76	99.63	99.70	100.58	98.75	99.27	99.28	99.59
<i>Trace elements (ppm)</i>										
Cr	98	505	65	130	409	104	604	259	270	118
Cu	16	52	14	14	38	18	40	43	32	10
Co	77	81	86	72	70	92	52	82	76	80
Pb	100	34	142	180	206	182	266	726	148	172
Ni	55	342	341	84	67	159	114	224	245	95
Zn	242	280	271	219	211	262	226	258	220	243

Mo	6	22	16	8	8	24	—	18	12	10
V	200	240	260	240	260	200	14	240	260	200
Ba	384	418	898	762	262	484	198	198	532	190
Sr	214	211	194	202	213	207	208	191	230	200
Mg/Mg + Fe ⁺	0.45	0.45	0.45	0.60	0.57	0.31	0.61	0.37	5.60	0.44
CaO/MgO	1.59	0.91	0.86	1.00	0.72	2.19	1.50	3.15	1.30	1.58
CaO/Al ₂ O ₃	0.42	0.63	0.47	0.37	0.75	0.43	0.66	0.79	0.69	0.59
K/Sr	42	35	95	76	10	79	2	25	34	21
Ca/Sr	262	250	247	214	376	255	346	451	330	340
K/Ba	24	18	21	28	8	34	2	24	15	22
Na/K	3.12	1.77	1.45	1.87	9.77	1.60	36.21	3.42	2.07	5.90
<i>CIPW norms</i>										
qz	2.46	15.12	—	4.62	—	—	12.6	4.38	—	—
or	6.12	5.04	13.34	11.12	1.67	11.68	—	3.34	5.56	2.70
ab	32.49	15.20	30.39	32.49	24.10	29.34	13.62	18.34	19.39	27.77
an	29.75	21.41	16.12	21.68	26.69	24.46	34.47	50.30	30.02	20.36
co	—	—	—	—	—	—	—	—	—	—
di	8.40	12.22	13.12	6.47	23.32	8.66	13.22	24.61	20.15	13.98
hy	9.1	14.70	9.40	14.40	7.00	5.34	14.70	6.0	3.70	1.76
ol	—	—	0.28	—	5.60	3.73	—	—	9.94	6.46
mt	7.19	11.37	9.05	4.87	6.50	7.19	5.34	5.57	4.18	10.90
he	—	—	—	—	—	—	—	—	—	—
il	3.04	3.95	6.38	2.74	3.04	6.10	0.46	3.65	4.10	5.78
ap	0.67	0.34	0.34	0.34	—	0.67	—	0.34	0.56	0.67

Table 4. Average compositions of anorthosites and gabbros compared.

	1	2	3	4	5	6	7	8
SiO ₂	44.35	45.87	48.51	53.29	49.36	53.52	48.40	49.58
TiO ₂	0.05	0.22	0.31	0.34	0.17	0.45	1.60	1.98
Al ₂ O ₃	31.34	28.62	26.62	26.42	30.32	25.03	16.40	15.50
Fe ₂ O ₃	1.06	1.05	0.66	0.86	0.97	1.11	3.20	4.98
FeO	0.82	2.65	2.55	1.67	1.08	1.17	7.40	6.10
MnO	0.02	0.06	0.14	0.03	0.11	0.03	0.17	0.15
MgO	1.12	3.36	3.38	1.09	1.33	0.76	5.60	6.00
CaO	18.18	14.43	15.21	10.26	13.07	10.46	9.80	8.94
Na ₂ O	1.22	1.80	2.23	4.32	2.72	4.97	2.50	2.88
K ₂ O	0.05	0.39	0.07	0.91	0.35	0.45	0.80	1.03
P ₂ O ₅	0.01	0.02	0.37	0.10	—	—	0.21	0.20
H ₂ O	0.72	0.53	0.28	0.71	—	—	0.70	—
Total	98.95	99.00	100.33	100.00	99.48	97.95	96.78	97.34
MgO/FeO	0.63	0.94	1.32	0.45	1.23	0.64	0.57	0.98
CaO/Na ₂ O	14.90	8.02	6.80	2.37	4.81	2.10	3.92	3.10

1. Average of 3 Sittampundi anorthosites (Subramaniam 1956)
2. Average of 3 Fiskenaesset anorthosites (Windley 1973)
3. Average of 18 Holenarasipur anorthosites (Naqvi and Hussain 1979)
4. Average of 48 Adirondack type anorthosite (Windley 1973)
5. Average of 4 anorthosites of Chilka Lake (Sarkar *et al* 1981)
6. Average of 10 Perinthatta anorthosite (present study)
7. Average of 127 analyses of gabbro (Manson 1967)
8. Average of 10 gabbro (present study).

(36–46%) and anorthite (38–50%) vary considerably. Normative diopside, hypersthene and olivine are present in six, four, and two samples respectively. Gabbro composition vary from quartz (2–19%) and hypersthene (6–14.7%) normative ones to olivine (0.28–9.24%) normative ones. Dominant presence of normative quartz and hypersthene in a majority of anorthosites and gabbros indicate the sub-alkaline composition of the rock types.

The sub-alkaline to calc-alkaline nature of the anorthosites and the dominantly calc-alkaline to slightly tholeiitic nature of gabbros is indicated on the A-F-M diagram (figure 2). The pegmatite plots close to the alkali apex. Two plots of anorthosites cross the calc-alkaline and alkaline dividing line of Irvine and Baragar (1971). Gabbros plot distinctly away from anorthosites and tend to show the presence of a range of chemistry from calc-alkaline to tholeiitic. However, a majority are essentially calc-alkaline. No clear fractionation trend is seen between anorthosites and gabbros. Similar features are demonstrated by gabbros while plotting on Na₂O + K₂O vs SiO₂ diagram (figure 3). The majority of the gabbros plot in the sub-alkaline field but close to the alkaline boundary and fall close to the fields of alkali rock series and high alumina basalt series of Kuno (1966).

To summarise further the characteristics of the composition of anorthosites and gabbros, they have been plotted on a K₂O-Na₂O-CaO diagram (figure 4). Anorthosites cluster close to calcium and show less variation with respect to alkalis indicating dominant participation of plagioclase in anorthosite. Only slight migration of gabbro

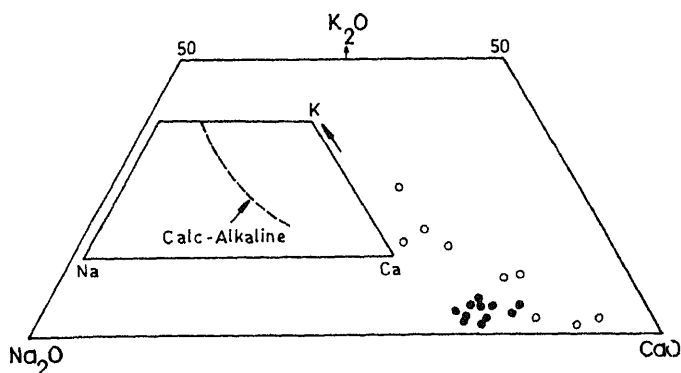


Figure 3. Plot of compositions of gabbro and anorthosite on K_2O - Na_2O - CaO variation diagram. Key as in figure 2. Calc-alkaline trend after Barker and Arth (1976).

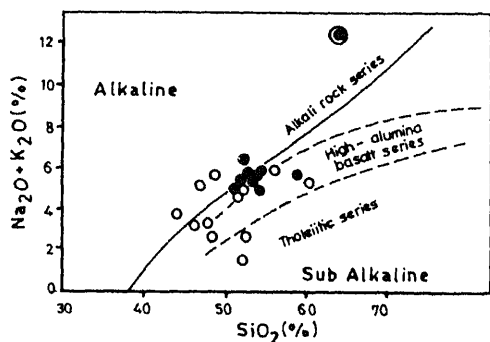


Figure 4. Total alkali versus silica diagram to distinguish alkaline and subalkaline groups (after Irvine and Baragar 1971). Fields of Kuno (1966) for alkali, high alumina basalt and tholeiite series are also shown. Symbols are as in figure 2.

diagram further illustrates that K-feldspar played no role in anorthosites and a very limited role, probably related to deformation and migmatization, in gabbros.

Major and trace element variations in relation to Mg number ($Mg/Mg + Fe^t$) are presented in figure 5. It is possible to identify highly fractionated ones with lower Mg numbers of less than thirty, and also less fractionated ones with higher Mg numbers of sixty in anorthosites and gabbros. The chemistry shows a $Mg/Mg + Fe^t$ ratio averaging around 0.36 for anorthosites and 0.48 for gabbros. Anorthosites and gabbros generally depict different trends of variations for all oxides against $Mg/Mg + Fe^t$ ratio (figure 5), and do not seem to be chemically related to each other. Except for K_2O , a generally smooth variation is observed for all the major oxides. A decrease of Al_2O_3 , TiO_2 and FeO' is observed with increase in $Mg/Mg + Fe^t$. Fractionation of plagioclase and clinopyroxene in anorthosite and in addition, orthopyroxene and magnetite in the gabbros is indicated by the diagram.

In comparison to the trace element content of Chilka Lake anorthosite ($V = 124$ ppm, $Pb < 25$ ppm, $Cr = 64$ ppm, $Co < 10$ ppm; Sarkar *et al* 1981), the Perinthatta anorthosites are depleted in V and enriched in Pb, Cr and Co. This feature

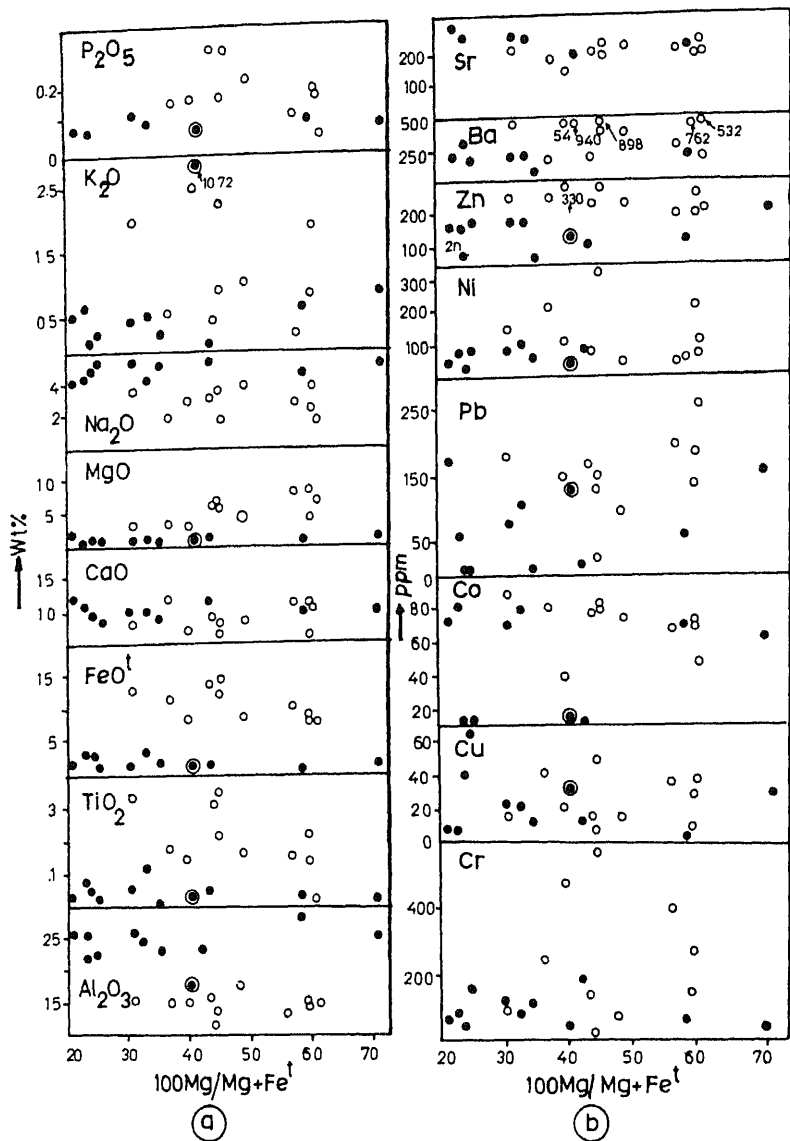


Figure 5. Major (a) and trace element (b) variations with $\text{Mg}/(\text{Mg} + \text{Fe})$. Symbols are as in figure 2.

may perhaps reflect the differences in the abundance of mafic mineralogy between the two massifs as Pb, Cr and Co partition in them. The decrease in V content of anorthosite (average 16 ppm) compared to Chilka Lake anorthosites may be explained by the early separation of the clinopyroxene of the Perinthatta anorthosite at higher temperature, as V will preferentially partition in clinopyroxene only at lower temperatures (Duke 1976). Ba content increases towards the higher feldspathic end of anorthosites. K/Ba also behaves coherently from the higher mafic end to the feldspathic

end. The Sr content does not show any significant variation (217–265 ppm) in anorthosite whereas a slight variation (172–230) is observed in gabbros.

5. Conclusions

In view of studying chemical characteristics of anorthosites and gabbros and to identify possible petrological relations, indicating any continuum in basic magmatic activity, field and geochemical studies were carried out. In many areas field and geochemical studies have indicated gabbroic parentage for anorthosites (Morse 1982). The presence of subhedral crystals of plagioclase in anorthosite suggested that the plagioclases were formed in a relatively calm environment, at depth. Chemistry indicated the calc-alkaline composition of anorthosites and gabbros. The low K_2O content, cumulate plagioclase and the mafics in anorthosite are in favour of a deep environment of origin. This point also disproved the crustal anatexis for the origin of anorthosites and favours the present general consensus that anorthosites are mantle derivatives (Morse 1982).

The geochemistry of anorthosite and gabbro indicate that there is no petrogenetic continuum between anorthosites and gabbros. The absence of continuous smooth variation in several major oxides of the two rocks and presence of different trends of variation further suggests that they might have evolved from discreet magma sources. The ratios of lower $Mg/Mg + Fe'$, CaO/Na_2O and higher Sr content of anorthosites supports the derivation of anorthosite from plagioclase rich parent magma. The chemical features of anorthosites are also in contrast to Archaean layered anorthosites, which have higher ratios of $Mg/Mg + Fe'$, CaO/Na_2O and lower content of Sr (50–100 ppm), suggesting derivation from ultrabasic magma. Invoking the presence of alkaline feldspathic magma as parental may serve to explain the absence of genetically related ultramafic and mafic rock associates with anorthosites and partly the evolution of highly alkaline syenite of Peralimala. Gabbros are presumably derived from a magma of basaltic composition. The regional chemical variations suggest the formation of at least three different magma types, at different levels in the crust and mantle, during the Proterozoic period (Ravindra Kumar 1984).

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Base metal exploration by gravity surveys in a part of the Cuddapah basin (India)—a case study

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Abstract. A closely spaced gravity survey was conducted in the Chikativaripally area in the Cuddapah basin, Andhra Pradesh, India for base metal exploration. Analysis of the data using two-dimensional prismatic models suggest a doubly plunging folded structure in the area and also its associated mineralization.

Keywords. Cuddapah basin; base metal; exploration; gravity; mineralization.

1. Introduction

The Cuddapah basin is one of the most prominent areas in Peninsular India which presents a complicated geological setting. This basin poses many problems to geoscientists regarding economic mineral exploration. The basic geological and geophysical surveys were carried out by a number of workers (King 1872; Glennie 1932; Coulson 1933; Narayanaswamy 1959; Reddy *et al* 1967; Qureshy *et al* 1968; Balakrishna *et al* 1970 and Murthy *et al* 1978). These studies are confined to systematic geological mapping and regional geophysical surveys over this basin.

In the Zangamrajupalle–Varikunta area of the Cuddapah basin lead, zinc mineralization occurs in the shale-dolomitic limestone. Though the mineralized belt extends for over 45 km, reasonable concentration of the ore has been so far observed only at Chikativaripalle–Gollapalle area (Krishnaswamy 1982).

In this context the gravity and *sp* surveys were carried out for base metal exploration, as these methods have become important for delineation of base metal zones in recent years (Turner and Gibb 1979). In particular, the surveys are aimed at identifying major tectonic features in the Chikativaripalle area, which is a potentially mineralized zone. There are certain remarkable features in this area related to cross-folding and en-echelon folding (Narayanaswamy 1971).

This paper presents gravity studies in this area which are compared with *sp* and geochemical data and the model is confirmed from borehole data. An imaginary 3-D model of this area is presented.

2. Geological setting

The area around Chikativaripalle village (14°44'00":78°53'30") comprises a part of the eastern Cuddapah basin with rock formations belonging to the Cumbum formations of the Nallamalai group of the Cuddapah super group. The rock types are carbonaceous shales, dolomitic limestones, green shales, and pink shales (figure 1). The

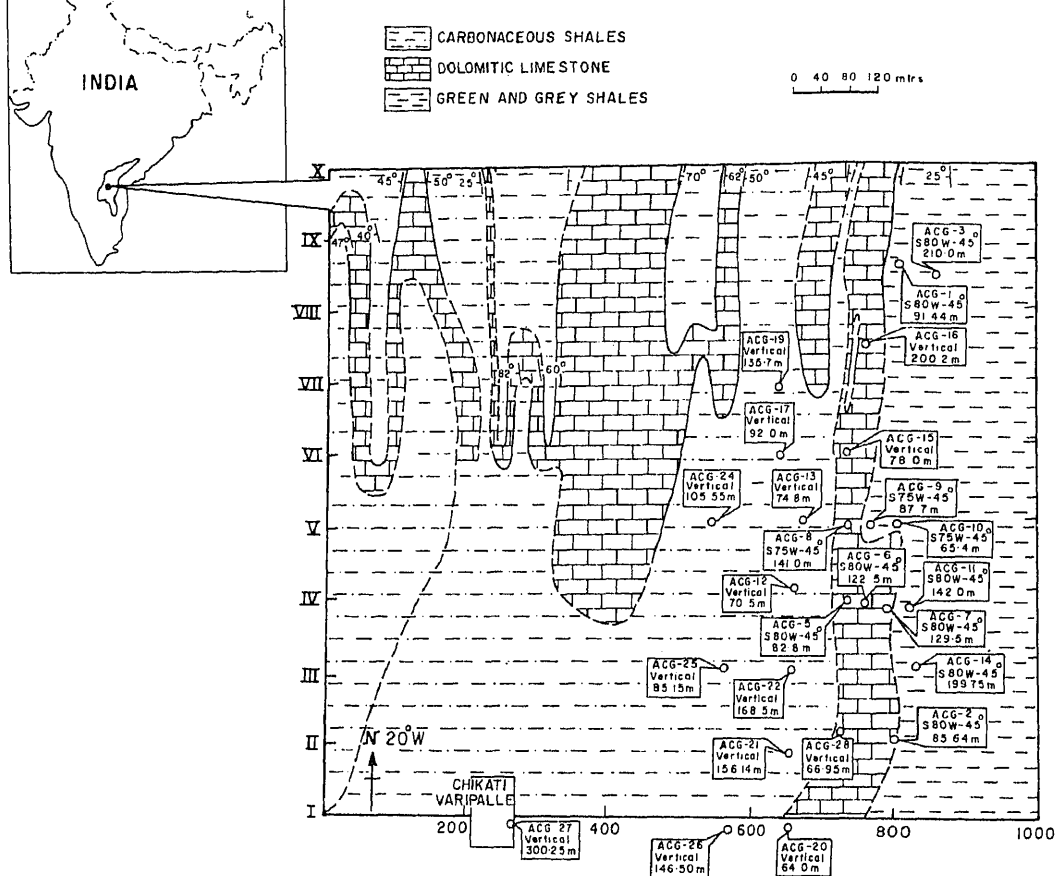


Figure 1. Geological map of the Chikativaripalle area, Cuddapah basin, Andhra Pradesh (Courtesy, GSI)

Chikativaripalle area, where the detailed gravity surveys were carried out, comprises the following stratigraphic sequence (Narayanaswamy 1971 and Krishnaswamy 1982).

- | | |
|------------------------|-----------------------------------|
| Cumbum sub-group | 1. Carbonaceous shales (25–30 m) |
| of the Nallamalai | 2. Massive dolomite (40–50 m) |
| series of the Cuddapah | 3. Green shales (20–60 m) |
| super group | 4. Pink shales ($\approx$ 200 m) |

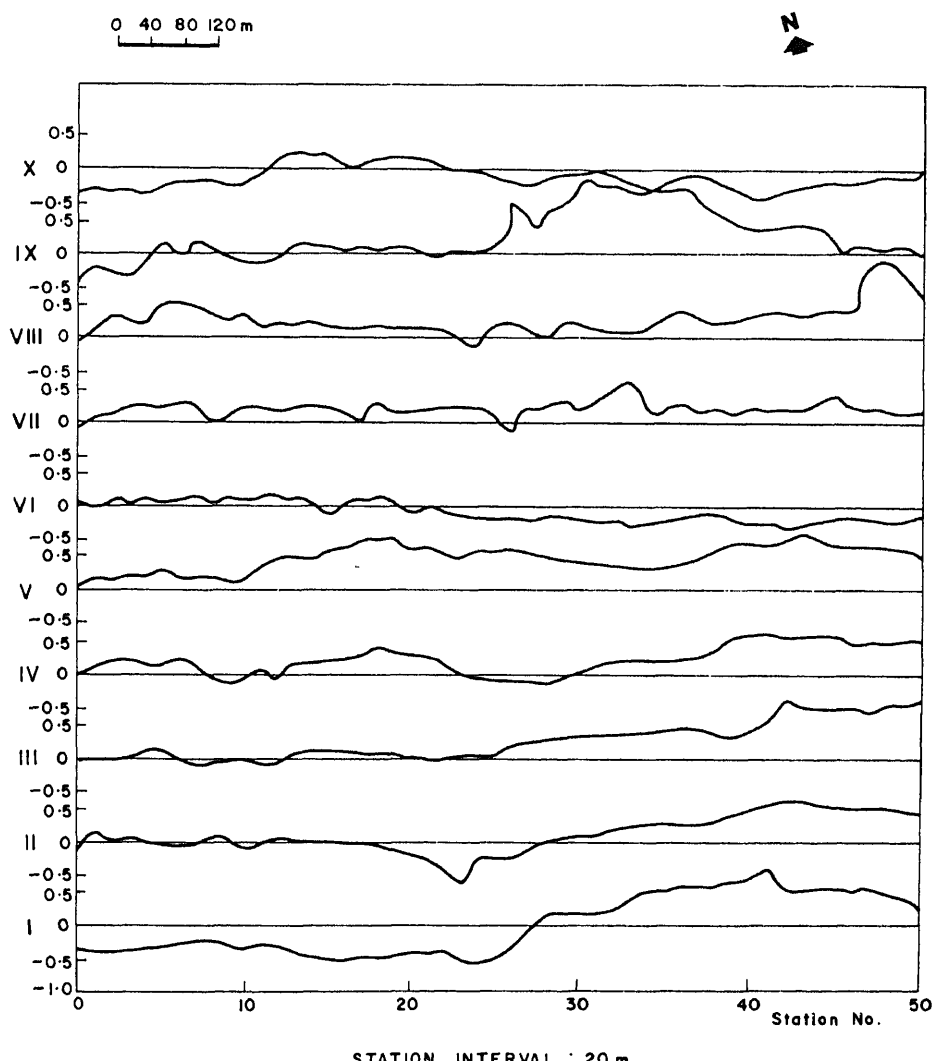
The general strike of the formations of the area is $N10^{\circ}W-S10^{\circ}E$ with minor variations. These foldings have resulted in the frequent repetitions of the beds. The regional plunge is $15^{\circ}-20^{\circ}$ towards south. Most of the folds are asymmetrical and some of them are overturned.

In synclinal structures younger carbonaceous shales and dolomites are present, while the anticlinal portions are represented by green and pink shales. The youngest member of the shale group, namely the overlying carbonaceous shale is present in and around

the Chikativaripalle village. The lead, zinc mineralization is seen as discontinuous stratabound veins or chert zone and copper is found at the base within the dolomite member (Krishnaswamy 1982). The sulphides in this area are found to be syngentic in origin and have clear lithostratigraphic control with subsequent enrichment by remobilization.

3. Data acquisition and analysis

In Chikativaripalle area the detailed gravity and sp surveys were carried out over an area of 1 km². Ten traverses were laid out along N70°E at 100-metre intervals and observations were made at every 20 m. All ten traverses of gravity observations are shown in figures 2, 3, 4 and 5, and the values are tied to a single station situated on



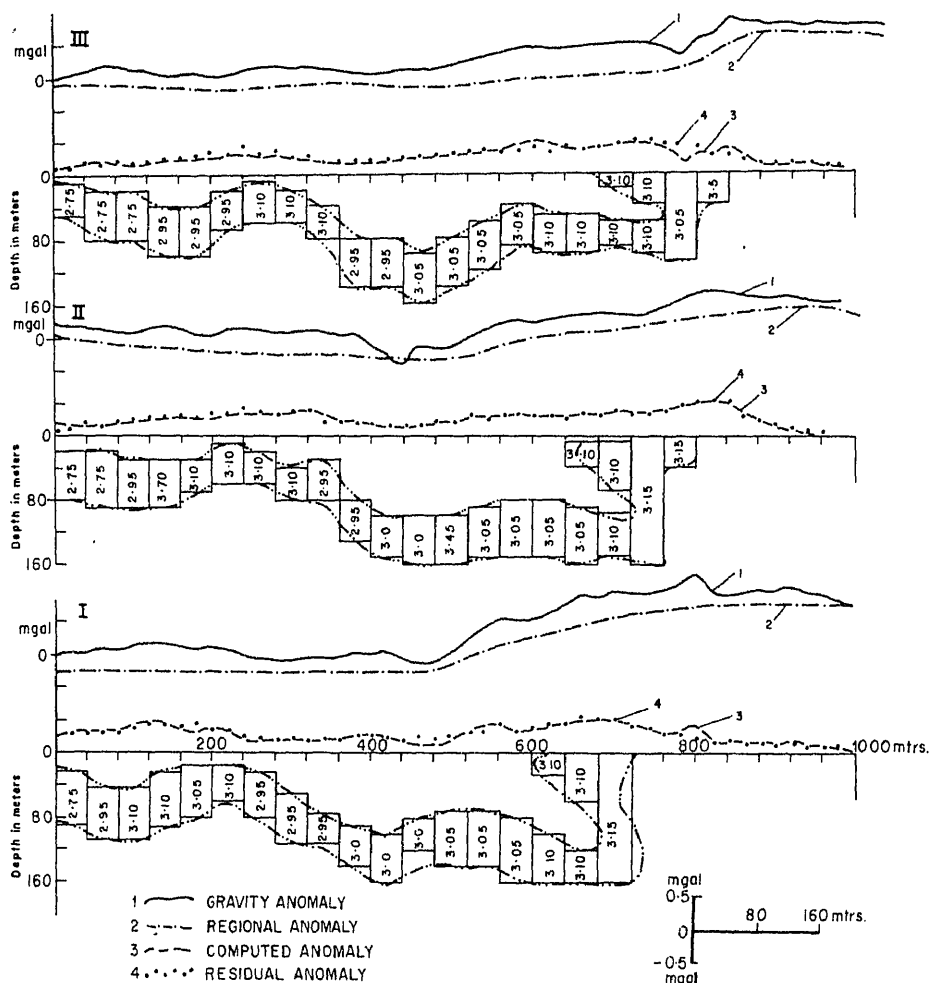


Figure 3. Observed gravity values (relative Bouguer), regional residual and computed residual and inferred gravity model along the traverses (I to III).

dolomitic limestone formation. The contoured $\sigma\rho$ data is presented in figure 7. The accuracy of gravity values is about ± 0.07 mgal, while the accuracy of elevations is about ± 0.5 cm.

The densities of different types of rocks encountered in the surveys were determined by means of a direct reading densitometer (Bhimasankaram *et al* 1975). Density determination on more than 200 samples from the surface; and 100 core samples from different geological formations suggest that the average densities are of 2.65 gm/cm^3 for carbonaceous shales and green shales; 2.78 gm/cm^3 for barren dolomitic limestones and 3.17 gm/cm^3 for mineralized dolomitic limestones. Positive density contrasts of 0.13 gm/cm^3 between shales and dolomitic limestones (barren) and of 0.39 gm/cm^3 between mineralized and barren dolomitic limestones formations exist.

Gravity data of the present detailed work were inverted to get depths to the bottom

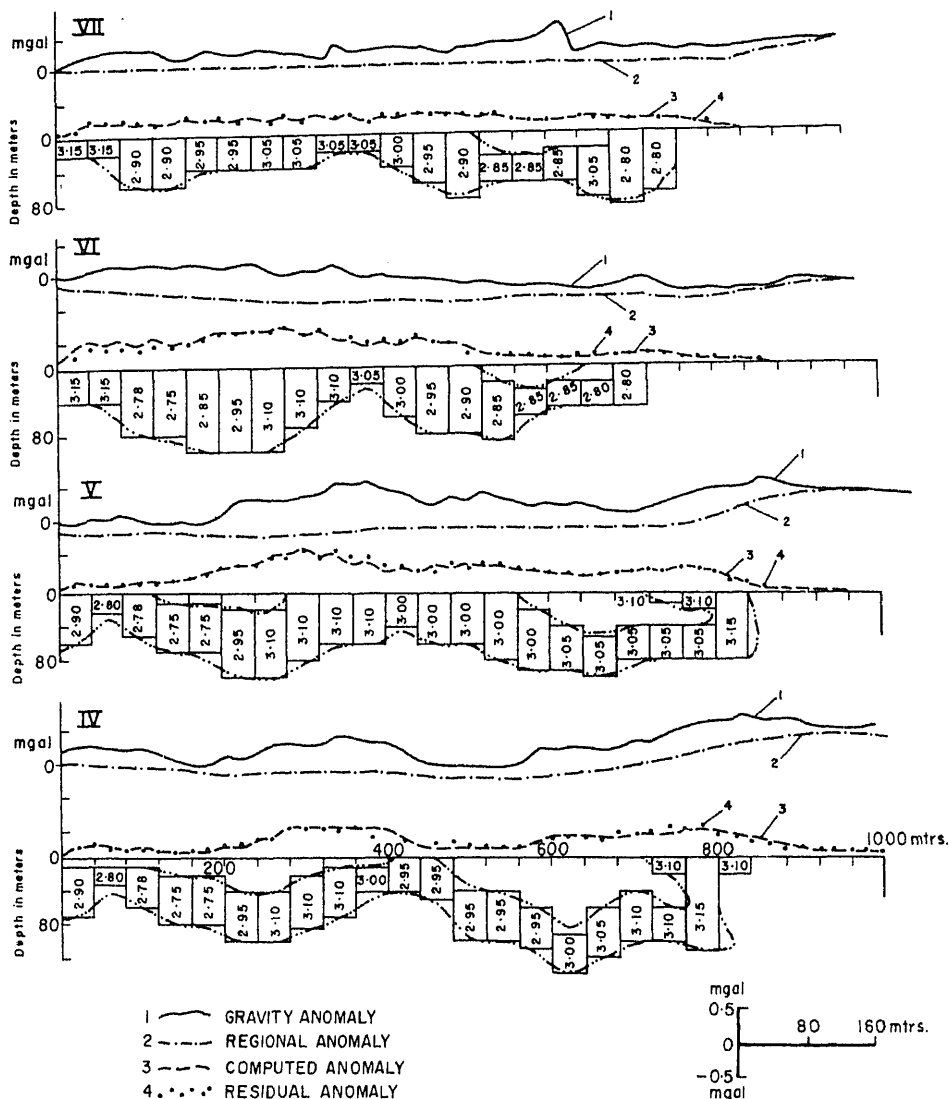


Figure 4. Observed gravity values (relative Bouguer), regional residual and computed residual and inferred gravity model along the traverses (IV to VII).

of the dolomitic limestone horizon. The geology and available borehole data were utilized to fix the regionals along ten detailed gravity traverses. The level of the regionals along all gravity profiles was fitted by assuming uniform conditions. To obtain the residual anomaly along each traverse a graphical smoothening technique was adopted (Rao and Murthy 1978). The resulting regional and residual data are shown in figures 3, 4 and 5. Residual gravity values thus obtained over all the gravity traverses were further considered for gravity interpretation. Since the variation of

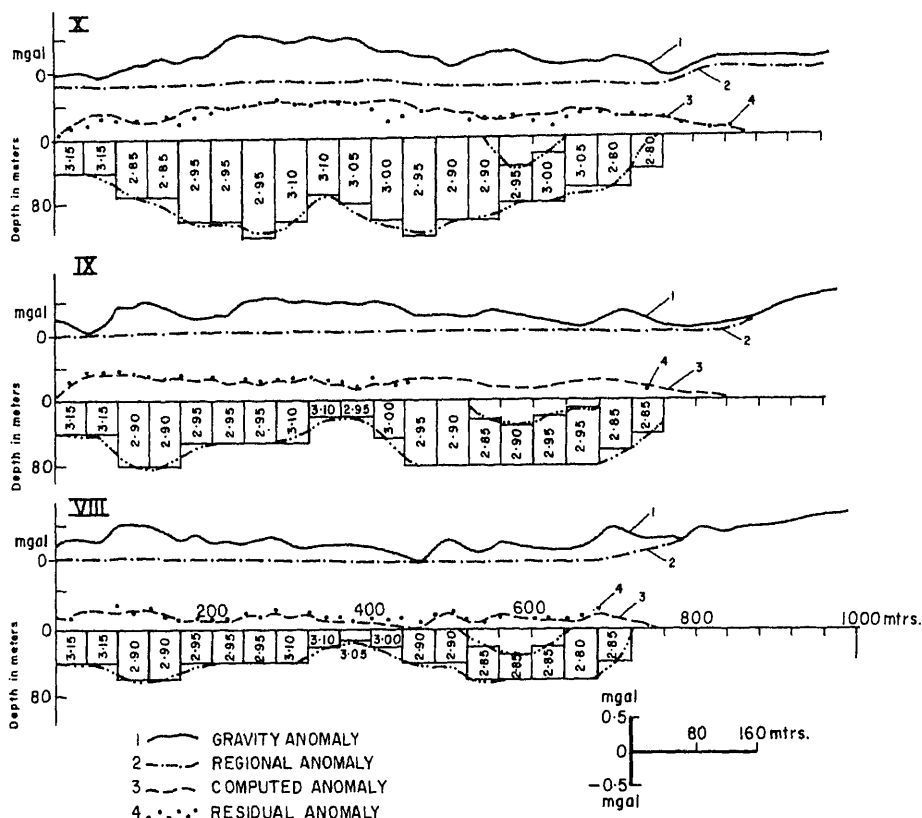


Figure 5. Observed gravity values (relative Bouguer), regional residual and computed residual and inferred gravity model along the traverses (VIII to X).

densities along the traverse is large compared to the variation in the thickness of the dolomitic limestones the quantitative analysis is complex. To overcome this hurdle prismatic models with variable densities are used to get the residual Bouguer anomaly. It may be noted that in the first four profiles the dolomitic limestone formations are not exposed except at the eastern end of the profiles, whereas in the remaining profiles the formations are totally exposed. Borehole data which is available along first nine profiles, helped in selecting suitable models, namely vertical prisms and the density contrast of each prism. The gravity values are computed for the model by the formula given below (Heiland 1968):

$$\Delta g(x) = \sum_{k=1}^N 2G\sigma_k \left[x \cdot \ln \left(\frac{D_k^2 + x^2}{d_k^2 + x^2} \right)^{1/2} - (x - b_k) \cdot \ln \left(\frac{D_k^2 + (x - b_k)^2}{d_k^2 + (x - b_k)^2} \right)^{1/2} \right. \\ \left. + D_k \left(\tan^{-1} \frac{x}{D_k} - \tan^{-1} \frac{(x - b_k)}{D_k} \right) \right. \\ \left. + d_k \left(\tan^{-1} \frac{x}{d_k} - \tan^{-1} \frac{(x - b_k)}{d_k} \right) \right],$$

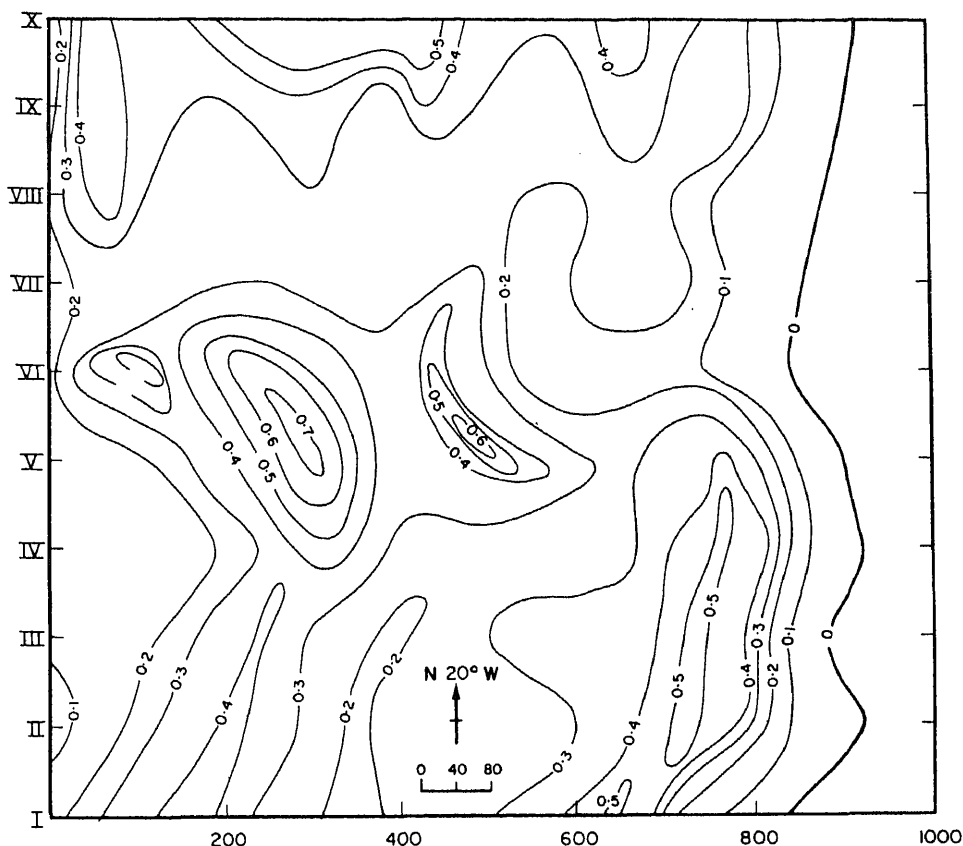


Figure 6. Gravity residual contour map (contour interval 0.1 mgal).

where Δg represents the observed/computed gravity effect, b is half width of the prism, G is the universal gravitational constant $= 6.678 \times 10^{-8}$ cgs units. σ is the density contrast in gm/cm^3 , d is the depth to the top of the prism and D is the depth to the bottom of the prism.

The computed effect due to the prism chosen with different parameters was compared with the residual gravity values within the permissible error in the range of 5–10%. This was repeated by varying the parameters of the models to get the close agreement with the residual data (Nettleton 1976). The same procedure was extended for all the profiles.

To delineate the disseminated nature of the base metals, the residual gravity contours were drawn and is shown in figure 6. The SP anomaly contours (figure 7) corroborate in general the gravity residuals.

4. Identification of tectonic setting using gravity and SP data

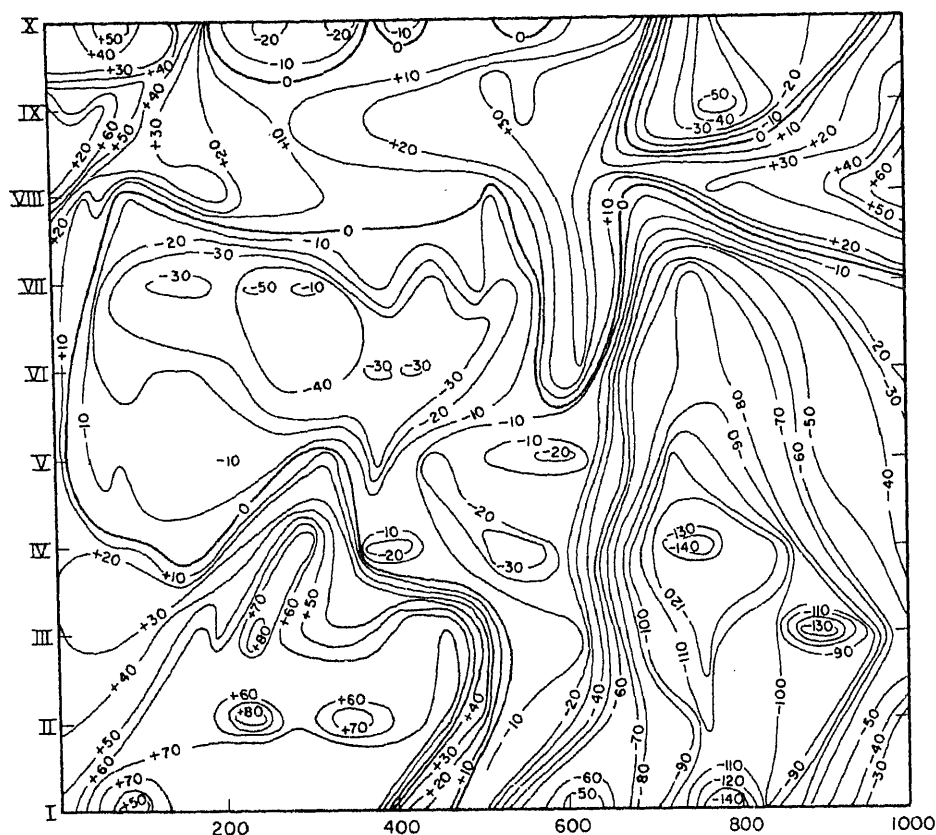


Figure 7. SP contour map (contour interval 10 mv).

eastern side the residual contour seem to indicate that the overturned fold is narrow and fairly straight, which is confirmed from the borehole data. The central region depicts that the overburden effect is relatively high when compared to the eastern zone. The density contrast at the eastern and western ends is relatively higher where mineralization is present. It may be noted that the density of formations overlying and underlying the dolomitic limestone is 2.65 gm/cm^3 as was obtained from core samples.

The overturned fold is significant at the eastern end of the profiles I to V, whereas the intensity of the folding pattern is diminishing towards the north which can be seen on profiles VI to X. The crests and troughs can be seen along the traverses I to IV and along the other traverses the dolomitic limestones are exposed towards the north of the area. From the contour map of the SP data, a low value (-140 mv) can be seen at the eastern end of the first four traverses.

Based on the quantitative estimation of the two dimensional gravity models a model for the entire area of investigation is presented in figure 8. The surface level which is considered as the base station is situated between traverses V and VI. The folding

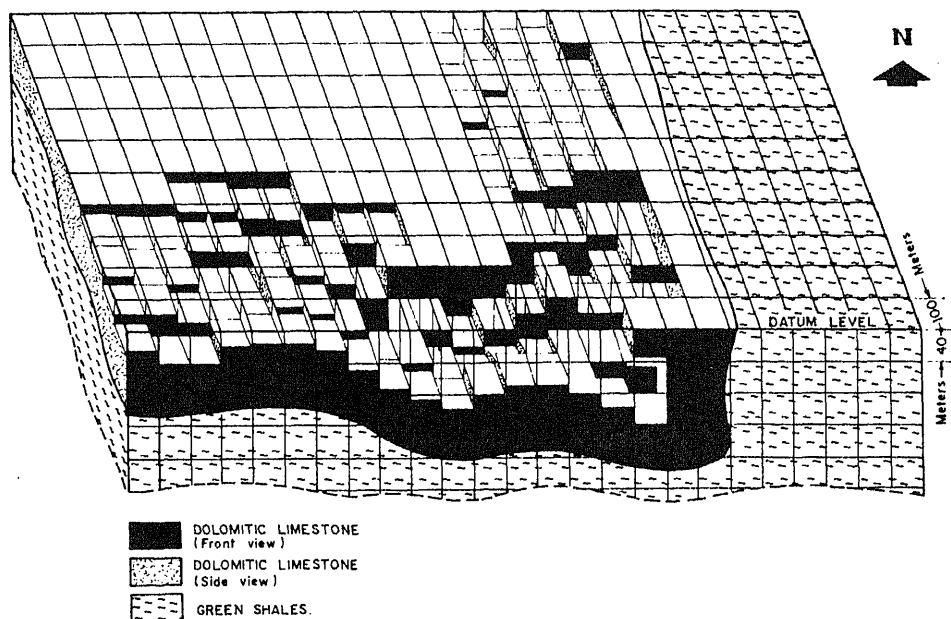


Figure 8. Three-dimensional block model of the Chikativaripalle area (Cuddapah basin), India.

5. Mineralization

The lead, zinc and copper deposits at the Chikativaripalle area are of a disseminated type which are localized in narrow bands of folded dolomites occurring within the shales and along steeply dipping overturned zones at the eastern part of the traverses I–V. The increase in density of the blocks in the anticlinal portions of the dolomite band indicates mineral concentration of disseminated nature.

The *sp* data obtained over a disseminated sulphide ore body in the Chikativaripalle area revealed a negative anomaly of the order of -70 mv to -140 mv. However, the carbonaceous shales which are present in the synclinal portions show positive *sp* values of the order of 10 to 30 mv. The dolomites which are devoid of any mineralization show values close to zero. From this it may be concluded that negative *sp* values are observed only over the mineralized zone in this area. It is evident from the contour map of the *sp* anomaly, that low values of the order of -40 to -140 prevail over the mineralized portions, particularly near the eastern end of the first four traverses.

Also it is revealed from the geochemical analyses that indicated prominent anomalies are due to the presence of sulphides of Cu, Zn and Pb between 720–840 m depth along the profile with the maximum concentration at 780 m (figure 9). The analysis of geochemical data shows that the soil over these zones contains 8–64 ppm Cu, 40–640 ppm Zn and 22–8120 ppm Pb.

The values of all these elements are maximum in the soils over mineralized dolomitic bands. This is the case where bodies are exposed to weathering. In general, Zn content is

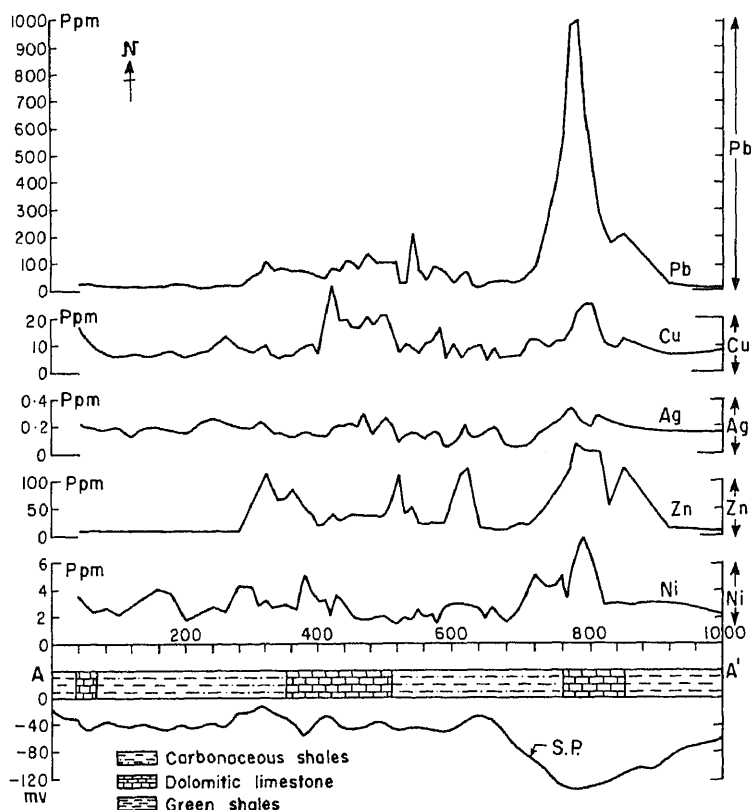


Figure 9. Graph showing variation of Cu, Pb, Zn, Ni and Ag elements along traverse IV.

Zn content is higher in shales than barren dolomites it is mainly due to contamination from the adjacent mineralized zone. Ag content is slightly higher in both mineralized and barren dolomites as compared to shales. The major sulphide minerals of the ore are chalcopryrite, pyrite associated with calcite, sphalerite and galena.

6. Conclusions

From the above analysis, residual gravity data closely agree with the SP and are confirmed by geochemical and borehole data. It may be concluded that the gravity method can be effectively used for indirect detection of sulphide deposits.

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Evolution of the landscapes in the Calcutta-Sundarban coastal areas of South Bengal, India

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Abstract. Studies of the geomorphological features for appraisal of the geomorphic set-up and its related operating processes have been carried out in parts of South Bengal (the Calcutta-Sundarban delta/flood plain region). Different structural features reveal three morphotectonic lineaments in the sediments of South Bengal. Physiographically the area can be divided into three different units from north to south which also corroborates the chronology of the delta progradation. The subsurface geology indicates an upward coarsening sequence of the deltaic environment in a cyclic order. The bore-hole data reveal repeated upheaval and subsidence of the depositional basin (Bengal basin) owing to neotectonic movements. Occurrence of peats and calcretes at different levels is indicative of such horizontal movements and also corroborates the world over eustatic changes in the quaternary period. Scrutiny of the landsat imageries reveals four different zones of geomorphic surfaces. Analyses of the aerial photographs enables a more detailed classification of these zones. Successive strand lines corresponding to the different stages of evolution of the land mass and the landscapes were delineated. Thus, the concentric pattern of a five-tier lobe surfaces on the deltaic plain visibly emerges which is inferred to have formed gradually along with progradation of the delta in the recent past (presumably over the last 5000 years).

Keywords. Delta; flood-plains; Bengal basin; eustatic change; physiographic units; delta-lobe.

1. Introduction

The present geomorphological work was carried out over a section of South Bengal which forms a part of the largest delta of the world—the Gangetic delta (or Ganga delta as it will be called hereinafter). The geomorphic set-up, processes and historical development of the present day landforms of the Calcutta-Sundarban area have been brought to light in this analysis. Since there is a lack of extensive studies of this kind over the present area of this deltaic region, there is much scope in the evaluation of the drainage pattern and its distribution, subsoil condition, siltation and discharge of the major rivers, nature of progradation of the delta front, etc. Such studies may find application in agricultural, industrial, and urban/semiurban development and may have important bearing on the overall land use distribution.

The work was carried out in the laboratory taking recourse to remote sensing methods, i.e., using landsat data obtained through visual interpretation of black and white imageries plotted directly on a 1 : 1 m scale. A map was finally prepared from a mosaic of aerial photographs and also stripwise interpretation of the same. Necessary field checks were carried out to supplement the final conclusion (figure 1).

It will be relevant to recall at this stage the work of Fox (1931), Coulson (1940), Morgan *et al* (1959), Kailasam (1954), Biswas (1959), Furguson (1863), Chatterjee *et al* (1964), Ghosh (1964), and Nivongi (1975). In the view of these authors the present land

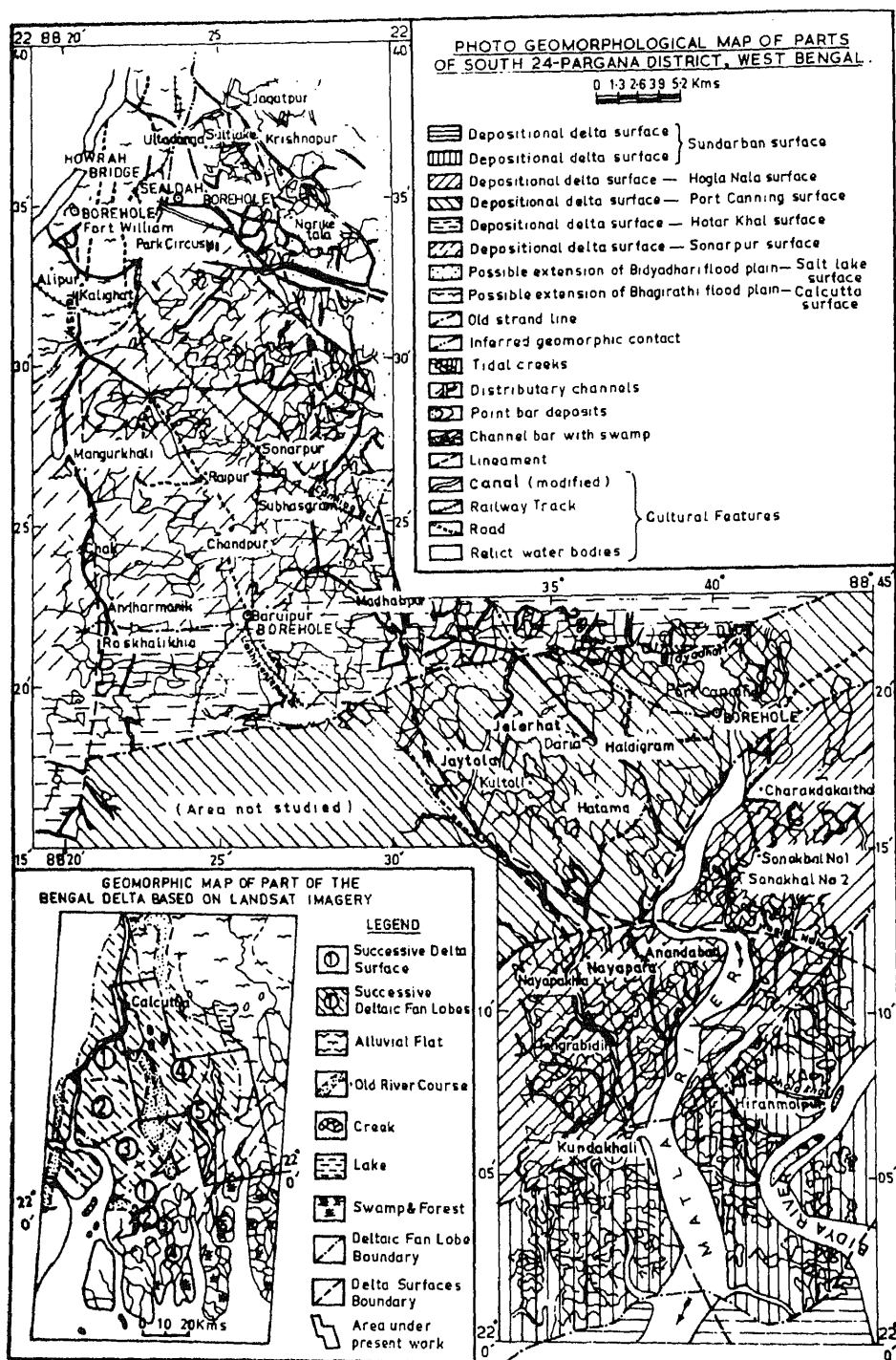


Figure 1. Photo geomorphological map of the area studied. Inset: Geomorphic map of the Bengal delta based on landsat imagery.

mass owes its origin to the action of the river Ganga; and the formation of the present land mass approximately dates as far back as 5000 years B.P. During the last 600 years the main river changed its course between the present Ganga (the Hooghly), Matla, and the Bidyadhari river courses (Fox 1931). The average elevation of the deltaic plain varies between 5.8 to 6.1 metres above the mean sea level with several lowlying depressions; and the digitate distributions in the lower reaches originated from the tidal effects (Biswas 1959). A scrutiny of the drainage pattern reveals the important fact that all the drainages are consequent and parallel on the northwest of the Ganga delta gradually changing to the dendritic and anastomosing type towards southeast (Fox 1931). The Sundarban tidal flats also exhibit growth of an anastomosing creek pattern.

Babu (1976) examined the ancient levees, old channels and morphostructures of a part of the present area from aerial photo studies, to put the old strand line over to the south of Calcutta. Recently, Rakshit (1980) correlated the geomorphic units and stages of development of the Sundarban deltaic plains with the help of older strand line remnants, and subdivided the entire Ganga delta area into four major geomorphic units. Both Babu and Rakshit emphasise the presence of certain tectonic lineaments across the deltaic plains without, however, providing further details.

In the following sections, a comprehensive attempt has been made to reinvestigate the evolution of the land forms and to classify them. Analyses of the sub-surface geology of the area have been put forward to corroborate the findings by remote sensing studies.

2. Geologic and tectonic setting of the area

The deltaic region of South Bengal is covered solely by the Quaternary sediments carried and deposited by the river Ganga and its distributaries. The region is conspicuous for it forms an integral part of the largest delta of the world. The sediments are constituted of the flood plain deposits as well as deltaic deposits, and may be subdivided into two major groups after the Geological Survey of India (1974).

Quaternary —	[	recent to sub-recent — newer alluvium
		Pleistocene — older alluvium

The Pleistocene deposits comprise clay, sand, silt and calcareous nodules (kankar) with assorted boulders, pebbles, and gravels which are locally cemented. These sediments occurring on the surface are characterised by their reddish brown or mottled colour apparently caused by high degree of oxidation.

Recent sediments of the area are represented by sand, silt and clay with assorted boulders and pebbles. This sequence is found to have fine to coarse texture in which silt and clay are dominant. The type lithounits of this class are dark, loosely compacted with high moisture content. The flood plain deposits and deltaic plain deposits have shown strong lithological similarity.

Information on the tectonic features of this thick pile of sediments is rather scant although the structure and tectonism of the Bengal basin evoke curiosity and keen research interests. The problems are many and yet to be resolved satisfactorily. The basin was formed during the late Jurassic period, and major movements seem to have taken place during late Cretaceous, Eocene, and Miocene with regression of the sea during the late Miocene and early Pliocene times. However, Morgan and McIntire

140

(1959) hold that primary faulting has significantly influenced the Quaternary geology of the Bengal basin. They attributed the changes in the course of the Ganga and the Brahmaputra to faulting and consequent tilting of the fault blocks, and summarized that faulting and structural uplift continued into the recent epoch. Consequently the dispositional environment of the Bengal basin changed sharply from a stable shelf to deeper basin conditions over a zone stretching between the Hooghly and the Calcutta-Ranaghat line (Eocene Hinge line). Babu (1976) and Rakshit (1980) indicated the presence and possible identification of some tectonic lineaments across the Bengal coastal sediments. In the present study three distinct morphotectonic lineaments in the area have been identified from aerial photographs and satellite imageries on the bases of tonal differences, sudden break in slope, and alignment of the creeks, rivers and rivulets and old stream channels (figure 1). These lineaments, identified by morphological aspects, remain to be correlated with the geometry, deformation episodes, and tectonic history of the Bengal basin. It is interesting to note the latest informations provided by Nandy *et al* (1983), on the possible extension of the structural episodes of the Tripura-Mizoram Fold Belt below the alluvium of the Ganga delta within adjacent Bangladesh. They depict a relevant overall plate tectonic framework within the present day set-up of the stress field in the eastern part of the Bengal basin, which may also have relevant application in the evolutionary history of the present area.

3. Description of the land forms

Broadly, the area forms part of the vast, lowlying, alluvial and deltaic plains with a very gentle topographic slope towards south and south-east. Depending upon the distinct soil conditions and land form features the whole area may be subdivided into three broad physiographic units, from north to south.

i. East Calcutta and adjoining lowlying riverline plains dominated by marshy bogs, infilled tidal creeks, and partly filled modified remnant distributaries—most of this region, i.e. east Calcutta and Salt Lake area have been reclaimed for urban development.

ii. The Baruipur-Canning, and the adjoining alluvial flats consisting of partly filled, moderately spaced, interlinked tidal creeks and remnant distributaries—the area experiences intensive seasonal cultivation.

iii. The thickly vegetated (mangroove forest) Sundarban marshy flat land at the lowest levels with estuaries and closely interlinked anastomosing tidal inlets—this unit is marked by characteristic features like the tidal creeks or inlets and intertidal flats. The intertidal flats are clad with a dense forest of various types of trees and mangroove elements and the tidal sand bars reflect the alternating environment of deposition through which the coastal emergence took place.

Creek textures are different at different physiographic surfaces. In the Sundarban surfaces the creek texture is compact but towards north, it becomes widely distributed. Within the older surfaces creeks are totally or partly filled up by suspended sediments from the infilled tidal waters or are converted into irrigation canals.

Occurrence of partly in-filled channels, swamps and bogs are prevalent in the Ganga flood plain. The existence of some buried channels across the flood plains have been identified. Landsat imagery indicates a levee extending from south Calcutta to Bakultala which perhaps indicates an ancient channel oriented approximately normal

to the general shore lines. Intermediate discontinuity and shift of this levee indicating intermittent change of the then river courses, may be due to coastal movements or excessive silting.

It will be interesting to note here that a remarkable feature of the Ganga flood plain that has been clearly revealed in the present study is that Calcutta metropolitan development and urbanisation has so far followed the levee of the east bank of the Ganga, gradually extending its reach on the back swamp region towards the far eastern side, i.e. the newly laid East Calcutta Metropolitan By-pass. But surprisingly enough, there is no such distinct levee-backswamp pair along the western bank of the river. The reason for this contrast may be attributed to the ancient conjunction of the river Damodar with the Ganga. A scrutiny of the photo-imageries indicates that heavy bed-load of the ancient Damodar course occurring as a fan destroyed the backswamp on the western bank of the Ganga near Calcutta.

4. Sub-surface geology

The sub-surface geology of the area may be best illustrated by a panel diagram (figure 2) which indicates the correlation of the borehole logs through different geomorphic units in the flood plain deposits as well as in the deltaic plain deposits.

It is interesting to note the frequent occurrence of decayed wood, especially of fossil bones, down to the lowest beds reached in the Fort-William borehole which raise serious doubts about the deposits being entirely, if at all, marine. At the bottom of the bore hole (data given by Medlicott, as quoted in Coulson 1940) the pebbles are of large size and are of primary rocks. If these are no indication of the local base of the deltaic deposits, they at least point to the presence of rocks *in situ* much nearer than were suspected from the surface features.

The bore hole data at Port-Canning, 40 km SE of Calcutta (Coulson 1940) reveal that during the late Quaternary period the sea level changed, consequent to which frequent floods disturbed the lower part of the delta. The floods were thought to be responsible for the huge monotonous clay bodies sometimes intermingled with sands (Niyogi 1975). The individual sand beds might be distributary sand produced by the abandoned channels. During the time gap between two successive floods the sediments, mainly clay, got exposed to the sun, producing yellow ochres.

From the bore hole data at Baruipur (Chatterjee *et al* 1964) it is found that the clay bodies vary in texture and colour and are often mixed with sand. Individual beds are characterised by marked lenticularity and variations in the texture of the sediments are probably the reflections of the different phases of velocities of the transporting medium.

Correlation of these data indicates a more or less continuous deposition of the two major lithological units one arenaceous and the other argillaceous in a seemingly uninterrupted but alternating sequence. The lithological criteria suit a deltaic environment in which the upward coarsening sequence in a cyclic order fits well with the geology of the area. The correlation also exhibits vertical and lateral variations in sedimentary facies within flood plain and delta plain areas.

It appears that during the Quaternary the present area underwent upheaval and subsidence due probably to neotectonic movements. Niyogi (1975) observes that the Bay coastal plain underwent a steady but slow uplift throughout the Quaternary. Such

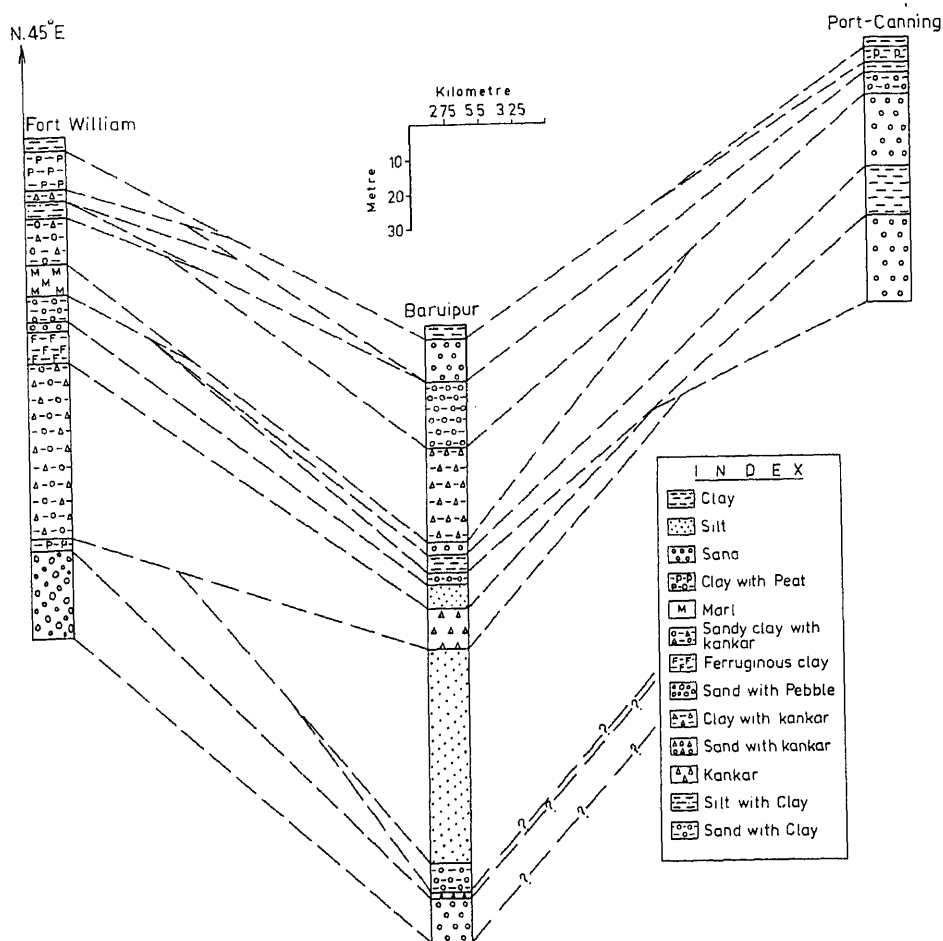


Figure 2. Correlation of lithologs showing vertical and lateral variations of sedimentary facies.

the presence of upright tree-trunks as far below as 114 m depth. Independent of the tectonic movements, there were oscillations of the sea level throughout the Quaternary. It is suggested that the sea level oscillated from -13 to $+5$ m from 8000 to 4000 years B.P., and it then approached the present level (Geyh *et al* 1979). The repeated occurrence of peat and calcareous nodules (calcretes or kankars) and ochres at different levels leads to certain important inferences like: (i) during the Quaternary there were large areas extensively covered with plants, (ii) due to sudden subsidence vegetable matter got trapped under the superincumbent load of sediments, (iii) due to prevalent pH condition ranging between 3.5 and 4, microbial activity converted the vegetable matter into peat, and (iv) calcretes and ochres formed under stable conditions when surface (soil) remained dry and above the water level for a considerable period of time (Dastidar and Ghosh 1964).

In this section, attempts have been made at understanding the processes as well as visualising a possible evolution of the deltaic region from the recorded data. Babu (1976) holds the view that the old strand line, immediately south of Calcutta being associated with ancient levees, old channels, and morphostructures, is the main contributing factor which shaped the present deltaic plain. He further observes that this deltaic plain mainly consists of tidal flats and delta lobes with a few islands and high surfaces. Rakshit (1980) reconstructed the geomorphic set-up of the entire Ganga delta area based on landsat imagery which constitutes four major geomorphic units, (i) a rounded to elliptical shaped area with subradial to radial drainage pattern, (ii) river flood-plain area, (iii) alluvial plain and backswamp area and (iv) deltaic plain area which includes distinct delta lobes, areas between successive strand lines, and the entire stretch of the Sundarban tidal flats.

We feel that a more categorical classification of the area on a larger scale is necessary through interpretation of aerial photographs coupled with landsat imagery data. After careful analyses of the geomorphic features and environmental interaction a dual-mode classification was made, one with the help of landsat imagery, and the other with the help of aerial photos corresponding to the first one but containing more detailed informations (table 1). Even though aerial photographs provide details of land forms and features within the units, they do not, however, yield a synoptic view of the micromorphology, particularly in regard to the configuration of the delta lobes, nature of the delta fronts, and patterns of the strand lines. The scrutiny of the landsat images, on the other hand, provide us with better understanding of the morphological set-up of the delta lobes and delta front areas.

Depositional surfaces in the deltaic plains gradually prograde as the delta front migrates outward i.e. S-SE, with ever varying configurational geometry in space and

Table 1. Classification of the geomorphic stage units as depositional surfaces

Classification based on landsat imageries (figure 1 inset)	Classification based on aerial photographs (figure 1)
1. Shore line deltaic plains (surface) representing tidal flats, viz, 1 to 5	1. Depositional delta surface 2. Depositional delta surface 3. Depositional delta surface—The Hogla Nala Surface 4. Depositional delta surface—The Port Canning Surface
2. The Bhagirathi-Bidyadhari delta lobe surface, viz, 1 to 5	5. Depositional delta surface—The Hotar Khal Surface 6. Depositional delta surface—The Sonarpur Surface
3. The Bidyadhari-Bhagirathi alluvial plains superposed on delta lobe	7. The Bhagirathi (present Hooghly) flood plain —The Calcutta Surface 8. The Bidyadhari and its flood plain —The Salt Lake Surface
4. The vast alluvial flat, East of delta lobe (Flood plains of Ichamati-Bidyadhari)	(not studied in detail)

time by sedimentation as well as through coastal emergence. As such, the youngest unit is composed of the shore line deltaic plains representing the present Sundarban tidal flats which can be further classified into four different delta surfaces with the help of photo imagery. Successive older surfaces are separated by successive ancient strand lines which have been dissected by a number of valleycuts. Different surfaces are marked by geomorphic stage features like tidal creeks or inlets and intertidal flats. In Sundarban surfaces, the tidal sand bars reflect the alternating environment of deposition. The intertidal flats are covered by dense forest of various types of flora mainly mangroove elements. In these frontal coastal areas the interface interaction of the sweet water discharge of rivers with the sea water has indirect control over the type of vegetation (Sanyal *et al* 1984).

The big delta fan lobes of the Bhagirathi-Bidyadhari system are characterized by low relief 5-tier concentric surfaces (figure 1, inset, table 1) as successive alluvial aprons with numerous anabranching distributary channels. These lobate forms, which are essentially flood water deposits of this channel system at a much lower stream gradient are the oldest recorded features of the delta remnants and are indicative of the fact that the shore-line was once close to Calcutta. This 5-tier delta lobe complex of south Bengal is flanked by the river Bidyadhari (at present an inactive distributary) in the east. On the other hand, the river Ganga (Hooghly), the main drainage course in the region, with an open meandering pattern in the alluvial plains, flanks the west. The area around Calcutta is flat, slightly raised over the open end of the oldest delta lobe. These five lobes are shown in the inset of figure 1 and the details are given in table 2.

The Bhagirathi flood plain is a low, marshy land of appreciable relief contrast. The flood plain quite frequently resulted in the surface deposition of the finest sediments whenever quiescence prevailed and this continued for a long span of time. The flood plain deposits, across which the river presently flows, suffered dessication and sufficient mechanical compaction—observations which are supported by the development of localized ferruginous concentration within the superficial clay beds and sharp decrease in the total moisture content of the underlying clays from the overlying sediments (Dastidar and Ghosh 1964).

The branching out of the Ganga as the Hooghly and the Bhagirathi near Calcutta was achieved probably quite early in the history of the delta development. The ancient course of the Bidyadhari starts from a salt water lake just East of Calcutta, and flowed into the Sundarban first as Matla and then as the river Thakuran. Its headward connection with the Hooghly is possibly represented at present by the Krishnapur canal (original course of the Bidyadhari). The Bidyadhari probably became inactive either

Table 2. Geometric aspect of the successive delta lobes

Lobe no.	Length (km)	Maximum width (km)	Exposed perimeter (km)
1	70		
2	90	30	160
3	102	35	80
4	110	45	130
5	100	70	140
		23	100

due to the growth of sand bars at the head, or to a higher rate of silting along the course. A narrow linear belt of superposed flood plain deposits of the Bidyadhari on the deltaic plain proper at places obscures the surficial expressions of the delta-lobes.

6. Discussion

From the foregoing discussion it is evident that the effects of relative emergence of the delta area and marginal submergence of the shoreline gave rise to the development of the older strand lines and the rising of the backswamp (reed swamp, mangroove swamp, etc.) in delta plains where the land surface is being built up more rapidly through sedimentation and accrued to the positive landforms. These paleostrandlines are sites for potential petroleum accumulation since they signify the locales of deposition of organic sediments. Present sites for petroleum exploration by ONGC, e.g. Port-Canning, are located over these strandlines. It is seen that most of the rivers of the deltaic plain are increasingly being silted up, and because of this process of channel filling, many streams are transformed into inactive distributaries and abandoned channels in marshy areas occasionally leaving behind the scars of the abandoned river beds and associated older levee banks. Salt marshes/mangroove swamps in tidal flats and inland reed swamps (like "roseau" on the Mississippi delta, Thornbury 1969) may have influenced the pattern of sedimentation in the back-swamps and the accumulation of organic muds and peats. The agricultural potential of these areas is of great importance. Recent studies show that subaqueous portions of any delta are characterised by a high rate of stream borne sedimentation into relatively still-standing bodies of water, building up to create new land. In such prograding delta, geologists have had to direct their attention to the whole 3-dimensional complex of the deltaic deposits in order to understand the evolution of the deltaic sequence. In the present case, the course of changes of the morphology of the deltaic plain through time may be depicted as follows:

- (i) incidence of sediment laden flood waters which promotes vertical accretion, particularly on channel margin.
- (ii) effect of marshy land vegetation (mangroove, reeds, etc.) which colonized the depositional terrain, and
- (iii) The extent of continuing subsidence, due to compaction of the already deposited underlying sediments including peat horizons or crustal depressions of isostatic adjustment.

More detailed and elaborate study is needed to further the objectives of the present work with particular bearing on the navigational problems of the Calcutta Port, industrialisation on the banks of the Ganga, agricultural land use pattern, scope of fisheries in the backswamp regions, seasonal floods and draught problems, and overall urban development.

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Numerical hysteresis model for intermittent studies in unsaturated soils

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Abstract. In the present study, the use of one of the recent dependent domain models of capillary hysteresis in the numerical analysis of intermittent infiltration and redistribution of water in two types of soils (a sand and Rubicon Sandy Loam) has been shown. The numerical results for both the soils have been presented in terms of pressure head depth, moisture content depth and the pressure head-moisture content relationships. The capillary hysteresis model has been found to be very useful for the prediction of both wetting and drying scanning curves of various orders.

Keywords. Capillary hysteresis model; unsaturated soils; scanning curves; intermittent conditions.

1. Introduction

In groundwater recharge problems, the study of soil moisture-depth profile under rapid intermittent surface flux conditions requires the inclusion of a capillary hysteresis model in the numerical simulation of flow processes. Under intermittent conditions, the effect of hysteresis is very significant and therefore it is *a priori* necessary to have the complete knowledge of higher order wetting and drying scanning curves. Watson and Perrens (1973) developed an interpolative capillary hysteresis model for the evaluation of higher order scanning curves from experimentally measured primary scanning curves of soil-moisture. The method is limited in application since it is quite impossible to measure all the scanning curves (Everett 1967) in the laboratory. In this regard, the conceptually based physical models are quite suitable for prediction of scanning curves from a limited set of experimental data.

Among the conceptually based domain models, Mualem and Dagan (1975) developed a dependent domain model by accommodating the concept of dependency of the state of neighbouring pores (Everett 1967; Topp 1971b) in the independent domain model of Mualem (1974) by including parameters like blockage against air-entry, $P_a(\theta)$, and blockage against water-entry, $P_w(\theta)$. Mualem (1977) presented a model to evaluate the hysteresis curve from either the boundary drying or the boundary wetting curve. Parlange (1976) proposed a simple extrapolative conceptual domain model of capillary hysteresis using the Mualem diagram. The model requires only the knowledge of the boundary loop and its derivative to predict all primary and other higher order wetting and drying scanning curves.

The use of the non-independent domain model of Pouloussis and Childs (1971) in the numerical analysis of intermittent flow study is shown by Lees and Watson (1975).

The purpose of the present study is to show the use of the dependent domain model of Mualem and Dagan (1975) in the numerical study of intermittent infiltration and redistribution of water in two types of soil. The types of soil selected are a sand and Rubicon Sandy Loam which are subject to the assumptions that the medium is homogeneous, isotropic, isothermal and rigid in behaviour. It is also shown that, in the subsequent cycles of intermittent sequences of flow processes, how the necessary higher order scanning curves are evolved for the surface node.

2. Numerical method

The water transport equation for unsaturated soil may be written as

$$C(h) \frac{\partial h}{\partial t} = \frac{\partial}{\partial Z} \left[K(h) \frac{\partial h}{\partial Z} \right] + \frac{\partial K}{\partial Z}(h), \quad (1)$$

where C is the specific moisture capacity, h the pressure head and Z the depth with positive direction upwards, t the time, and K the unsaturated hydraulic conductivity.

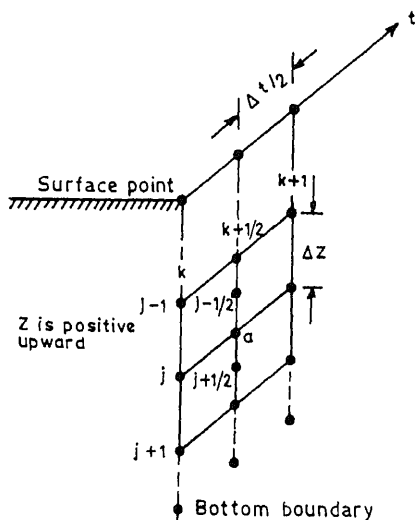
The numerical solution of the above equation can be obtained by a finite difference technique following Crank-Nicholson's approach. The method involves the use of a three-point central difference grid (figure 1) with the current value being evaluated at the mid-point. The finite difference form of the (1) is given by

$$A_j h_{(j-1, k+1)} + B_j h_{(j, k+1)} + C_j h_{(j+1, k+1)} = D_j; \quad (1 \leq j \leq N) \quad (2)$$

where the equation coefficients A_j , B_j , C_j and D_j are given by

$$A_j = -K_{(j-1/2, k+1/2)}, \quad (3)$$

$$B_j = \frac{2}{r} C_{(j, k+1/2)} - A_j - C_j, \quad (4)$$



$$C_j = K_{(j+\frac{1}{2}, k+\frac{1}{2})}, \quad (5)$$

$$D_j = A_j[h_{(j,k)} - h_{(j-1,k)}] + C_j[h_{(j,k)} - h_{(j+1,k)}] \\ + 2\Delta Z(A_j - C_j) + 2C_{(j,k+\frac{1}{2})} \cdot [h_{(j,k)}/r], \quad (6)$$

where

$$r = \Delta t / \Delta Z^2. \quad (7)$$

Equation (2) represents a set of N linear equations for N nodes along the depth axis which is given by

$$\begin{aligned} A_1 h_0 + B_1 h_1 + C_1 h_2 &= D_1 \\ A_2 h_1 + B_2 h_2 + C_2 h_3 &= D_2 \\ A_3 h_1 + B_3 h_2 + C_3 h_3 &= D_3 \\ &\text{-----} \\ A_N h_{N-1} + B_N h_N + C_N h_{N+1} &= D_N \end{aligned} \quad (8)$$

The matrix obtained in (8) is known as the 'tridiagonal matrix'.

To facilitate the solution of (3) through (7), the 'auto-selection iteration technique' was employed in a computer program developed following the approach elaborated by Whisler and Watson (1968). In (1), the $K(h)$ relationship is hysteretic, it was desirable to obtain the capillary conductivity values, from a $K(\theta)$ relationship. The specific moisture capacity was obtained by taking a gradient along the hysteretic path of a particular node at a desired pressure head value from a $h(\theta)$ relationship. Moreover the $C(h)$ curve is very unstable and oscillatory in nature and becomes discontinuous on the boundary drying curve within the air-entry region. To avoid these difficulties it is assumed that $d\theta/dh$ should be constant within a $\Delta h = (h_i - h_{i-1})$ interval, but in doing so the resulting $C(h)$ histogram becomes discontinuous between successive h intervals. So, as a remedy the average $C(h)$ values were obtained in adjacent intervals and then interpolated at the required point. At no time should the $C(h)$ value be negative or zero. It should have always a finite positive value.

2.1 Mualem-Dagan model of capillary hysteresis

It is sufficient for continuity of discussion at this point to outline the general form of the equations.

2.1a Entry into the scanning curve system from the boundary wetting curve: The equation for drying scanning curve is given as:

$$\theta \left[\begin{array}{ccc} \psi_1 & \text{---} & \psi_N \\ \psi_{\min} & \psi_2 & \psi \end{array} \right] = \theta_w(\psi_1) - \bar{P}_d(\theta) \cdot [1 - H(\psi)] \cdot [L(\psi_N) - L(\psi)] \\ + \sum_{j=1}^{(N-1)/2} \bar{P}_d(\theta_{2j}) \cdot [1 - H(\psi_{2j})] \cdot [L(\psi_{2j+1}) - L(\psi_{2j-1})]. \quad (9)$$

In the above equation N should be odd and ≥ 1 .

The equation for wetting scanning curves is given as:

$$\theta \left[\begin{array}{cccc} \psi_1 & & & \psi \\ \psi_{\min} & \psi_2 & \text{---} & \psi_N \end{array} \right] = \theta_w(\psi_1) + \bar{P}_d(\theta_N) \cdot [1 - H(\psi_N)] \cdot [L(\psi) - L(\psi_N)] \\ - \sum_{j=1}^{N/2} \bar{P}_d(\theta_{2j}) \cdot [1 - H(\psi_{2j})] \cdot [L(\psi_{2j-1}) - L(\psi_{2j})] \\ + \sum_{j=1}^{(N/2)-1} \bar{P}_d(\theta_{2j+1}) \cdot [1 - H(\psi_{2j+1})] \cdot [L(\psi_{2j+1}) - L(\psi_{2j+1})]. \quad (10)$$

In (10) N should be even and ≥ 2 .

2.1b *Entry into the scanning curve system from the boundary draining curve:* The equation for the drying scanning curve is written as:

$$\theta \left[\begin{array}{cccc} \psi_{\max} & \psi_2 & & \psi_N \\ \psi_1 & & \text{---} & \psi \end{array} \right] = \theta_d(\psi_1) - \bar{P}_d(\theta) \cdot [1 - H(\psi)] \cdot [L(\psi_N) - L(\psi)] \\ + \sum_{j=1}^{N/2} \bar{P}_d(\theta_{2j-1}) \cdot [1 - H(\psi_{2j-1})] \\ \times [L(\psi_{2j}) - L(\psi_{2j-1})] \\ - \sum_{j=1}^{(N/2)-1} \bar{P}_d(\theta_{2j+1}) \cdot [1 - H(\psi_{2j+1})] \\ [L(\psi_{2j}) - L(\psi_{2j+1})]. \quad (11)$$

In (11) N should be even and ≥ 2 .

The equation for the wetting scanning curve is given as:

$$\theta \left[\begin{array}{cccc} \psi_{\max} & \psi_2 & & \psi \\ \psi_1 & & \text{---} & \psi_N \end{array} \right] = \theta_d(\psi_1) + \bar{P}_d(\theta_N) \cdot [1 - H(\psi_N)] \cdot [L(\psi) - L(\psi_N)] \\ + \sum_{j=1}^{(N-1)/2} \bar{P}_d(\theta_{2j-1}) \cdot [1 - H(\psi_{2j-1})] \cdot [L(\psi_{2j}) - L(\psi_{2j-1})] \\ - \sum_{j=1}^{(N-1)/2} \bar{P}_d(\theta_{2j+1}) \cdot [1 - H(\psi_{2j+1})] \cdot [L(\psi_{2j}) - L(\psi_{2j+1})]. \quad (12)$$

In (12) N should be odd and ≥ 1 .

2.2 Description of the physical system

The porous materials selected for the present study are a sand (data obtained from Perrens and Watson 1977, who called it no. 17 sand) and Rubicon Sandy Loam (from Topp 1969). At a saturated moisture content of $0.292 \text{ cm}^3/\text{cm}^3$, No. 17 sand has a capillary conductivity of 0.8 cm/min and an air-entry value (Childs 1960) of -34.5 cm .

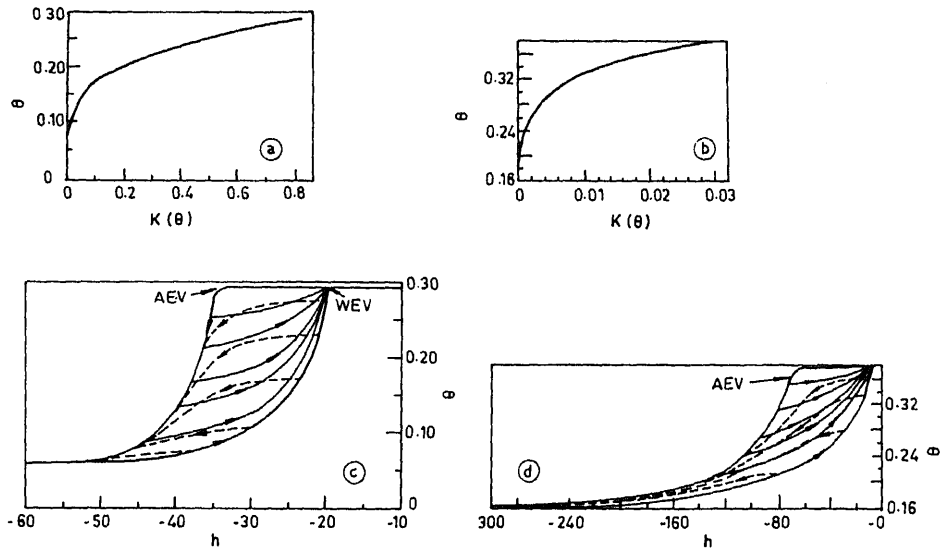


Figure 2. The hydrologic characteristics of the porous media. a. $K(\theta)$ for sand, b. $K(\theta)$ for sand, c. $h(\theta)$ for Rubicon Sandy Loam, and d. $h(\theta)$ for Rubicon Sandy Loam.

Similarly Rubicon Sandy Loam has a capillary conductivity of 0.03 cm/min at a saturated moisture content of $0.38 \text{ cm}^3/\text{cm}^3$ with an air-entry value of -30.0 cm .

The analysis presented is the unsteady flow to a semi-infinite and initially uniformly dry profile for a 70.0 cm column of No. 17 sand, whereas for the Rubicon Sandy Loam a water table was set at a depth of 120 cm below the surface of an initially partially dry profile. In each case the entire column was divided into equal intervals in the vertical Z -direction with each node separated by 1 and 1.2 cm for No. 17 sand and Rubicon Sandy Loam respectively. This yields 71 nodes for the sand and 101 nodes for Sandy Loam. The hydrologic characteristics $K(\theta)$ and $h(\theta)$ for the sand and Rubicon Sandy Loam are shown in figures 2a and b (Watson and Perrens 1973) and figures 2c and d (Topp 1969).

2.3 Evaluation of the domain functions

A capillary hysteresis subprogram has been developed which is based on the domain model of Mualem and Dagan (1975). This subprogram generates the various primary, secondary and other higher order redrying and rewetting scanning curves. To effect this a separate subroutine is used for the evaluation of distribution functions $H(h)$, $P_d(\theta)$ and $L(h)$ for both sand and Rubicon Sandy Loam. For the evaluation of these functions apart from the boundary loop, one primary drying curve for each soil originating from $(h_d = -20.2 \text{ cm}; \theta_d = 0.27 \text{ cm}^3/\text{cm}^3)$ for the sand and $(h_d = -6.0 \text{ cm}; \theta_d = 0.36 \text{ cm}^3/\text{cm}^3)$ for the Rubicon Sandy Loam are also selected. In addition the distribution function $P_d(\theta)$ is also derived for Rubicon Sandy Loam using one primary

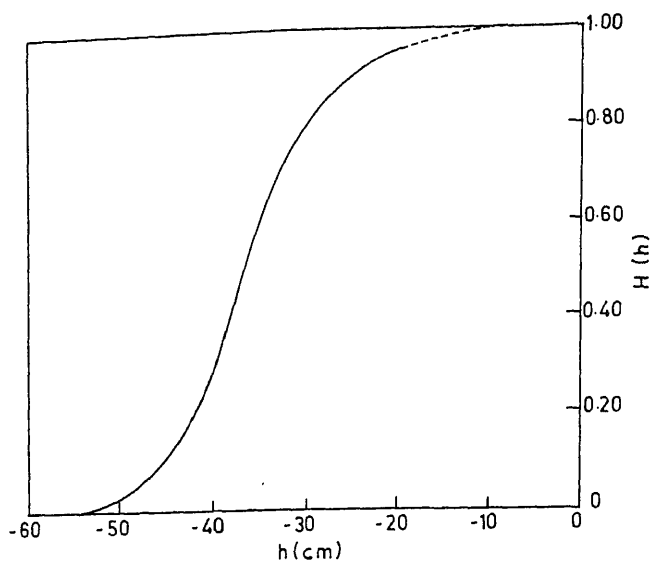


Figure 3. The distribution function, $H(h)$, for sand.

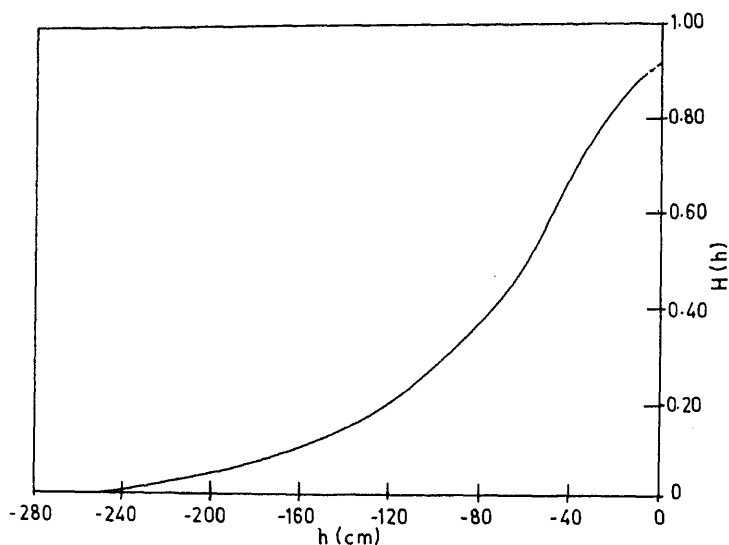


Figure 4. The distribution function, $H(h)$, for Rubicon Sandy Loam.

2.4 Flux pattern

2.4a *Sand*: In the case of sand, the surface flux (F_s) applied and the time of change of boundary conditions constrained during the $4\frac{1}{2}$ cycles of intermittence are as follows:

$$F_s = -0.5 \text{ cm/min}, \quad 0 \leq t \leq 10 \text{ min}, \quad Z = 0.$$

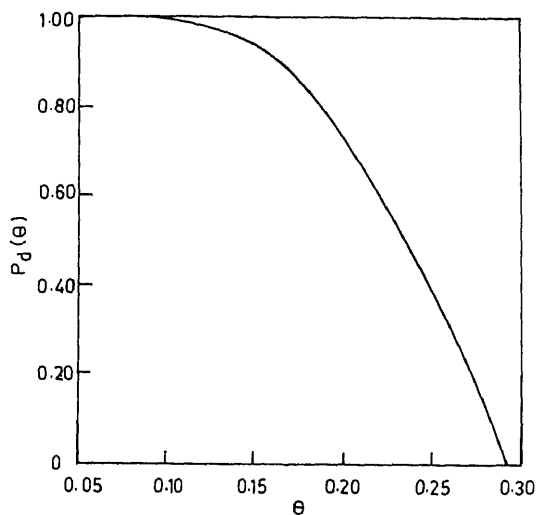


Figure 5. The function for pore blockage against air-entry, $P_d(\theta)$, for sand.

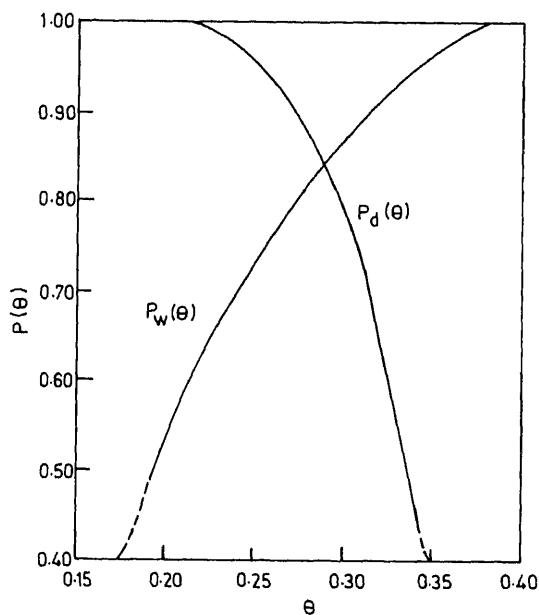


Figure 6. The function for pore blockage against air-entry, $P_d(\theta)$, and water-entry, $P_w(\theta)$, for Rubicon Sandy Loam.

$F_s = 0$,	$21 \text{ min} \leq t \leq 25 \text{ min}$,	$Z = 0$.
$F_s = -0.5 \text{ cm/min}$,	$25 \text{ min} \leq t \leq 25.3 \text{ min}$,	$Z = 0$.
$F_s = 0$,	$25.3 \text{ min} \leq t \leq 26.3 \text{ min}$,	$Z = 0$.
$F_s = -0.5 \text{ cm/min}$,	$26.3 \text{ min} \leq t \leq 26.4 \text{ min}$,	$Z = 0$.
$F_s = 0$,	$26.4 \text{ min} \leq t \leq 26.7 \text{ min}$,	$Z = 0$.
$F_s = -0.5 \text{ cm/min}$,	$26.7 \text{ min} \leq t \leq 36.7 \text{ min}$,	$Z = 0$.

2.4b Rubicon Sandy Loam: The surface flux applied during the $2\frac{1}{2}$ cycles of intermittence in Rubicon Sandy Loam is:

$F_s = -0.02$ cm/min,	$0 \leq t \leq 100$ min,	$Z = 0$.
$F_s = 0$,	$100 \text{ min} \leq t \leq 200$ min,	$Z = 0$.
$F_s = -0.015$ cm/min,	$200 \text{ min} \leq t \leq 250$ min,	$Z = 0$.
$F_s = 0$,	$250 \text{ min} \leq t \leq 300$ min,	$Z = 0$.
$F_s = -0.02$ cm/min,	$300 \text{ min} \leq t \leq 350$ min,	$Z = 0$.

3. Numerical results and discussions

3.1 Sand

The numerical results for pressure head-depth, moisture content-depth and pressure head-moisture content relationships have been presented in figures, 7, 8 and 9, respectively. The numerals on figures 7 and 8 represent the infiltration profiles (the solid lines) at the end of 10 min (for the 1st cycle), 21 min (2nd cycle), 25.3 min (3rd cycle), 26.4 min (4th cycle) and 36.7 min ($4\frac{1}{2}$ cycles), and the redistribution profiles (the dashed lines) at the end of 20 min (for 1st cycle), 25 min (2nd cycle), 26.3 min (3rd cycle), 26.7 min (4th cycle) for both pressure head-depth and moisture content-depth relationships.

In the 1st cycle of intermittence, the 10 min pressure head and moisture content profiles in figures 7 and 8 show that during the 1st infiltration event of 10 min the soil has got wet up to a depth of -26.0 cm with an entry of 4.32 cm^3 of water. The pressure head and moisture content were -20.63 cm and $0.26 \text{ cm}^3/\text{cm}^3$, respectively. During the 10 min redistribution event of this cycle (see the 20 min profiles in figures 7 and 8), the soil drains under the influence of gravity and causes the near surface region to lose water. The draining water accumulates below the wet front to a depth of about -40 cm (figure 8). At the end of this event the pressure head at the surface decreased to a value of -42.1 cm with moisture content as $0.1 \text{ cm}^3/\text{cm}^3$.

It has been observed that during the redistribution event of the 1st cycle there is a development of negative moisture content gradient ($d\theta/dZ$) at $Z = 0$ (figure 8). If Z_{wf} is considered to be the wet front depth at the end of the 1st infiltration event, then it would be found that the reduced gradient ($\bar{\theta}_z$) will be negative on the draining part of the profile and zero near but above Z_{wf} . Below Z_{wf} it develops a positive gradient, which shows that the drained water is accumulating below Z_{wf} . At the maximum moisture content (θ_m) in the profile, $d\theta/dZ = 0$. Further as Z_{wf} increases, $\bar{\theta}_z$ decreases, the downward movement of water being greater. Also the redistribution rate is faster for a greater value of Z_{wf} .

The 21 and 25 min profiles for pressure head and moisture content are shown in figures 7 and 8, respectively, which developed during the infiltration and redistribution events of the 2nd cycle of intermittence. The pressure head and moisture content values for the surface node during the infiltration event were -21.6 cm and about $0.23 \text{ cm}^3/\text{cm}^3$, respectively, and those during the redistribution event were -40.6 cm and $0.12 \text{ cm}^3/\text{cm}^3$. The total volume of water which entered the soil is 5.26 cm^3 . The depth to which the wet front reached during the infiltration and redistribution events

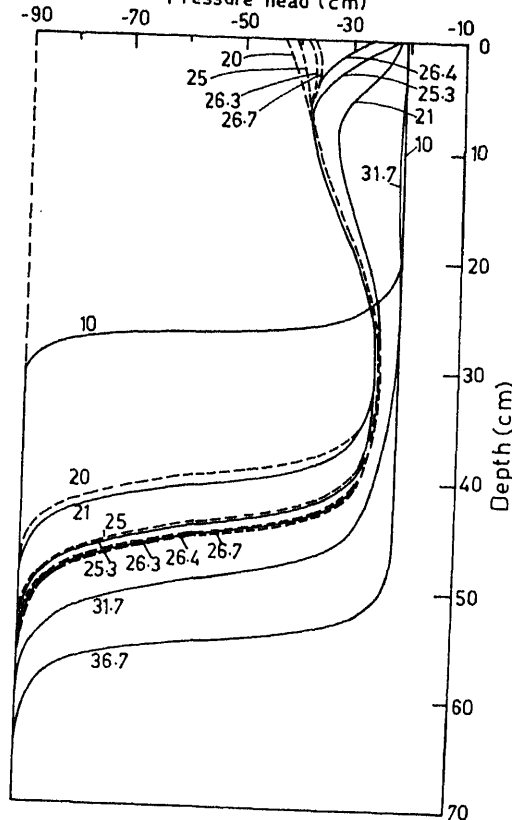


Figure 7. The pressure head depth profiles for $4\frac{1}{2}$ cycles of intermittence in sand. Solid lines represent the infiltration and the broken lines the redistribution.

It has been found that during the 2nd cycle of intermittence the top — 10 cm of soil wets up quickly with positive moisture content gradient and it can be seen in the moisture depth profile of 21 min (figure 8) that a 'peak' forms around — 10 and 30 cm of depth. The gradient ($\bar{\theta}_z$) is sufficient to transmit the water below this peak. Further, a feature to be noted in the moisture depth profile of 25 min (figure 8) is that though the gradient ($\bar{\theta}_z$) has become negative in the top few centimetres, yet below this depth the water drains along the 20 min profile, which is indicative of the fact that in sand the transmission zone is marked by this peak (see figure 8).

The pressure head and moisture content profiles of 25.3 and 26.3 min, in figures 7 and 8, developed during the infiltration and redistribution events of the 3rd cycle of intermittence; and those developed during the infiltration and redistribution events of the 4th cycle of intermittence are shown by the 26.4 and 26.7 min profiles also in figures 7 and 8. The pressure head and moisture content values for the surface node during the infiltration event of the 4th cycle were — 26.4 cm and $0.17 \text{ cm}^3/\text{cm}^3$, respectively and those during the redistribution event were — 38.0 cm and $0.12 \text{ cm}^3/\text{cm}^3$. The depth to which the wet front moved down at the end of the 4th cycle is — 46.0 cm (see 26.7 min profile in figure 8), and 5.6 cm^3 of water had entered the soil column. The collation of

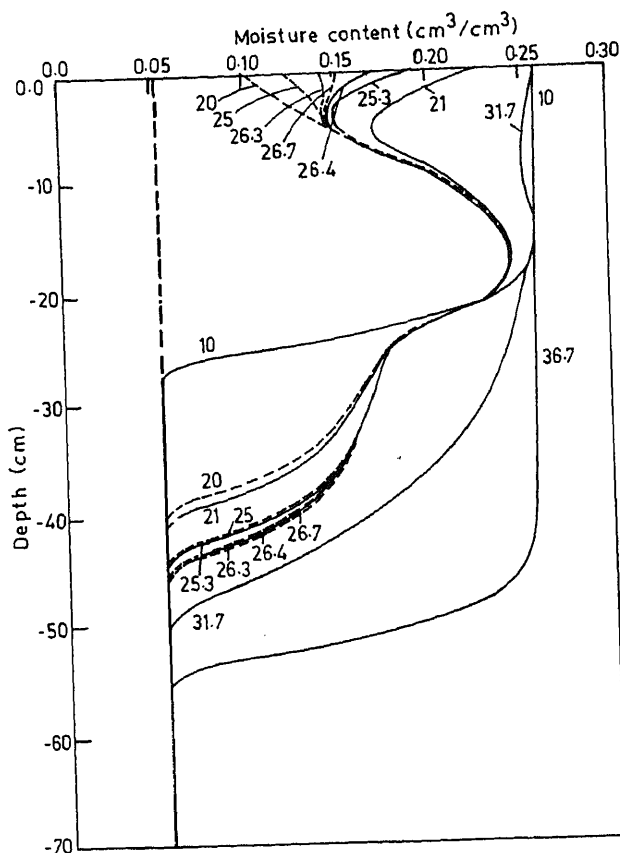


Figure 8. The moisture content-depth profiles for $4\frac{1}{2}$ cycles of intermittence in sand. Solid lines represent the infiltration and the broken lines the distribution.

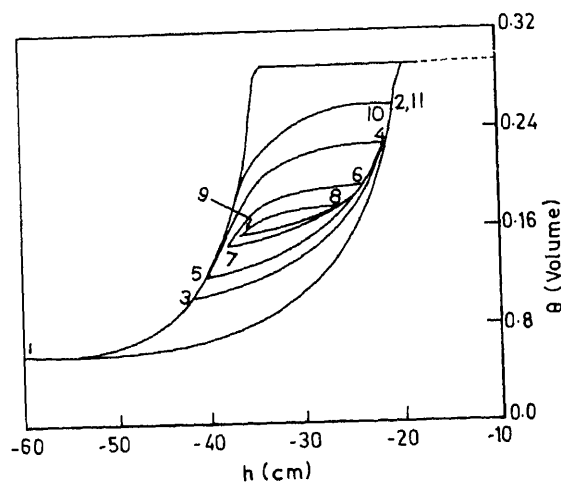


Figure 9. The $h(\theta)$ path of the surface node during the $4\frac{1}{2}$ cycles of intermittence of sand. The numerals 1, 2, 3, ..., 11 correspond to cumulative times, $t = 0, 10, 20, 21, 25, 25.3, 26.3, 26.4, 26.7, 31.7$ and 36.7 min, respectively.

the 26.3 and 26.7 min profiles of figure 8 indicates that during the 4th cycle the peak still persists and remains in its position whereas the wet front moves down slowly.

In the final half cycle of intermittence, only the infiltration event has been considered. The 31.7 and 36.7 min profiles of this event in figures 7 and 8 show that the peak disappears with the development of a positive moisture content gradient. In fact, the 36.7 min profile is in equilibrium with constant pressure head and moisture content values, and water infiltrates under unit hydraulic gradient with a capillary conductivity value equal to the surface flux. The values of pressure head and moisture content at the end of this event were -20.63 cm and $0.26 \text{ cm}^3/\text{cm}^3$, respectively. The wet front reached a depth of about -56.0 cm with 7.65 cm^3 of water entering the soil column.

Figure 9 shows the history of imbibition and drainage of the surface node after a number of intermittent infiltration and redistribution sequences. The numerals 1, 2, 3, ———, 9 correspond to the time $t = 0, 10, 20, 21, 25, 25.3, 26.3, 26.4$ and 26.7 min respectively at which pressure head and moisture content reversals have occurred. It can be ascertained from figure 9 that upon continued infiltration during the final infiltration event the current quarternary rewetting scanning curve closes the inner and outer loops at points 8, 6, 4, and 2 and advances to the points 10 and 11 which correspond to the times 31.7 and 36.7 min respectively. Moreover as the infiltration continues, the surface node remains at the point 11 and the bottom nodes start to come to equilibrium with the surface node. Thus the hysteresis model is quite efficient in generating the various order of scanning curves of either wetting or draining (figure 9).

3.2 *Rubicon Sandy Loam*

The model is also tested with Rubicon Sandy Loam for intermittent conditions. The flux applied at the surface of the soil column has always been less than the saturated value of hydraulic conductivity (see flux pattern). The numerical results for pressure head-depth, moisture content-depth and pressure head-moisture content relationships have been given in figures 10, 11 and 12, respectively. The numerals on figures 10 and 11 represent the infiltration profiles (solid lines) at the end of 100 min (for 1st cycle), 250 min (2nd cycle) and 350 min ($2\frac{1}{2}$ cycles), and the redistribution profiles (dashed lines) at the end of 200 min (for 1st cycle) and 300 min (2nd cycle) for both pressure head-depth and moisture content-depth relationships.

The zero time profiles in figures 10 and 11 are for the initial equilibrium state. In these figures the 100 min and 200 min profiles are those developed during the infiltration and redistribution events respectively, of the 1st cycle of intermittence. During these events the changes in pressure head and moisture content profiles occurred to a depth of about -50.0 cm. The 200 min moisture content depth profile shows that the drained water has redistributed below the depth of -27 cm (see figure 11).

In the second cycle of intermittence, the infiltration and redistribution were set for 50 min each. The pressure head profiles for this cycle have been shown by the 250 min for infiltration and 300 min for redistribution events (see figure 10); and corresponding moisture content profiles are shown in figure 11. The 300 min moisture content profile

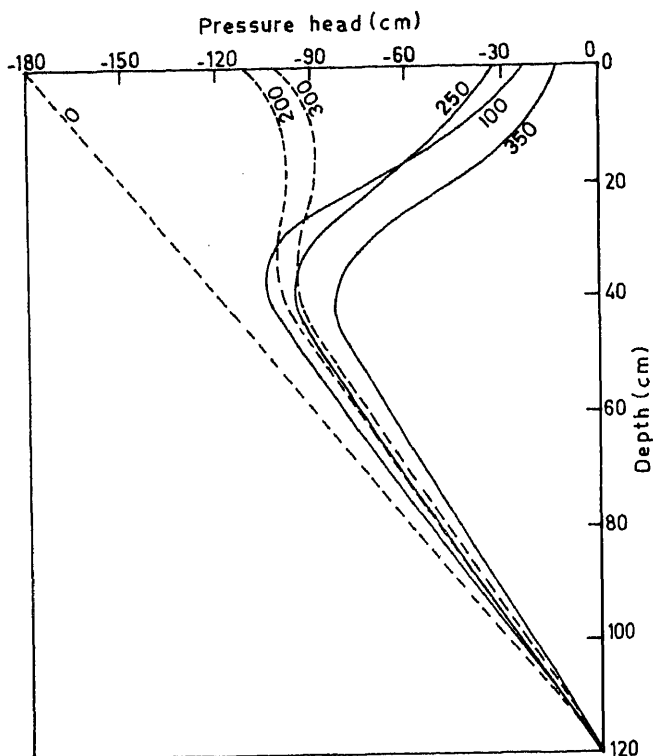


Figure 10. The $h(Z)$ profiles for $2\frac{1}{2}$ cycles of intermittence in Rubicon Sandy Loam. Solid lines represent the infiltration and the broken lines the redistribution.

profiles in figures 10 and 11), and during 50 min of infiltration the wetting has occurred mainly within -46.0 cm of the soil. The pressure head and moisture content values for the surface node during each of the above mentioned events can be read from figures 10 and 11.

In figure 12, the pressure head–moisture content relationship for the surface node is shown. The numerals 1, 2, ---, 5 on the diagram correspond to the time 0, 100, 200, 250 and 300 min, respectively, at which reversals have occurred. Upon continued infiltration the current rewetting curve (between 5 and 4 in figure 12) closes the tertiary (between 5 and 4) and secondary (between 3 and 2) loops at points 4 and 2 respectively, and advances beyond to the point 6 on the primary wetting curve (between 1 and 6) because the surface flux is greater now than during the initial infiltration.

Finally, it should be noted that for the prescribed surface flux and timings of the events, the steady state condition is not reached in Rubicon Sandy Loam.

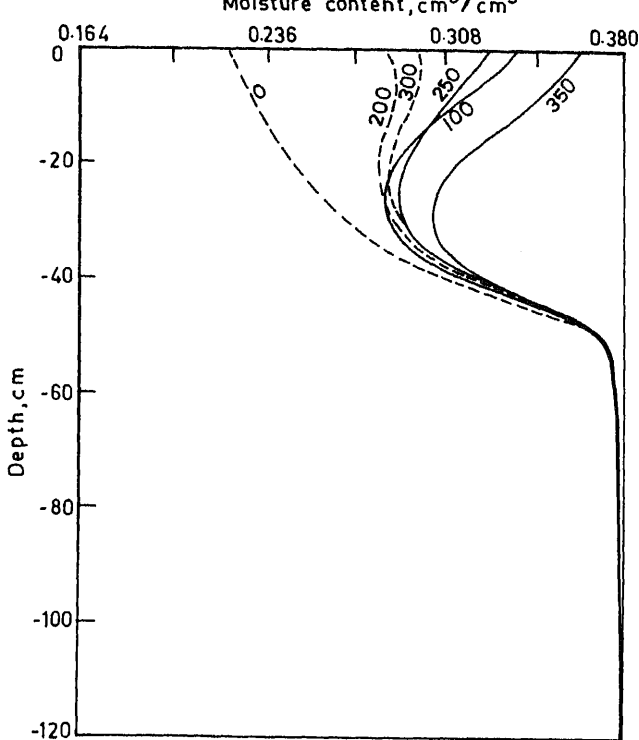


Figure 11. The $\theta(Z)$ profiles for $2\frac{1}{2}$ cycles of intermittence in Rubicon Sandy Loam. Solid lines represent the infiltration and the broken lines the redistribution.

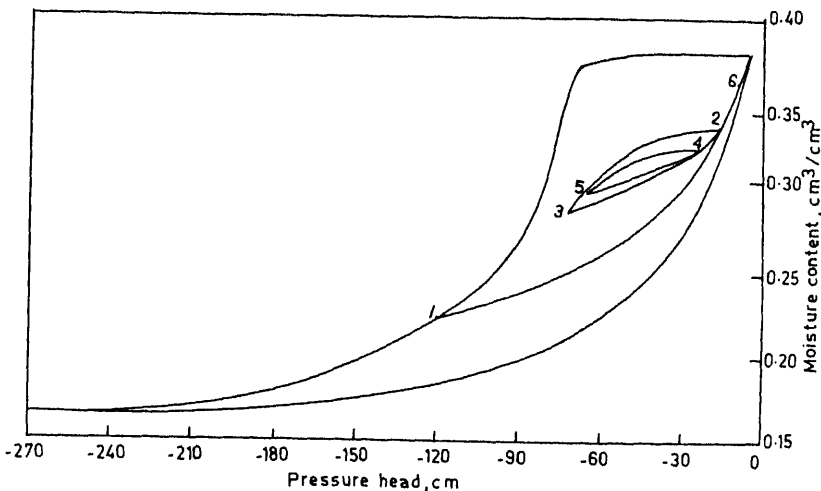


Figure 12. The $h(\theta)$ path of the surface node during the $2\frac{1}{2}$ cycles of intermittence in Rubicon Sandy Loam. The numerals 1, 2, ----, 6 correspond to the cumulative time, $t = 0, 100, 200, 250, 300$ and 350 min, respectively.

In studies of intermittent drainage, the Mualem-Dagan model of capillary hysteresis is found to be quite satisfactory with regard to the evaluation of primary and other higher order scanning curves. This prompts one to use further the Mualem and Dagan (1975) model for the problems of rapid intermittent flow situations.

Acknowledgements

The computations were carried out on CDC computers at the School of Civil Engineering, University of New South Wales, Sydney. The author is thankful to Prof. C L Singh, for his cooperation and help.

List of symbols

A_j, B_j, C_j	
and D_j	= the equation coefficients,
C	= the specific moisture capacity,
F_s	= the surface flux,
$h, h(j, k)$	= the pressure head,
$h(\theta)$	= the moisture content-pressure head relationship,
$L(h)$	= the distribution function,
$H(h)$	= the distribution function,
K	= the unsaturated hydraulic conductivity,
h_d	= the pressure head value at the point of reversal to draining,
h_w	= the pressure head value at the point of reversal to wetting,
$P_d(\theta)$	= the blockage against air-entry (the distribution function),
$\bar{P}_d(\quad)$	= $P_d(\quad) \cdot P_w(\quad)$
$P_w(\theta)$	= the blockage against water-entry (the distribution function),
θ_d	= the moisture content value at the point of reversal to draining,
θ_w	= the moisture content value at the point of reversal to wetting,
θ_m	= the maximum moisture content,
$\bar{\theta}_z$	= $d\theta/d(Z/Z_{wf})$, the reduced gradient,
$d\theta/dh$	= the gradient of $h(\theta)$ relationship,
t	= time,
Z	= the depth,
Z_{wf}	= the wet front depth.

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Radionuclide depth profiles in Dhajala chondrite

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Abstract. Spallation produced radionuclides ^{10}Be , ^{22}Na and ^{26}Al and low energy neutron capture radionuclide ^{60}Co have been measured in several fragments of the Dhajala chondrite, their shielding depth in space having been established by measurement of cosmic ray track density. These data enable us to obtain depth profiles of production rates of these nuclides within the meteoroid body. These profiles are discussed in terms of their dependence on size of the meteoroid and variations of galactic cosmic ray fluxes in time and space. The slowing down density of neutrons of $0.1 \text{ neutrons/cm}^3 \text{ sec}$ is determined near the centre of the Dhajala meteoroid based on ^{60}Co .

Keywords. Cosmic rays; meteorites; cosmogenic nuclides; nuclear tracks; Dhajala.

1. Introduction

The production rate of nuclides formed by nuclear interactions of cosmic rays with the meteoritic material depends, among other parameters, on the size of the meteoroid. This dependence is poorly understood although profiles of a few nuclides in meteorites of known size (such as Bansur, Madhipura, Keyes, St. Severin, San Juan Capistrano and Udaipur etc.) have been measured (Wright *et al* 1973; Schultz and Signer 1976; Englert and Herr 1978; Bhattacharya *et al* 1980; Bhandari *et al* 1978; Finkel *et al* 1978). These measurements are limited to small meteorites, with effective preatmospheric radius, R_E , between 6 and 30 cm. Profiles of some nuclides have also been measured in Kirin with $R_E = 85\text{--}100 \text{ cm}$ but the data do not show the expected systematics and indicate complex exposure history (Honda *et al* 1980, 1982). It has, therefore, not been possible to estimate reliable production rates in large meteoroids ($R_E > 30 \text{ cm}$). In this context, Dhajala with $R_E \simeq 50 \text{ cm}$, offers an opportunity of establishing production rates in a big meteoroid. With this purpose this study of cosmogenic isotopes and tracks has been carried out in several fragments of this meteorite.

Dhajala is a H3 chondrite which fell on 28 January 1976 in Gujarat in western India and has been described by Bhandari *et al* (1976). Over 500 fragments weighing more than 60 kg were recovered. Multiple fragmentation in the lower atmosphere resulted in a wide range of sizes from 10 g to 12 kg. The fall and recovery have been described elsewhere (Bhandari and Ballabh 1977; Lal and Trivedi 1977; Bagolia *et al* 1977; Bhandari *et al* 1978). The results of this study are presented in this paper.

Table 1. Radiochemical and counting data for various radionuclides

Sample Code	Chemical form	Amount of salt (1)		Mode of detection	Counting efficiency %	Background (cpm $\times 10^3$)	signal	Specific activity (dpm/kg)	Average sp. activity (4)
		expected (mg)	counted (mg)						
¹⁰ Be									
DH-9	BeO	65.2	28.20	β^-	40	54.16 $\pm$ 3.0	82.7 $\pm$ 3.3	18.2 $\pm$ 0.7	18.2 $\pm$ 0.7
T-92	BeO	65.2	27.1	β^-	40	40.8 $\pm$ 1.8	98.2 $\pm$ 3.8	17.3 $\pm$ 0.7	17.3 $\pm$ 0.7
T-272	BeO	65.2	15.2	β^-	40	40.8 $\pm$ 1.8	69.7 $\pm$ 5.0	19.2 $\pm$ 1.3	19 $\pm$ 1
T-9-1	BeO	109.8	9.0	β^-	28.6	40.8 $\pm$ 1.8	42.3 $\pm$ 3.0	18.9 $\pm$ 1.4	
			27.0		41.1	156 $\pm$ 6.1	64.1 $\pm$ 10.1	19.4 $\pm$ 3	
T-11	BeO	110.0	25.0	β^-	40	40.8 $\pm$ 1.8	100 $\pm$ 14.0	21.2 $\pm$ 3	21.8 $\pm$ 1.5
T-68	BeO	73.2	50.0	β^-	24	33 $\pm$ 2.7	102 $\pm$ 5.0	24.7 $\pm$ 1.2	17.3 $\pm$ 4.3
T-273	BeO	73.2	22.8	β^-	40	74 $\pm$ 1.8	70 $\pm$ 16.6	17.3 $\pm$ 4.3	
		73.2	24.9	β^-	40	54 $\pm$ 3.5	70 $\pm$ 8.7	21.5 $\pm$ 2.4	
²² Na (2)	NaCl	73.2	24.9	β^-	40	40.8 $\pm$ 1.8	148 $\pm$ 7.0	21.6 $\pm$ 1	21.5 $\pm$ 2.4
		933	420	$\beta^+ - \gamma$	9.74	5.63 $\pm$ 0.93	114.6 $\pm$ 4.3	20 $\pm$ 0.81	20.8 $\pm$ 0.9
DH-9	NaCl	1590	220	$\beta^+ - \gamma$	12	7.69 $\pm$ 0.51	12.9 $\pm$ 1.25	115 $\pm$ 11	115 $\pm$ 11
T-272	NaCl	1373	302	$\beta^+ - \gamma$	12	7.69 $\pm$ 0.51	10.2 $\pm$ 1	135 $\pm$ 13	135 $\pm$ 13
T-273	NaCl	1297	112	$\beta^+ - \gamma$	9.8	5.66 $\pm$ 0.66	11.8 $\pm$ 1.1	105 $\pm$ 11	105 $\pm$ 11
T-9-1	NaCl	112	112	$\beta^+ - \gamma$	10.8	5.66 $\pm$ 0.66	7.6 $\pm$ 1.7	71 $\pm$ 16	73 $\pm$ 11
							8.0 $\pm$ 1.7	75 $\pm$ 16	

DH-9	Al_2O_3	847.2	562.4	$\beta^+ - \gamma$	9.0	7.65 $\pm$ 1.75	71.2 $\pm$ 3.5	51 $\pm$ 2.5	51 $\pm$ 2.5
T-92	Al_2O_3	868.9	396.7	$\beta^+ - \gamma$	9.24	19.75 $\pm$ 1.75	55.1 $\pm$ 4.4	50 $\pm$ 4	49.5 $\pm$ 2.5
T-273	Al_2O_3	1136.5	296.1	$\beta^+ - \gamma$	9.66	7.2 $\pm$ 1.06	52.7 $\pm$ 2.7	49 $\pm$ 3	49 $\pm$ 2
T-9-1	Al_2O_3	66.9	30.6	$\beta^+ - \gamma$	13.5	5.64 $\pm$ 0.47	34.2 $\pm$ 2.7	48.8 $\pm$ 3.0	45 $\pm$ 4
T-11	Al_2O_3	7081.0	326	$\beta^+ - \gamma$	9.24	10.70 $\pm$ 1.20	72.8 $\pm$ 2.8	48.8 $\pm$ 1.9	56 $\pm$ 3.4
T-68	Al_2O_3	652.5	237.2	$\beta^+ - \gamma$	9.24	3.33 $\pm$ 0.20	52.5 $\pm$ 0.5	45 $\pm$ 4	54 $\pm$ 3
^{60}Co (3)						19.35 $\pm$ 1.75	59.2 $\pm$ 3.6	55.9 $\pm$ 3.4	
DH-9	Co_2O_3	43.0	42	$\beta^- - \gamma$	2.54	5.84 $\pm$ 0.75	30.0 $\pm$ 2.3	53.9 $\pm$ 3.2	
T-92	Co_2O_3	29.9	24.7	$\beta^- - \gamma$	4.20	11.01 $\pm$ 1.3	30.3 $\pm$ 2.2	81 $\pm$ 6	83 $\pm$ 5
T-272	Co_2O_3	40	38.2	$\beta^- - \gamma$	4.20	11.01 $\pm$ 1.3	45.6 $\pm$ 3.5	85 $\pm$ 7	
T-273	Co_2O_3	43.5	36.8	$\beta^- - \gamma$	2.38	7.73 $\pm$ 0.51	37.5 $\pm$ 1.5	58 $\pm$ 3.5	58 $\pm$ 3.5
T-9-1	Co_2O_3	54.5	36.8	$\beta^- - \gamma$	2.43	5.79 $\pm$ 0.5	9.0 $\pm$ 1	25 $\pm$ 2.7	25 $\pm$ 3
					2.38	5.79 $\pm$ 0.5	7.5 $\pm$ 1.5	19.6 $\pm$ 3.9	19 $\pm$ 2.3
					2.27	4.80 $\pm$ 0.51	6.9 $\pm$ 0.8	18.4 $\pm$ 2.3	6.6 $\pm$ 2.3

- (1) Weight of samples processed are DH-9 (39 g), T-92 (40 g), T-272 (42.3 g), T-273 (52.3 g), T-9-1 (52.6 g), T-11 (326 g) and T-68 (30.0 g).
 (2) The mean dates of counting of ^{22}Na are DH-9 (22.1.1979), T-272 (5.11.1980), T-273 (15.1.1980), T-9-1 (28.9.1978 and 28.1.1979) which have been used for decay corrections.
 (3) The mean dates of counting of ^{60}Co are DH-9 (15.10.1979 and 28.2.1980), T-92 (15.12.1979), T-272 (10.3.1979), T-273 (27.8.1978 and 20.5.1979) and T-9-1 (20.9.1978) which have been used for decay corrections.
 (4) At the time of fall.

products but covering a half-life range of 2.56 to 0.72×10^6 years and ^{60}Co , a 5.6 yr half-life radionuclide produced by capture of secondary thermal and epithermal neutrons. The track density has been measured in each fragment to estimate the preatmospheric shielding depth. Some preliminary results have been reported earlier (Bhandari *et al* 1978). A simultaneous study of these four radionuclides enables us to deduce the fluxes of different components of nuclear-active particles of primary cosmic rays and their secondaries as a function of depth in the meteoroid as well as to determine spatial and temporal variation in cosmic ray intensity in the interplanetary space.

2. Experimental technique and results

The radioactivity of various nuclides was measured, after radiochemical separation of the elements of interest, by counting with low-background radiation detectors. The procedure has been described in detail earlier (Potdar 1981; Bhandari *et al* 1978). Briefly about 10 to 50 g of a Dhajala fragment was crushed and some olivine grains were picked up for track analysis. These grains were mounted in epoxy, polished and etched in Swamy's etch (Krishnaswamy *et al* 1971) for 6 hr for track density measurements. The rest of the sample, after addition of Be carrier, was repeatedly digested with HF and HClO_4 , dried and the residue was dissolved in 9 M HCl. Iron was removed by ether extraction and cobalt was separated by anion exchange column. The elements Be, Na and Al were separated on a cation exchange column. Be was purified by extracting with theonyl trifluoro acetone and counted on a low background gas flow GM counter with NaI(Tl) active anti-coincidence shield. ^{26}Al and ^{22}Na were counted as oxide and chloride, respectively on a β - γ coincidence spectrometer (Bhandari 1969). ^{60}Co was counted on the same system as oxide or after plating as metal on copper planchets.

The radiochemical and counting data for ^{10}Be , ^{22}Na , ^{26}Al and ^{60}Co are summarized in table 1. There is some uncertainty in ^{22}Na value of sample T-9-1 because the efficiency of recovery of sodium exceeded 100%. However for other samples the values are reasonable. Track density data, measured in olivines, are given in table 2. Sample T-11 is the same as studied earlier by Bhandari *et al* (1978). Repeated countings have improved the accuracy of the results and the final values are included in the table. Radioactivity and track density measurements in some other spot samples of three fragments (T-92, T-68 and T-67) have been reported by Bhandari *et al* (1978) and Bagolia *et al* (1977). Here we have made measurements in different spot samples. Therefore the values reported earlier (Bhandari *et al* 1978) and those given in table 2 are different, presumably because of depth gradient. The differences occur only for large fragments like T-92 (11.6 kg) and T-67 (2.7 kg), whereas in T-68 which is small, the values are similar.

3. Discussion

A large amount of work has been done on cosmogenic rare gases, tracks and radionuclides in several Dhajala fragments. This work has resulted in understanding the exposure history of Dhajala. We first summarize the available estimates of the exposure age and then use track data to determine shielding depths of various fragments analyzed for radionuclides to estimate its preatmospheric size.

Table 2. Shielding depths of various Dhajala fragments based on their track density.

Sample code	ρ , Track density in olivine ² (/cm ²)	Normalized TPM ¹ (per cm ² M. yr.)	Shielding depth ³ (cm)
T-272	9.6×10^3 (59)	3.4×10^3	16.5
T-92	7×10^2 (10)	2.5×10^2	29
DH-9	2.7×10^2 (7)	9.43×10^1	≥ 42
T-11	1.71×10^4 (35)	6.0×10^3	14
T-68	3.94×10^4 (15)	1.4×10^4	11
T-67	2.36×10^5 (58)	8.2×10^4	6.2

¹TPM = tracks/cm² .M. Yr normalized to the mineral groups (pyroxenes + feldspar) by multiplying ρ olivine by 2.9 and taking the exposure age as 8.3 M. yr

²Number of tracks are given in parenthesis.

³The shielding depths have been estimated from the production rate profiles of Bhattacharya *et al* (1973).

Table 3. Exposure age of Dhajala calculated from ²¹Ne concentration in different fragments.

Fragment	²¹ Ne (10 ⁻⁸ cc STP/g)	NeR	Track density (t/cm ²)	Shielding depth (cm)	P(²¹ Ne) ⁽¹⁾	Neon exposure age (M. yr) ⁽⁴⁾
DH-9	2.47 ⁽²⁾	1.08	2.7×10^2	≥ 42	0.325	7.6
T-67	2.613 ⁽³⁾	1.12	4.5×10^5	6 ± 2	0.305	8.5
T-11	3.175 ⁽³⁾	1.09	3.5×10^4	14 ± 1	0.355	8.9

Adopted exposure age = 8.3 M. yr

⁽¹⁾Bhandari and Potdar, (1982); ⁽²⁾Heusser *et al* (1978); ⁽³⁾Gopalan *et al* (1977); ⁽⁴⁾The error in ²¹Ne measurement is 10 % and consequently the exposure ages will also have atleast this error.

3.1 Exposure age of Dhajala

The exposure age of Dhajala has been determined from rare gas concentration, mainly ³He and ²¹Ne. Heusser *et al* (1978) believe that there has been some loss of ³He but ²¹Ne ages should be reliable. Gopalan *et al* (1977) and Heusser *et al* (1978) estimate its exposure age to be 6 and 4.75 M. yr, using production rate corrected for shielding. Since then a more accurate estimate of ²¹Ne production rates as a function of shielding depth has been computed (Bhandari and Potdar 1982), where it has been shown that the production rates are very sensitive to the size of the meteoroid. Using production rates of ²¹Ne (for $R_E = 50$ cm, as discussed below) the exposure ages of three fragments DH-9, T-67 and T-11 have been calculated to be 7.6, 8.5 and 8.9 M. yr, as given in table 3. We adopt here the mean value of 8.3 M. yr for the exposure age of Dhajala.

3.2 Shielding depths of various samples and preatmospheric size of Dhajala

Bagolia *et al* (1977) examined over 250 fragments and found the lowest track density of 6.5×10^2 /cm² in fragment T-9. Assuming that this is the most shielded fragment and

estimated a preatmospheric radius of 38 ± 2 cm for the Dhajala meteoroid. Now we have found another fragment DH-9 which has even lower track density of $2.7 \times 10^2/\text{cm}^2$ (table 2) and the highest ^{60}Co activity of 83 ± 5 dpm/kg (table 1) measured so far in any Dhajala fragment (Heusser *et al* 1978). The exposure age of Dhajala has also been revised to 8.3 M. yr, based on the recent production rates of ^{21}Ne (Bhandari and Potdar 1982) as discussed above. These data yield the track production rate of $32/\text{cm}^2$ M. yr for olivines in this fragment. Equivalent production rates for (feldspar + pyroxene) group of minerals were obtained by multiplying this value by 2.9 (Bhandari *et al* 1980), which corresponds to the centre of a 45 cm meteoroid according to the calculated depth profiles of track production rate given by Bhattacharya *et al* (1973). Allowing for the possibility that this fragment may also not have been located at the centre of the meteoroid, a radius of 45 to 50 cm appears to be appropriate for the Dhajala meteoroid.

We have also calculated shielding depths of samples from other fragments using this procedure. These are given in table 2. In the case of T-11, T-67 and T-272 the shielding depths could be determined to an accuracy of ± 2 cm whereas in other cases there is a greater uncertainty (about ± 5 cm) due to poor statistics of track density.

3.3 Depth profiles of radionuclides

The available data on different radionuclides in various fragments of Dhajala (Evans *et al* 1982; Heusser (Private Communication.) Bhandari *et al* 1978 and table 1) are summarized in table 4. The activity ratio of some radionuclides, e.g. $^{10}\text{Be}/^{60}\text{Co}$, $^{22}\text{Na}/^{26}\text{Al}$ and $^{10}\text{Be}/^{26}\text{Al}$ is also included in this table. Largest variations are observed for the ^{60}Co activity which varies by a factor of ~ 13 (6.6 to 83 dpm/kg between fragment T-67 and DH-9). The smallest variations are observed for ^{10}Be and ^{26}Al . The ^{10}Be activity varies between 17 and 22 dpm/kg whereas the ^{26}Al activity ranges between 45 and 56 dpm/kg, possibly with slightly higher values around the shielding depth of 15 cm. The depth variation of activity for different nuclides is shown in figures 1 and 2. The activity of ^{60}Co and $^{10}\text{Be}/^{60}\text{Co}$ ratio is found to be very sensitive to depth. Using this depth profile (figure 2) and the observed values of the ^{60}Co (or $^{10}\text{Be}/^{60}\text{Co}$) in the sample, we can determine the shielding depth i.e. in T-9-1, T-273, and T-75, where track density data are not available. The deduced shielding depths are given in table 4. The profiles given in figures 1 and 2 can be used to estimate the production rates within the meteoroid body, as we shall discuss below.

3.4 Production of low energy nuclides and evaluation of the spectral hardness parameter

The production (P_i) of a nuclide in spallation reaction at a depth X in a spherical rock of radius R depends on the flux $\phi(E)$ of the interacting particles of energy E , and is given by

$$P_i(X, R_E) = \sum_j \int_{E_T} N_j \cdot \sigma_{ij}(E) \cdot \phi(E, X, R) dE, \quad (1)$$

Table 4. Summary of cosmogenic radionuclide and particle track data in Dhajala samples

Radio- Nuclide/ Ratio	T-9-1	T-273	T-272	T-92	DH-9	T-11	T-68	T-67 (a)	T-75 (a)	DH-9 (b)
^{10}Be (dpm/kg)	21.8 ± 1.5	20.8 ± 0.8	19.0 ± 0.8	17.3 ± 0.7	18.2 ± .7	17.3 ± 4.3	21.5 ± 0.24	—	—	—
^{22}Na (dpm/kg)	73 ± 11	105 ± 15	135 ± 13	126 ± 12 ^(e)	115 ± 11	—	—	103 ± 6	117 ± 2	110 ± 8
^{26}Al (dpm/kg)	45 ± 4	49 ± 2	—	49.5 ± 2.5	51 ± 2.5	56 ± 3.4	54 ± 3	45 ± 2	44 ± 2	48 ± 6
^{60}Co (dpm/kg)	6.6 ± 2.3	19.0 ± 2.3	25 ± 3	58 ± 3.5	83 ± 5	15 ± 2	—	6.6 ± 0.3	15.3 ± 0.9	77 ± 6
$\left[\begin{smallmatrix} ^{10}\text{Be} \\ ^{60}\text{Co} \end{smallmatrix} \right]$	3.30 ± 1.17	1.09 ± 0.14	0.76 ± 0.10	0.29 ± 0.02	0.22 ± 0.01	1.15 ± 0.32	—	—	—	—
$\left[\begin{smallmatrix} ^{22}\text{Na} \\ ^{26}\text{Al} \end{smallmatrix} \right]$	1.62 ± 0.28	2.14 ± 0.22	—	2.55 ± 0.27	2.25 ± 0.24	—	—	2.28 ± .06	2.65 ± 0.13	2.29 ± 0.3
$\left[\begin{smallmatrix} ^{10}\text{Be} \\ ^{26}\text{Al} \end{smallmatrix} \right]$	0.48 ± 0.05	0.42 ± 0.02	—	0.34 ± 0.02	0.36 ± 0.02	0.31 ± 0.08	0.40 ± 0.02	—	—	—
Track density (cm ⁻²)	—	—	9.6 × 10 ³	7 × 10 ²	2.7 × 10 ²	1.7 × 10 ⁴	3.9 × × 10 ⁴	2.3 × 10 ⁵	—	2.7 × 10 ²
Shielding depth (cm)	6	15	16.5	29	≥ 42	14 ± 1	11	6 ± 2	14	≥ 42

^(a)Evans *et al* (1982); ^(b)G. Heusser (Private communication); ^(c)Bhandari *et al* (1978)

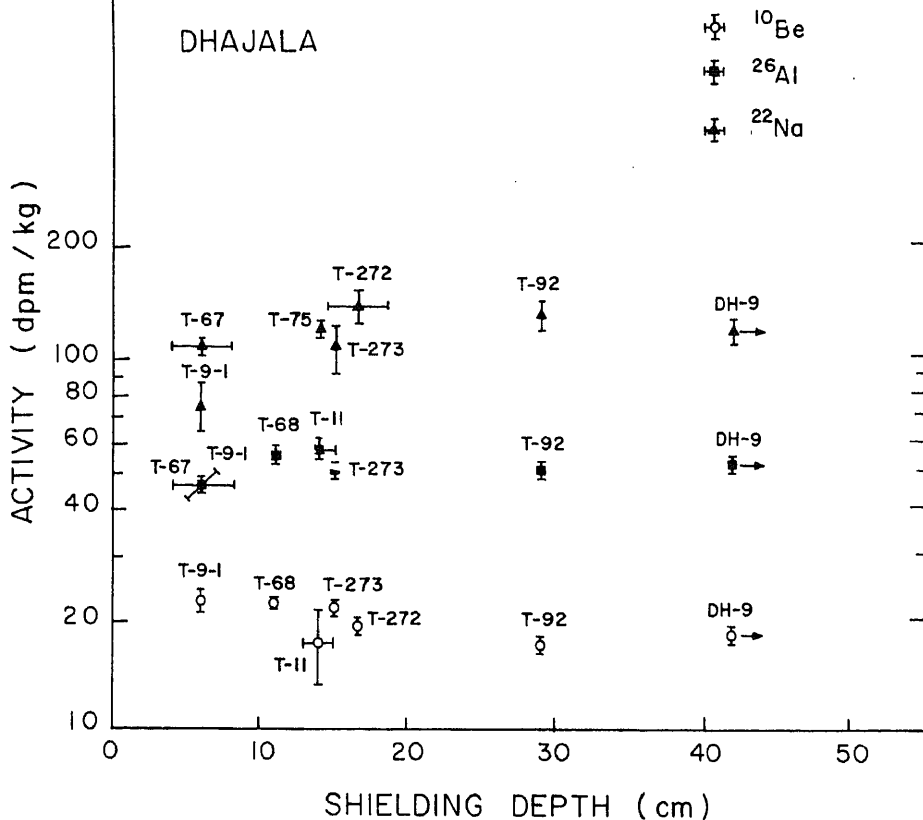


Figure 1. Measured activity of ^{10}Be , ^{26}Al and ^{22}Na in different Dhajala fragments plotted as a function of shielding depth determined from the observed track densities. The error in shielding depth in some cases is shown by the horizontal bar; in other cases it is ± 5 cm.

where N_j is the target element abundance and σ_{ij} is the excitation function, above threshold energy E_T . For rocks of irregular shape, we can use an effective radius R_E by equating its mass to a spherical body. If $P_i(X, R_E)$ and σ_{ij} are known, we can calculate the flux $\phi(E)$, the spectral shape of which is characterized by the hardness parameter α in the Reedy-Arnold model by the following relation

$$\phi(E, X, R_E) = k(X, R_E) [\alpha(X, R_E) + E]^{-2.5}, \quad (2)$$

where k is the normalizing constant related to J_G (the integral flux of nucleons above 1000 MeV) by the following relation

$$k(X, R_E) = \frac{3 J_G(> 1000 \text{ MeV}, X, R_E)}{2} [\alpha(X, R_E) + 1000]^{1.5}. \quad (3)$$

The value of J_G is taken to be $1.7/\text{cm}^2 \text{ sec. } 4\pi$ for the primary cosmic rays in free space and is calculated for other depths using known attenuation coefficients following the model of Reedy and Arnold (1972). Based on the measured activity profiles of ^{53}Mn in

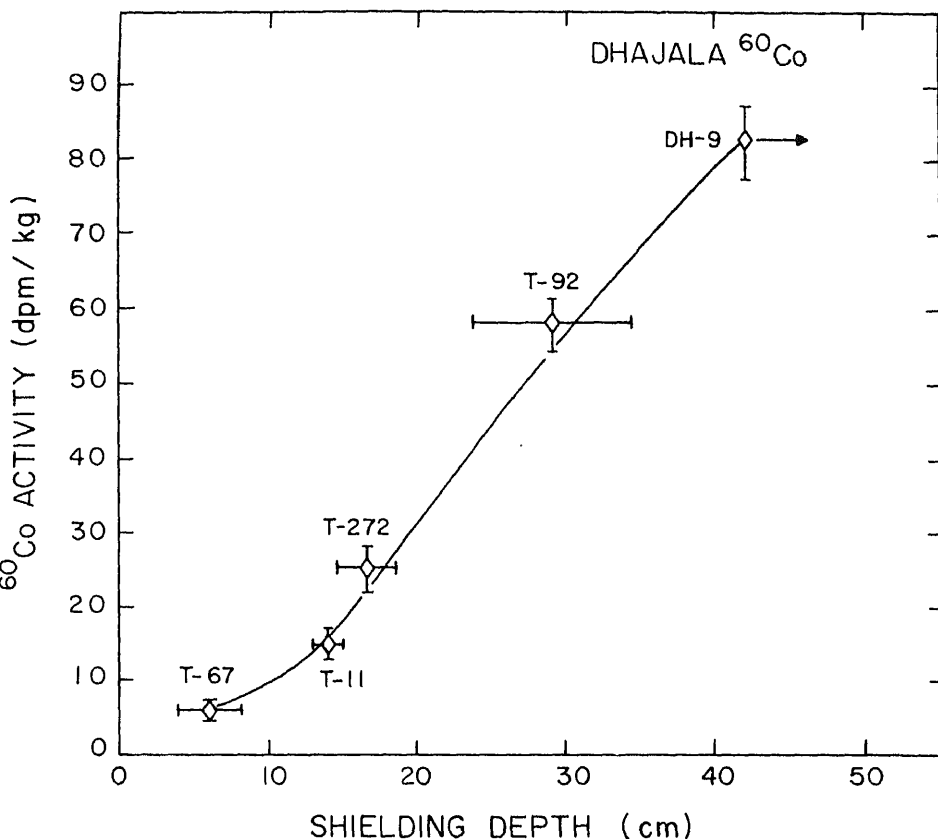


Figure 2. Depth profile of ^{60}Co in the Dhajala chondrite (thick line) obtained from the measured ^{60}Co activity in different fragments. Shielding depth and errors are as in figure 1.

several chondrites of known sizes (6–25 cm), Bhattacharya *et al* (1980) calculated α (X) and developed a model for isotope production in meteorites in low energy interactions which was successfully used for estimating the production rates of neon isotopes (Bhandari and Potdar 1982) and ^{26}Al (Bhandari 1981). We now extend this model to meteoroids of Dhajala size ($R_E \approx 50$ cm) by calculating α from the measured profiles of ^{26}Al (table 1, figure 1). Following the above procedure $\alpha(X, 50)$ has been computed and is shown in figure 3 together with α for chondrites of other sizes. It has been shown that using the values of α given in figure 3 and appropriate σ_{ij} , N_{ij} etc., the production profile of any nuclide can be computed for a particular flux of GCR particles. These calculations agree fairly well with observed values in several chondrites for ^{54}Mn , ^{53}Mn , ^{26}Al , ^{22}Na , ^{22}Ne , ^{21}Ne , ^{10}Be etc (Bhandari 1981; Bhandari and Potdar 1982). However, the observed activities of two radionuclides ^{54}Mn and ^{22}Na in Dhajala are much higher

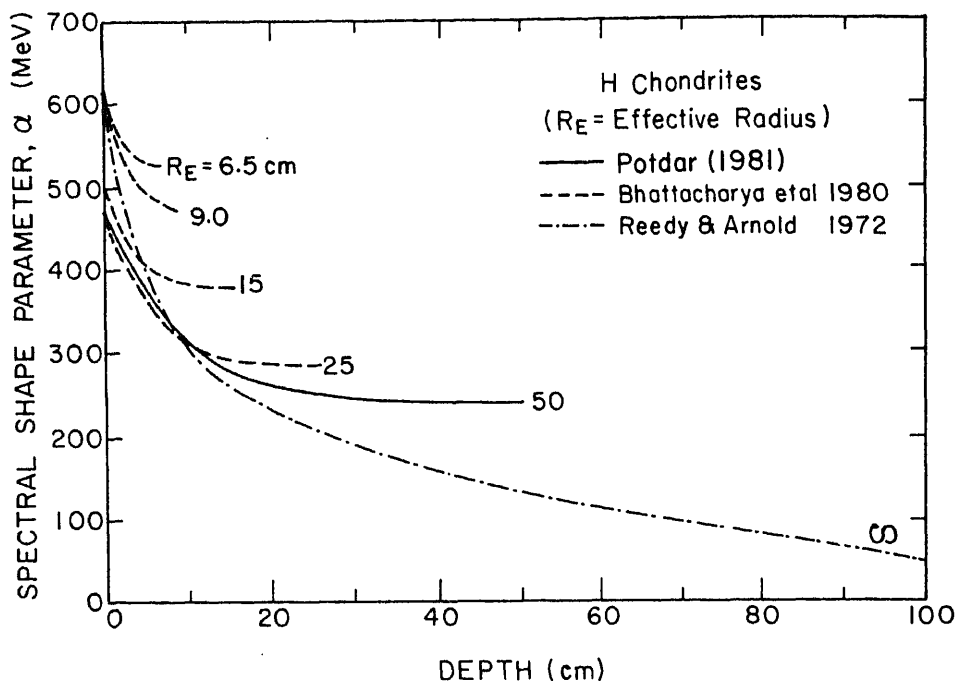


Figure 3. Spectral shape parameter α as a function of depth in chondrites with $R_E = 6.5, 9, 15, 25$ cm (Bhattacharya *et al* 1980) and 50 cm (present work) and in moon (Reedy and Arnold 1972).

^{54}Mn and ^{22}Na has been attributed to possible higher GCR flux in this region of the heliosphere (Bhandari *et al* 1978) as discussed below.

3.5 Spatial and temporal variation of cosmic rays

The question of spatial and temporal variation of cosmic rays based on induced radioactivity in meteorites has been discussed extensively in literature (Reedy *et al* 1983; Bhandari *et al* 1978; Schaeffer *et al* 1981). The production of a particular isotope in a meteorite depends on the size, depth, shape and fragmentation and exposure history leading to large uncertainties in calculated production rates. It is, therefore, difficult to ascertain small anomalies in observed activities, which can be attributed to possible changes in the cosmic ray flux with time or in the interplanetary space. With the evaluation of the spectral shape of secondary nucleons as a function of size and depth, discussed in 3.4, a major source of uncertainty has been removed and it is possible to calculate the expected activity for some nuclides e.g. ^{26}Al , ^{53}Mn , ^{22}Na etc with reasonable accuracy, atleast in case of meteorites with simple, one stage, exposure history. These calculations show that the activity ratios of isotopes such as $^{22}\text{Na}/^{26}\text{Al}$ are nearly independent of size of the meteoroid and depth within it and can serve as a useful index of time variation in cosmic ray flux over the mean lives of the radioisotopes. We have measured the depth profiles of ^{26}Al and ^{22}Na (Gardner *et al* 1978). The ratio $^{22}\text{Na}/^{26}\text{Al}$

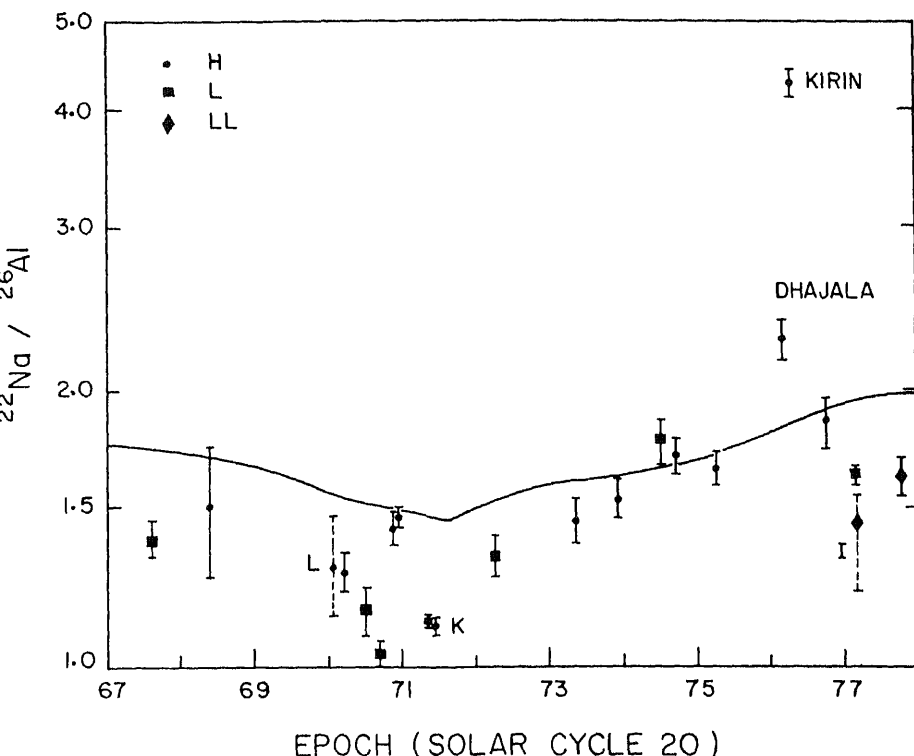


Figure 4. Calculated $^{22}\text{Na}/^{26}\text{Al}$ activity ratio in spherical H chondrites with radius of 50 cm (thick line) as a function of time during solar cycle 20. The curve is based on measured cosmic ray fluxes and the model of isotope production in meteorites (Potdar and Bhandari 1979; Bhandari 1981). The data points refer to different chondrites belonging to H, L and LL groups and are taken from Evans *et al* (1982) and table 4. The error bars (continuous) and where more than one sample has been measured, the range of measurements (dash) are shown for various chondrites. Points for Kirin, Dhajala, Kabo (K), Innisfree (I) and Lost City (L) are marked.

ies between 2.14 and 2.6 (except for T-9-1 which is unreliable, as discussed earlier) with a mean value of 2.35 compared to 1.8 expected for fall in 1976 (figure 4). This unusually high ratio of $^{22}\text{Na}/^{26}\text{Al}$ has already been noted by Bhandari *et al* (1978) and Evans *et al* (1982). We compare this value with the ratios observed in other meteorites and discuss its implications.

The ^{22}Na activity in a meteoroid is expected to vary with the phase of the solar cycle due to differential modulation of galactic cosmic rays. The fluxes of GCR protons (> 1 GeV) from the observed cosmic ray spectra at 1 A.U. were computed by Potdar and Bhandari (1979), and based on these fluxes, the activity of ^{22}Na and ^{26}Al as a function of time of fall was calculated. The computed time variation of $^{22}\text{Na}/^{26}\text{Al}$ for H group meteoroid with $R_E \approx 50$ cm for the period 1967 to 1978 is shown in figure 4. Apart from the Dhajala meteorite, the experimental data of Evans *et al* (1982) for several other meteorites are shown in this figure. The departure of experimental values from the calculated curve during the period 1967–71 can partly be due to the fact that GCR fluxes have not been measured before 1965 which is important for ^{22}Na

production as its activity is cumulative. Small differences can also arise because of target element abundances in different meteorites. The agreement between experimental points with calculated curves improves with time except for a few meteorites namely Kabo, Dhajala and Kirin. Kirin is a large body with possible multiple exposure history and will not be discussed here.

In case of Kabo, which fell at the time of solar maximum, the observed activity ratio is much lower than the computed value. As pointed out before, the activity ratio is nearly independent of the depth of the sample or size of the meteorite. This is also supported by observations in Innisfree, where nine samples from different locations (Evans *et al* 1982) give values of $^{22}\text{Na}/^{26}\text{Al}$ within a narrow range. The discrepancy between the observed and calculated values must then be due to either multiple exposure of meteorites or their peculiar orbits. The orbits of other meteorites are not known but in case of Dhajala, Ballabh *et al* (1978) have computed its probable orbit based on the observed trail in the atmosphere. From figure 2 of Bhandari *et al* (1978), it is clear that the aphelion of the meteorite was in the southern hemisphere and it spent considerable fraction of its period off the ecliptic covering 20°N to 38°S heliolatitudes. It spent the period of 10 to 100 days before its fall beyond 10°N and only the last ten days close to the ecliptic. The observed ^{22}Na activity is 30% higher than the calculated value. Apart from ^{22}Na , an excess activity of ^{54}Mn (28% higher), after appropriate correction for solar cycle was made, was observed by Bhandari *et al* (1978). These excess activities were attributed to $33 \pm 7\%$ higher GCR flux at heliolatitudes $15\text{--}40^\circ\text{S}$ covered by the orbit of the Dhajala chondrite during the solar minimum. The present more extensive data support this conclusion.

It may be noted here that the observed $^{22}\text{Na}/^{26}\text{Al}$ ratio in L and LL groups is generally lower than in the H group. The model (Bhattacharya *et al* 1980; Bhandari 1981) indicates that the ratio of saturation activity is similar for L, LL and H groups as target element differences has a negligible effect on the ratio. The explanation for the observed differences may therefore lie either in the exposure history or in orbits of these meteorite groups, where GCR fluxes may be different.

3.6 Production rates of ^{60}Co and neutron source function

The neutron capture nuclides have a different source function in meteoroids as was first shown by Eberhardt *et al* (1963) and more recently computed by Spergel *et al* (1982). Experimental data on depth profile of ^{60}Co is only available for Moon (Wahlen *et al* 1973) and a few meteorites e.g. Allende (Bourot-Denise and Pellas 1981) and Kirin (Honda *et al* 1980; Bhandari 1981). Dhajala data (table 4) allow a comparison of experimental and calculated profiles (figure 5). It is clear from this figure that the experimental profile in Dhajala is lower than the computed profile of Eberhardt *et al* (1963) for a body of radius 50 cm or even 40 cm. Both the production level and the depth variation are different. The observed activity initially increases slowly with depth up to 20 cm and then increases rapidly. The observed maximum production extrapolated to the centre of 50 cm body is about 100 dpm/kg as compared to the calculated production rate of 200 dpm/kg given by Eberhardt *et al* (1963).

The increase in ^{60}Co activity, above the average activity over a solar cycle, at the time of fall is calculated to be only 1.4% for Dhajala (Potdar and Bhandari 1979). Therefore

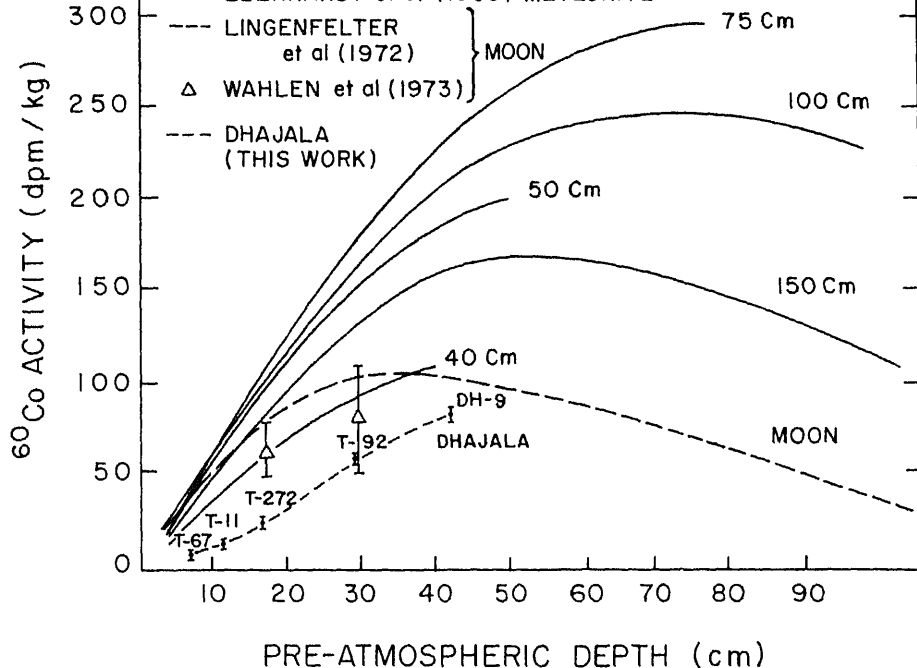


Figure 5. Comparison of experimental ^{60}Co depth profile in Dhajala with profiles calculated by Eberhardt *et al* (1963) for meteorites having different radii and the moon (Lingenfelter *et al* 1972). The lunar data points of Wahlen *et al* (1973) are also shown.

Dhajala be much smaller than 40 cm in view of the track data (Bagolia *et al* 1977). Two other possible reasons which can explain this difference are (i) Leakage of neutrons in slowing down processes from bodies of different sizes. Eberhardt *et al* (1963) considered this phenomenon but in Dhajala and in several other meteorites (Fireman 1966; Potdar 1981) as well as in Moon (Wahlen *et al* 1973), the activity of neutron capture isotopes is lower than the calculated value near the surface. (ii) The effect of the irregular shape of meteoroid has not been considered. The calculations have been made for spherical bodies whereas meteoroids having irregular shape will, in general, have much smaller effective radius for production by neutrons than by spallation. Further work on meteorites with different preatmospheric shapes and sizes is necessary to evaluate these effects.

The activity of ^{60}Co enables us to calculate slowing down density of neutrons. Using a value of 0.03 cm^{-1} for moderating power of chondrites and a cobalt content of 0.073% by weight (Potdar 1981), a value of $0.1 \text{ n/cm}^3 \text{ sec}$ for slowing down density of neutrons is obtained at the centre of the meteoroid. The measured ^{60}Co production of about 6 atoms/min. kg at shielding depth of 5 cm corresponds to slowing down density of $0.005 \text{ n/cm}^3 \text{ sec}$. The neutron exposure (over the exposure age of 8.3 M. yr) at the centre is $2.5 \times 10^{13} \text{ n/cm}^3$.

4. Summary and conclusions

The preatmospheric size of the Dhajala chondrite has been established to be equivalent to $R_E = 45\text{--}50 \text{ cm}$ based on extensive fossil track analysis. Depth profiles of ^{10}Be , ^{22}Na ,

^{26}Al and ^{60}Co illustrate the source function of cosmic ray nucleons and slow neutrons within the meteorite body. The slowing down density of neutron is found to be $0.1 \text{ n/cm}^3 \text{ sec}$ near the centre of the chondrite. The ^{22}Na production is in excess of the expected value supporting the earlier conclusion of relatively less modulation of GCR protons at high heliolatitude at the time of the solar minimum of 1976.

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Cosmogenic isotope and track records in meteorites and their implications to cosmic ray fluxes

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Abstract. Study of several cosmic ray effects, such as VH track density, spallogenic ^{26}Al and ^{53}Mn activity, ^{21}Ne and $^{22}\text{Ne}/^{21}\text{Ne}$ ratio, made in the same sample or in cores taken from different meteorites can identify parameters related to the exposure history of meteorites and cosmic ray flux variations. Meteorites with single or multiple exposure can be distinguished from a track production rate - $^{22}\text{Ne}/^{21}\text{Ne}$ correlation diagram and cosmic ray flux variations over 10^6 - 10^7 years can be deduced from a three-isotope correlation diagram of ^{26}Al , ^{53}Mn and ^{21}Ne . Isotopic data based on chondrites with simple, one-stage exposure are consistent with the same average galactic cosmic ray intensity over the past 2 million years as that during the past 10^7 years.

Keywords. Cosmogenic isotopes; meteorites; cosmic ray fluxes.

1. Introduction

The possibility that meteorites, having long term cosmic ray records in the form of isotopic products resulting from nuclear reactions and heavy nuclei (VH) tracks, can be used for monitoring cosmic ray fluxes in the past has been discussed since the early work of Arnold *et al* (1961). Calculations show that the rates of production of these cosmic ray effects depend significantly on the shielding depth within a meteorite (Bhattacharya *et al* 1980; Eberhardt *et al* 1966; Fleischer *et al* 1967). Also, the meteorites are not always exposed to cosmic rays in a fixed geometry because of multiple exposure on their parent bodies (Bhandari and Potdar 1982), fragmentation in space and space erosion. The effects of these processes have to be quantitatively assessed before the isotopic data in meteorites can be used to infer cosmic ray intensity. Because of these complications, there is considerable debate on the changes in the cosmic ray flux inferred from meteorite data. Nishiizumi *et al* (1980) and Müller *et al* (1981) have proposed that the cosmic ray intensity over the past 2×10^6 years was higher as compared to the average intensity before that. This is based on statistical analyses of ^{26}Al , ^{53}Mn and ^{21}Ne data after making empirical corrections for shielding depths but without considering the effects of various processes cited above. Recently Moniot *et al* (1983) have measured ^{10}Be , reanalysed the data on several isotopes and concluded that temporal variations in cosmic ray flux have not occurred.

Here we discuss isotopic and track density data in some chondrites restricting ourselves to measurements which have been made in the same sample. Whereas the

Table 1. Track density and isotopic data in chondrites.

Meteorite (class)	Pre-atmospheric radius (cm)	Track density $\equiv P$ ($\times 10^4/\text{cm}^2$) (A)	^{26}Al (dpm/kg) (B)	^{53}Mn (dpm/kgFe) (C)	^{21}Ne (10^{-8} ccSTP/g) (u) (D)	$^{22}\text{Ne}/^{21}\text{Ne}$ (u) (E)	Exposure age (M.Y.) (u)	Remarks (v)
Bansur (L6)	15 (e)	150 (e)	48 $\pm$ 5(h)	370 (e)	535	1.14	15	(A, B, C, D, E)
Barwell (L5)	17 (c)	104 $\pm$ 7(c)	63 $\pm$ 9(g)	203 $\pm$ 23(g)	1.18(g)	1.12	2.5	(A, B, C, D, E)
Bruderheim (L6)		25 $\pm$ 5.5(b)	78 $\pm$ 2(g)	466 $\pm$ 42(g)	11.5(g)	1.10(f)	23.2	(A, B, C, D, E)
Dalgaty		< 5 (h)	52 $\pm$ 2.9(s)	247 $\pm$ 20(j)	5.96	1.048	17.9	(A, B, C, D, E)
Downs (L4)								
Dhajala (H3)	50 (n)	50 $\pm$ 8.4(b)	45 $\pm$ 9(n)	270 $\pm$ 32(n)	3.17(m)	1.09(m)	8.3(b)	(A, B, C, D, E)
Eitter (H)		4.2 $\pm$ 1.3(h)	57 $\pm$ 2(p)	158 $\pm$ 9(j)	0.77	1.05	1.8	(A), (B, C, D, E)
Eva (L5)		170 $\pm$ 25(h)	42 $\pm$ 2.6(o)	17.29(u)	1.09	1.07	42	(A, B, C, D, E)
Edmonson (L6)		33 $\pm$ 8.8(h)	55 $\pm$ 3(o)	225 $\pm$ 23(o)	10.20(q)	1.15	26.7	(A, B, C, D, E)
Finney (L5)	12 (a)	160 $\pm$ 16(h)	50.2 $\pm$ 3.3(o)	375 $\pm$ 38(o)	7.13	1.058	17.5	(A, B, C, D, E)
Harrisonville (L)		240 $\pm$ 43(k)	68 $\pm$ 2.4(s)	355 $\pm$ 24(t)	2.58	1.124	6.2	(B, C) (A, D, E)
Keyes (L6)	30 (d)	50(f)	52(f)	393(f)	9.0	1.085	21	(A, B, C, D, E)
Madhipura (L)		3(f)	68(f)	480(f)	10.35	1.149(l)	15	(A, B, C, D, E)
Shaw (L)	6.5 (e)	600(e)	36 $\pm$ 4(h)	280(e)	5.37	1.12(g)	0.8	(A, B, C, D, E)
St. Severin (LL)	25 (d, e)	4.2 $\pm$ 1.4(h)	19.3 $\pm$ 1.4(o)	43 $\pm$ 7(j)	0.29(g)	1.13(f)	11.2	(A, B, C, D, E)
		210(f)	65(f)	381(f)	4.8(f)	1.10(f)		(A, B, C, D, E)
		10(f)	79(f)	420(f)	5.3(f)			(A, B, C, D, E)
Udaipur (H)	9 (e)	400(e)	38 $\pm$ 6(h)	320(e)	10.85(l)		22	(A, B, C, D, E)

(a) Bhandari *et al* (1980a), (b) Bhandari *et al* (1986), (c) Bagolia *et al* (1978), (d) Bhandari and Potdar (1982), (e) Bhattacharya *et al* (1980), (f) Bhandari (1984), (g) *et al* (1981), (h) this work, (i) Nishizumi *et al* (1980), (j) Englert and Herr (1978), (k) Bhandari *et al* (1980b), (l) Rao M. N. (unpublished), (m) Gopalan *et al* (1977), (n) Bhandari (1978), (o) Heimann *et al* (1974), (p) Herpers and Englert (1983), (q) P Englert (private communication), (s) Herpers, *et al* (1968), (t) Herpers and Englert (1983), (u) For and references see Schultz and Kruse (1983), (v) Measurements [A (track density), B (^{26}Al), C (^{53}Mn), D (^{21}Ne), E ($^{22}\text{Ne}/^{21}\text{Ne}$)] made in the same samples or in samples located at approximately the same shielding depth are grouped together within parentheses.

having multiple exposure and using the criteria developed earlier (Bhandari and Potdar 1982), we confine the study to chondrites with single, one-stage exposure history. A three-isotope (^{26}Al - ^{53}Mn - ^{21}Ne) diagram is proposed which is sensitive to cosmic ray intensity variation. The position of data points in such a diagram allows us to determine the time variation in cosmic ray flux during the past 10^7 years.

2. Experimental details and data sets

We have selected a few meteorites which have already been extensively studied. These are listed in table 1. Track densities in many of these meteorites was not known. This was measured in minerals, olivines and pyroxenes, following standard procedures described earlier (Bhandari *et al* 1972). The cosmic ray track density in olivine is always smaller than in pyroxenes by a factor of two to three. A value of 2.9 has been used here to obtain equivalent density in pyroxenes for the purpose of determining shielding depths from the calculated track production profiles. Samples from Bansur, Madhipura, Udaipur and St. Severin were taken from the cores which have been studied earlier by Bhattacharya *et al* (1980). Samples of Keyes chondrite from different cores were kindly provided by Dr D D Bogard. Aliquots of some of the other meteorites studied here were obtained from Dr P Englert. The track density data in all these samples are listed in table 1. In case of St. Severin the track density, ^{53}Mn , ^{21}Ne and $^{22}\text{Ne}/^{21}\text{Ne}$ have been measured along several cores (Tamhane 1972; Lal *et al* 1969; Bhattacharya *et al* 1980; Englert and Herr 1980; Schultz and Signer 1976). Some ^{26}Al measurements have been made (Marti *et al* 1969) but a depth profile is not available. We have therefore measured ^{26}Al in some samples of core CB4 and A III on a low level gamma-gamma coincidence detector assembly using two 7.5 cm $\times$ 7.5 cm NaI (Tl) activated crystals with a plastic scintillator active anticoincidence shield. The background of the system is 1.6×10^{-3} cpm and β^+ counting efficiency is 2.3% for a typical sample of 40 g. The

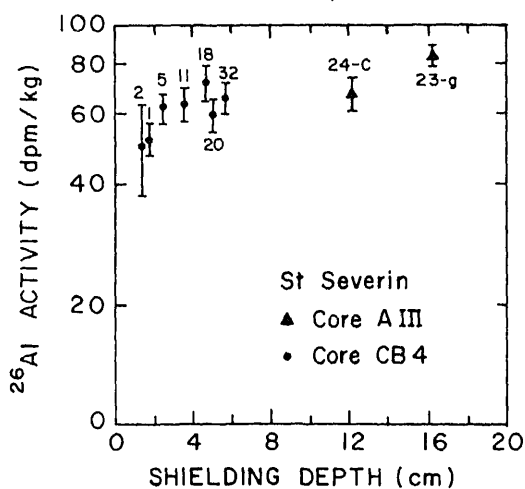


Figure 1. Preatmospheric depth profile of ^{26}Al in St. Severin core samples (Effective radius, $R_E = 25$ cm). The shielding depths are calculated from the observed track density. Sample number is marked on each point.

1978; Wright *et al* 1973). The $^{26}\text{Al}/^{53}\text{Mn}$ ratio in four samples of Keyes chondrite is 0.13 ± 0.01 . We use the ^{26}Al , ^{53}Mn , ^{21}Ne and $^{22}/^{21}\text{Ne}$ data in Dhajala samples obtained in our laboratory (Bhandari *et al* 1978; Gopalan *et al* 1977; Potdar 1981; Bagolia *et al* 1978). The $^{26}\text{Al}/^{53}\text{Mn}$ ratio in one sample measured is 0.16. In case of Keyes and St. Severin, the best fit curves drawn through the depth profiles summarised earlier (Bhandari 1984) have been used and two data sets, one corresponding to the near surface sample and other near the deepest point analysed are listed in table 1. In other cases we have used data reported by Müller *et al* (1981) and Englert and Herr (1978) as summarised by Moniot *et al* (1983). Shielding depths and approximate preatmospheric

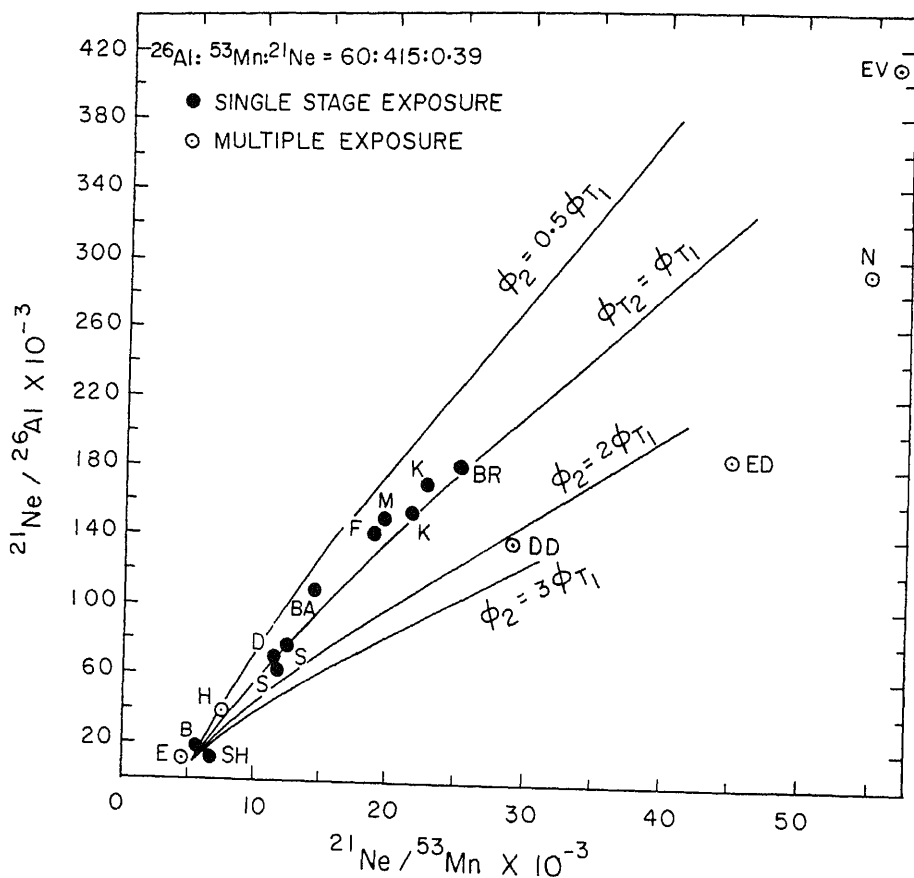


Figure 2. $^{26}\text{Al}/^{21}\text{Ne}$ vs $^{53}\text{Mn}/^{21}\text{Ne}$ correlation diagram for production ratios discussed in the text are plotted for various assumed changes in cosmic ray flux. $\phi_{T2} = \phi_{T1}$ represents the correlation curve for constant cosmic ray flux. If the cosmic ray flux has increased during the past 2 million years, the correlation curve will be below and if the flux has decreased, it will lie above the $\phi_{T2} = \phi_{T1}$ curve. B: Barwell, BA: Bansur, BR: Bruderheim, D: Dhajala, E: Etter, EV: Eva, ED: Edmonson, F: Finney, K: Keyes, M: Madhipura, N: Nogata, S: St. Severin, SH: Shaw, DD: Dalgety Downs, H: Harrisonville.

sizes, where possible, were calculated using the procedure given in Bhandari *et al* (1980a). All meteorites with long exposure ages (> 3 million years) have been included in table 1 and figure 2. The data on short exposure age meteorites cluster close to the line of constant cosmic ray flux as shown in figure 2 and hence do not indicate any variation in flux. Therefore, only a few typical cases are included.

3. Discussion

We will discuss these data in terms of variation of cosmic ray intensity. However, it may be useful to determine the changes expected in isotope production rates in meteorites for constant and variable fluxes first.

3.1 Isotope production in meteorites

The isotope production rate P_i in a meteorite depends on target element abundance (N_j), excitation function (σ_{ij}) and flux $\phi(E)$ of the primary and secondary nuclear-active particles above the threshold energy E_i . Since $\phi(E)$ depends on the development of nuclear cascade, it changes with the size of the meteorite (R_E , effective radius, if a spheroidal approximation is made) and shielding depth of the sample (X). Further, if the primary cosmic ray flux has been varying with time, time variation of $\phi(E)$ will lead to time variation in P_i . The average production over a time t is then given by

$$P_i = \left[\sum_j \int_{E_i} \int_{E_j} N_j \cdot \sigma_{ij}(E) \cdot \phi(E, R_E, X, t) dE dt \right] / \left[\int_0^t dt \right] \quad (1)$$

In case of stable isotopes, e.g. ^{21}Ne , the observed concentration, S_i , is given by

$$S_i = P_i \cdot t, \quad (2)$$

provided that no diffusive losses have occurred from the meteorite. On the other hand, in case of radioactive isotopes, e.g. ^{26}Al or ^{53}Mn , the observed activity, A_k , grows with time,

$$A_k = P_k [1 - e^{-t/T_k}], \quad (3)$$

where T_k is the mean life of the radioisotope. When $t \gg T_k$, i.e. at secular equilibrium $A_k = P_k$ and the exposure age of the meteorite can be calculated by measuring S_i and A_k if the production ratio P_i/P_k is independently known.

$$t = S_i / [A_k [1 - e^{-t/T_k}]^{-1} \times (P_i/P_k)] \quad (4)$$

In chondrites P_i and P_k change significantly (by upto a factor of 2) from sample to sample because of different shielding depths within a meteorite and its size. However, the ratio P_i/P_k is nearly the same (within $\pm 20\%$), if nuclides produced in similar reactions (or with similar nucleon energy) are considered. The exposure ages calculated from any isotope pair e.g. $^{21}\text{Ne}/^{26}\text{Al}$ or $^{21}\text{Ne}/^{53}\text{Mn}$ using (4) should therefore be identical for a given cosmic ray flux.

To determine P_i/P_k we consider three meteorites, St. Severin (LL), Keyes (L) and Dhajala (H), in which ^{53}Mn , ^{26}Al , ^{21}Ne , $^{22}\text{Ne}/^{21}\text{Ne}$ and tracks have been measured (Schultz and Signer 1976; Wright *et al* 1973; Englert and Herr 1980; Cressy 1975; Lal *et*

al 1969; Bhandari *et al* 1978; Bhattacharya *et al* 1980; Potdar 1981). These data form a complete set from which depth profiles have been constructed. The shielding depth in space for each sample was calculated from the track density values using the calculated production rates of Bhattacharya *et al* (1973). These profiles show that the *relative* production rates of ^{26}Al , ^{53}Mn , ^{21}Ne at any depth are nearly constant. The production rates of ^{26}Al : ^{53}Mn : ^{21}Ne are 60 atoms/min kg meteorite: 406 atoms/min kg Fe: 0.41×10^{-8} cc STP/g meteorite. M.Y. and their ratios are not very sensitive to depth or size of the chondrite. These rates are also similar to the model calculations of Bhandari (1981) which give production ratios of 60:415:0.39 (expressed in the units mentioned above) whereas the data of Englert and Herr (1978) in saturated meteorites yield values of 60:392:0.32. Using an altogether different set of experimental data Nishiizumi *et al* (1980) gave the production rates ^{26}Al : ^{21}Ne as 56 atoms/min kg meteorite (for H) or 60 atoms/min kg meteorite (for L and LL): 0.507 ± 0.039 (10^{-8} cc STP/g M.Y.) and ^{53}Mn : ^{21}Ne production rate as 414 atoms/min kg Fe: 0.302 ± 0.013 (10^{-8} cc STP/M.Y.). Moniot *et al* (1983) also favour a low production rate of ^{21}Ne of 0.28 ± 0.02 cc STP/g M.Y. based on ^{10}Be data. Thus there is considerable uncertainty in the production rate of ^{21}Ne . Here we adopt the production ratios in the proportion 60:415:0.39 given by Bhandari (1981) which depend slightly on target composition, shape and size of the meteoroid and shielding depth, though these effects are negligible, $< 20\%$ if we confine to ordinary chondrites. In case the cosmic ray flux changes without a significant change in the shape of the energy spectrum, the production ratio is given by

$$P_i/P_k = R_{ik} \bar{\phi}_i/\bar{\phi}_k, \quad (5)$$

where R_{ik} is the production ratio of the isotope for some arbitrary flux and $\bar{\phi}$ represents average flux for the period over which the isotope production is effective.

$$\text{Therefore } t = S_i/[A_k \{1 - e^{-t/T_1}\}^{-1} \cdot R_{ik} \cdot (\bar{\phi}_i/\bar{\phi}_k)] \quad (6)$$

We consider here two values of the exposure age t , calculated from $^{21}\text{Ne}/^{26}\text{Al}$ and $^{21}\text{Ne}/^{53}\text{Mn}$. In case the average cosmic ray flux has been the same over the mean life of ^{26}Al and ^{53}Mn , the two values of t should be the same and they should correlate as shown by the curve $\phi_{T_1} = \phi_{T_2}$ in figure 2. This figure also shows the correlation of t for some arbitrary change in flux in the past over periods (in million years) indicated by the suffix to ϕ . If the flux (ϕ_2) increases by a factor of two over the past two million years, compared to the flux (ϕ_{T_1}) before, i.e. $\phi_2 = 2\phi_{T_1}$, the exposure age correlation curve lies below the $\phi_{T_1} = \phi_{T_2}$ line whereas if the flux reduces to half, i.e. $\phi_2 = 0.5 \phi_{T_1}$, the curve lies above the $\phi_{T_1} = \phi_{T_2}$ line. The curves are significantly different for a factor of 2 or 3 variation in flux over 1 or 2 M.Y. period as shown in this figure so that such variations can be easily detected by comparing experimental data with these curves.

3.2 Complex exposure history of meteorites

It is assumed in the foregoing discussion that the meteorites are exposed to cosmic rays during their entire exposure period in a fixed geometry i.e. X and R_E do not alter with time. As far as the isotope production is concerned, change of X or R_E is equivalent to change of effective nucleon flux. Inherent in the mechanism by which meteorites are produced, such as in collisions of different bodies of the solar system (asteroids, for example), it is quite likely that the object is first irradiated to cosmic rays

buried in the asteroidal regolith (stage T_1) and then exposed as an independent body in space as a meteoroid (stage T_2). In such a case the nucleon flux experienced by the body in stage T_1 would be less than that in stage T_2 . Analysis of the rare gas data has revealed that about 28 % of meteorites show comparable exposure in stage T_1 and T_2 (Bhandari 1981). Therefore, in order to deduce information on cosmic ray variations from meteorite data, meteorites with complex exposure history must be excluded.

To distinguish the cases of multiple exposure and genuine cosmic ray variation we use the criterion suggested earlier by Bhandari and Potdar (1982). The tracks have a steep source function with path length of a few millimeters near the surface, increasing with shielding depth to a few centimeters at large depths (Bhattacharya *et al* 1973). The track production is therefore small in stage T_1 and their records in meteorites are mainly due to irradiation in stage T_2 . On the other hand, the spallogenic isotope production increases with depth, initially as a consequence of production of secondary nucleons, and then decreases with a path length of about 50 cm in chondrites (Arnold 1961; Bhandari and Potdar 1981; Reedy and Arnold 1972). The isotope production, for example of ^{21}Ne , therefore would be considerable in both the stages T_1 and T_2 . Furthermore in case of spallation neon isotope ratio $^{22}\text{Ne}/^{21}\text{Ne}$, it has been found that the ratio is sensitive to shielding depth (Bhandari and Potdar 1982; Eberhardt *et al* 1966; Cressy and Bogard 1976). In H chondrites, the ratio is about 1.24 near the surface

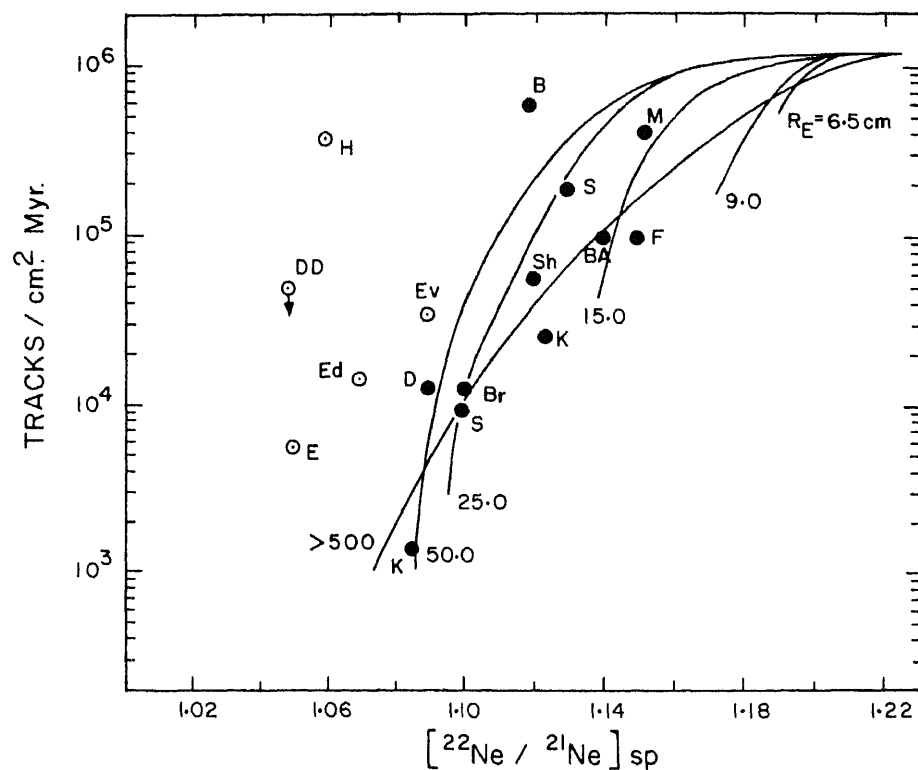


Figure 3. Track production rate vs $[^{22}\text{Ne}/^{21}\text{Ne}]_{\text{sp}}$ correlation diagram for various effective radii R_E , following Bhandari and Potdar (1982). The experimental data plotted here are given in table 1. Symbols are the same as in figure 2.

going down to about 1 at a depth of 2 meters. Track production rate vs $[^{22}\text{Ne}/^{21}\text{Ne}]_{\text{sp}}$ correlation diagram for single-stage exposure has been constructed for meteoroids of various sizes (figure 3). Any departure from the expected correlation for a particular size of the meteoroids can be taken to indicate multiple exposure history (Bhandari and Potdar 1982). We use this criterion in the following analysis to identify meteorites with single and multiple exposures. The track density and $[^{22}\text{Ne}/^{21}\text{Ne}]_{\text{sp}}$ for all the meteorites are plotted in figure 3. It is clear from this figure that there are six meteorites, namely, Bruderheim, Finney, Dhajala, Keyes, Shaw and St. Severin in which all the data are consistent with single stage exposure. Other meteorites such as Edmonson, Etter, Dalgety Downs and Harrisonville do not satisfy the criteria of single-stage exposure whereas Barwell and Eva are probably consistent with single exposure within the errors of measurement. The first stage exposure, T_1 , in these two cases, if any, has therefore been negligibly small.

3.3 Analysis of data

If we consider all the data sets, we find that the points lie on various curves, some on the line of constant flux $\phi_{T_1} = \phi_{T_2}$ and others like Dalgety Downs, Edmonson and Nogata towards the line representing $\phi_2 = 2\phi_{T_1}$ (figure 2). Clearly if there is a genuine cosmic ray flux variation, all meteorites must show this effect and the data points must lie on the line characteristic of the change of flux. This is not the case. All the well studied meteorites with single-stage exposures e.g. Bruderheim, Dhajala, Keyes and St. Severin lie close to the curve of constant cosmic ray flux $\phi_{T_1} = \phi_{T_2}$. The meteorites which deviate from this curve are those having complex exposure as indicated by figure 3. Therefore one can conclude that the apparent departure of data from constant flux line in figure 2 is only due to complex exposure of some of the meteorites and not due to increase in the galactic cosmic ray intensity 2 million years ago. Thus the isotope data in meteorites having simple exposure history do not indicate any variation over the past 1 or 2 million years compared to the intensity over the past 10^7 years.

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Absorption of radio waves during a solar eclipse

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Abstract. In this paper, the results of our observations on AI-method ionospheric absorption of radio waves on 1.8 and 2.2 MHz during the solar eclipse of 16 February 1980 are presented. The absorption decreased by about 41% and 46% of the normal value respectively at the above two frequencies at Ahmedabad following the maximum phase of the eclipse (about 77% of full disc) with a delay of 18 minutes. The quantity $A_T(f)$ which is a measure of $\int N \nu dh$ is now examined for better clarity of the influence of the changes in the E-layer. The results are discussed in relation to the observations of the ionizing radiations from the sun, changes in the electron density, recombination rate and absorption in the underlying D and E regions.

Keywords. Ionospheric radio wave absorption; solar eclipse.

1. Introduction

A rare event of total solar eclipse of about 150 sec duration occurred on 16 February 1980 and the belt of totality, about 120 km wide, crossed the plateau of South India along the path from Karwar (14.6°N, 74° E) to Puri (20° N, 86° E) during the interval 1008-1020 hr U.T. At Ahmedabad (23° N, 72.6° E), the maximum eclipse was 77%.

Figure 1 shows the map of India depicting the maximum eclipse and the time of its

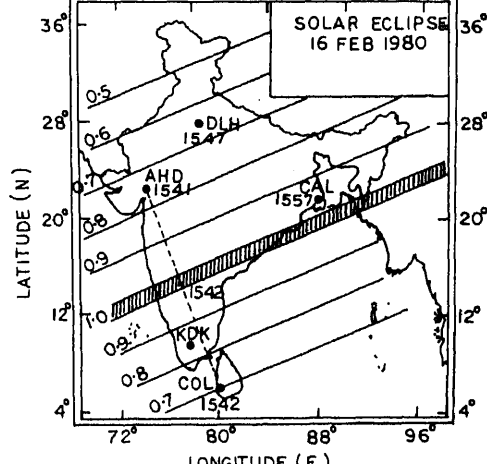


Figure 1. Map of India and Sri Lanka showing maximum percentage eclipse, contour-lines and the time of the maximum phase of solar eclipse at some selected places. AHD: Ahmedabad, DLH: Delhi, CAL: Calcutta, KDK: Kodaikanal, COL: Colombo and at midpoint of Ahmedabad-Colombo path in the totality zone.

occurrence in IST (U.T. + 5.5 hr). We were particularly interested in observing the changes in ionosphere which affect radio communication in SW band. Such observations have been made and interpreted in several ways earlier (Rawer 1956; Piggott 1956; Davies 1956; Landini *et al* 1966; Bischoff and Taubenheim 1967; Sechrist 1970). Some interesting results on ground-based and rocket-borne observations were reported for the annular solar eclipse of 20 May 1966 at a mid-latitude place in Greece. A preliminary report on some results of ionospheric radio effects at Ahmedabad during the eclipse of 16 February 1980 was published earlier (Jani *et al* 1982). Here, we present results of further studies made on the changes of ionospheric absorption on 1.8 and 2.2 MHz that took place during this solar eclipse.

2. Total radio pulse absorption and $A_T(f)$

Al-absorption (L , dB) of radio pulses was measured at intervals of 15 min on the day of eclipse and 30 minutes on ± 3 control days. The mean values on control days were almost the same as monthly median values at corresponding hours and were therefore taken as normal values.

As reported earlier (Jani *et al* 1982) the total absorption on the normal day and that on the eclipse day at the time corresponding to the lowest value of absorption 18 minutes after the maximum phase was as follows:

$$L_{tN}(1.8) = 32 \text{ dB}; L_{tE}(1.8) = 19.0 \text{ dB}; \text{decrease} = 41 \%,$$

$$L_{tN}(2.2) = 23 \text{ dB}; L_{tE}(2.2) = 12.5 \text{ dB}; \text{decrease} = 46 \%.$$

Subscripts t , N and E stand for total, normal and eclipse respectively. The fall in absorption began with the commencement of the eclipse, initially at a slower rate and then at a faster rate after the eclipse reached 40 % (i.e. 60 % unobscured disc). During the time interval of the eclipse both on the control days and on the eclipse-day, there was no low-type blanketing E_s which produce a decrease in total absorption by an amount nearly equal to the deviative absorption in the E -layer (Kotadia *et al* 1977).

Since the total absorption (L_t) is a mixture of non-deviative absorption (L_{nd}) especially in the D -region and deviative absorption (L_d) in the E -region which varies in a complex manner, it is appropriate for interpretation of results to follow the method proposed by George (1971), and Samuel and Bradley (1975) which gives a quantity called $A_T(f)$ expressed in dB.MHz². This takes into account a time-dependent function $\phi_n(f_r/foE)$ where f_r is the observing radio frequency, and foE is the ordinary-wave critical frequency of the E -layer. In the eclipse function $(1-f)\cos\chi$ for the eclipse-day, χ is the solar zenith angle (SZA) and f denotes fraction of the sun's disc eclipsed. Since both $(1-f)$, i.e. visible part of the disc and $\cos\chi$ decrease upto the time of maximum phase, and later change in reverse order, the slope of the $(1-f)\cos\chi$ curve is steeper before the maximum phase than that after it. As a result of such dissimilarity, the electron density $N(h)$ -profile and the height of reflection undergo changes in such a way, as explained by Leid (1956), that the total of two components L_{nd} and L_d varies in a complicated manner. The study of $A_T(f)$ which is a good measure of $\int N\nu dh$, ν being the collision frequency, greatly reduces the confusion arising from such a situation.

The values of $A_T(1.8)$ and $A_T(2.2)$ calculated for the eclipse-day and control days at corresponding times are shown in figure 2 along with the eclipse-SZA function. It may be

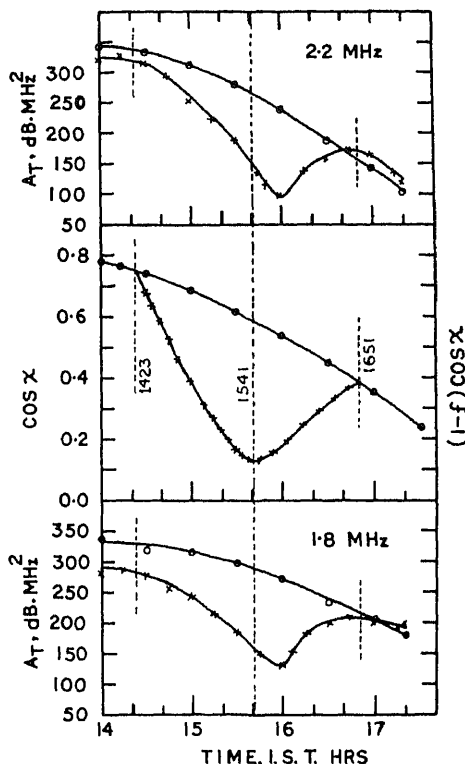


Figure 2. Variation of $A_T(1.8)$ and $A_T(2.2)$ during the solar eclipse of 16 February 1980 compared with its control days' mean value. Also shown are the time variation of $\cos \chi$ and eclipse-associated $(1-f) \cos \chi$. Times of first contact, maximum phase and last contact of the eclipse are marked by vertical dashed lines.

stated that the points for $A_T(f)$ lie on a smooth curve as compared to the scatter that was seen in total absorption reported earlier. But still, the delay time of about 18 min and the asymmetry between the variation of $A_T(f)$ before and after the maximum eclipse remain. Almost the same delay was found in the maximum drop of foE on the eclipse-day from the normal value. It fell from 3.5 MHz to 2.7 MHz, i.e. the E -layer maximum electron density NmE decreased by 45% accompanied by a rise in height hmE from 110 km to 120 km (Chandra *et al* 1981).

3. Unobscured solar disc and change in $A_T(f)$

In order to see the effect of the eclipse clearly as to how much was the reduction in $A_T(f)$ and in what manner it varied, the ratio A_{TE}/A_{TN} is plotted against time in figure 3. Also shown is the unobscured part $(1-f)$ of the solar disc, $\cos \chi$ being a common factor on control days and the eclipse-day. It may be seen that the fall in $A_T(f)$ in the beginning was small, but became rapid after the optically visible part of the disc fell below 60%. Values of the fall in $A_T(f)$ corresponding to the fraction of the solar disc left unobscured are given in table 1. Thus for a decrease in the visible part of the sun's

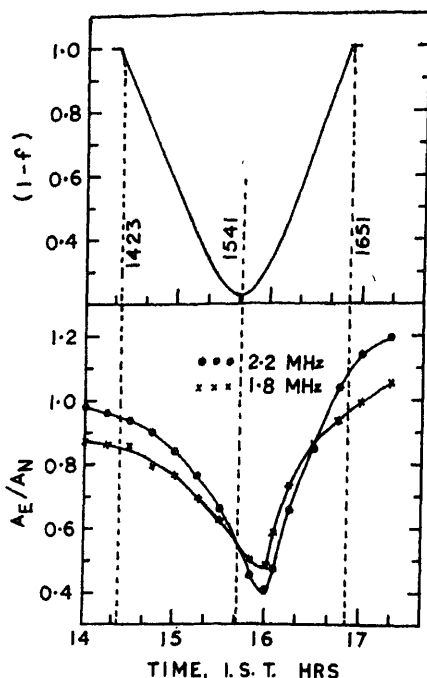


Figure 3. Variation of ratio A_{TE}/A_{TN} on eclipse-day on 1.8 and 2.2 MHz. The fraction of the solar disc not eclipsed is also shown for comparison.

Table 1. Decrease in $A_T(f)$ at instants of different visible parts of the solar disc relative to normal value for fully visible disc.

(1-f)	$A_T(1.8)$		$A_T(2.2)$	
	A_{TE}/A_{TN}	Decrease %	A_{TE}/A_{TN}	Decrease %
0.60	0.78	22	0.85	15
0.30	0.65	35	0.69	31
0.23	0.55	45	0.56	44
*	0.48	52	0.40	60

*Delayed effect 18 min after maximum phase.

disc from 0.60 to 0.30, or by a factor of 2, $A_T(1.8)$ dropped from 0.78 to 0.65, or by a factor of 1.2 and similarly $A_T(2.2)$ fell by a factor 1.23, but not by a factor $(1-f)^{1/2}$ or $(2)^{1/2}$ in this case. Further, for 23% open disc at maximum phase, $A_T(1.8)$ fell to 0.55 and $A_T(2.2)$ to 0.56 of normal. This means that for a further decrease in the visible part of the solar disc by a factor of 1.3, the change in A_T was faster than for the earlier part of the eclipse. After the delayed minimum, $A_T(1.8)$ almost half of normal, it recovered quickly to the normal value a little before the last contact within ± 15 dB MHz² or absorption within ± 2 dB limits. It should be pointed out that the variation of A_T with frequency

than that of $A_T(1.8)$. Thus, in contrast to the symmetry seen in the optical eclipse before and after the maximum phase, the changes in total absorption L_t as well as in $A_T(f)$ showed considerable asymmetry.

4. Discussion and conclusions

The decrease in the total ionospheric absorption L_t of about 41 % on 1.8 MHz and 46 % on 2.2 MHz, and the corresponding decreases of 52 % in $A_T(1.8)$ and 60 % in $A_T(2.2)$ from the normal value due to the eclipse following its maximum phase do not seem to be in conformity with the area of the solar disc left unobscured, nor is the rate of decrease in L_t or $A_T(f)$ in tune with the instantaneous changes in the obscuration of the solar disc. In the *E*-region, the maximum fall in NmE on the eclipse-day was 45 % of the normal. Let us examine how these results compare with known facts. The active regions on the sun emitting UV/EUV radiations responsible for ionization in the *D* and *E* regions are not uniformly distributed over the whole solar disc (Friedman 1959; Hall 1970, NOAA 1980). The small scatter seen in the changes of L_t and $A_T(f)$ in the initial stage of the eclipse could be a real effect indicating obscuration of irregularly distributed active areas or the so-called 'hot-spots' which on the average are smoothed out to normal. Secondly, the decrease in electron density $N(h)$ is not necessarily in direct proportion to the decrease in electron-ion production rate q or the ionizing radiation flux F at different altitudes, e.g. N varies as q in the lower *D*-region whereas N may vary as $(q)^{1/2}$ or in some other way in the upper *D* and lower *E* regions (Jespersen and Pedersen 1970; Haug *et al* 1970; Kane 1970). Such a dependence of N on q would cause comparatively greater decrease in L_{nd} in the *D*-region than increase in L_d in the *E*-region where height of reflection increases at a given frequency during the eclipse. Elwert (1958) and Taubenheim and Serafimov (1969) showed that 10 to 15 % of coronal x-rays remained unobscured even during totality of the solar eclipse, but according to Willmore (1970) and Haug *et al* (1970), the quiet-sun x-rays are not important for ionization at around 75 km altitude. All these factors concerning radiation and ionization suggest a greater decrease in L_{nd} than an increase in L_d . Hence, neither L_t nor $A_T(f)$ is likely to have any definite correlation with the unobscured part of the solar disc.

To substantiate the above conclusion, L_{nd} was found from a rough estimate of L_d by using Jaeger's (1947) method. The results obtained at the time corresponding to the maximum fall in foE are given in table 2, which shows a clear decrease in L_{nd} and an increase in L_d on the eclipse-day as compared to their normal values, the effect being

Table 2. Content of L_{nd} and L_d in total absorption on the eclipse day and control days at the time corresponding to maximum drop in foE during the eclipse.

f (MHz)	Control days normal		Eclipse-day	
	L_{nd}	L_d in % of L_{tN}	L_{nd}	L_d in % of L_{tE}
1.8	82	18	68	32
2.2	75	25	47	53

larger on 2.2 MHz than on 1.8 MHz. If, however, only the magnitudes in dB of the L_{nd} component on the normal and the eclipse days are considered, a decrease of 51% was found in it on 1.8 MHz and 65% on 2.2 MHz during maximum drop, mainly due to decrease in the D -region ionization. Incidentally, it is worth noting that the content of deviative component L_d was 53% in the total absorption observed on the eclipse-day at 2.2 MHz when foE dropped to 2.7 MHz. Thus, interestingly, the drop in $A_T(f)$ is greater than that in L_i by about 11–14%, but the remarkable point is that the drop (52%) in A_T on the lower frequency 1.8 MHz is very nearly the same as that (51%) in L_{nd} on the eclipse-day. The rapid decrease and recovery of $A_T(f)$ on 2.2 MHz is probably due to faster change in ionization in the upper D -region by the covering and uncovering of the solar ionizing radiations.

Rocket flights were made at SHAR (13° N, 80° E) and Thumba (8° N, 76° E) only on the eclipse-day of which one of the two flights at the former and three out of four at the latter were successful (Gupta and Chakravarty 1982). From the rocket-borne probe measurements, it was found that the electron density decreased significantly at all levels in the D and E regions as the eclipse progressed which also included the normal diurnal variation, but it was not possible to find how much was the decrease relative to that on the normal or control days. Acharya *et al* (1982) constructed $N(h)$ profiles at Udaipur (24.6° N, 73.6° E) so as to match the observed L_i on the control days and the eclipse-day at the time corresponding to the maximum phase of the eclipse (72%, 1547 hr IST). They showed a decrease in $N(h)$ by a factor of about 7 at 70 km altitude, but hardly by a factor of 1.5 at 80–85 km. From our experience in constructing model $N(h)$ profiles using ground-based measurements of ionospheric absorption, partial reflections and ionosonde data (Gupta and Kotadia 1979; Datta *et al* 1983) and the method adopted by Mechtly and Bilitza (1974) to fit rocket-borne measurements which is also incorporated in the international reference ionosphere (IRI) modelling, we constructed $N(h)$ profiles at Ahmedabad for different solar zenith angles on the normal and the eclipse days such that the calculated and measured L_i were in agreement. L_i from 70 km altitude upto the height of reflection of the radio waves at the two frequencies was calculated by the generalized magnetoionic theory of Sen and Wyller (1960). These $N(h)$ profiles are not presented here. Instead, decreases in $N(h)$ on the eclipse-day from its control days mean values at different altitudes are shown in figure 4 for the time corresponding to the

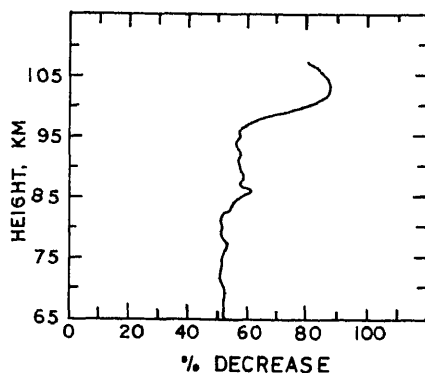


Figure 4. Percentage decrease of electron density on the eclipse-day from that of the control days' mean value at different heights.

lowest value of A_T to show clearly the direct connection of decrease in radiowave absorption with that in $N(h)$. It is seen that the D -region electron density in the height range 70–90 km decreased by 52–56% i.e. approximately by a factor of 2 and it is about the same as the decrease in $A_T(f)$ and also in L_{nd} , and this agreement seems to be better at the lower frequency. These results amply prove the reliability of our method for constructing $N(h)$ profile. Another thing to note is that there was about 90% decrease in electron density near 105 km altitude, though the decrease in NmE was only 45% of the normal value. This should be distinguished from the increase in hmE on the eclipse-day by about 10 km over the normal value of 110 km with the result that N decreased sharply at around 105 km. The increase in height of reflection and decrease in NmE result in the increase of deviative absorption L_d as shown before, more so at the frequency closer to foE .

Finally, the time-delay of 18 min in the absorption or $A_T(f)$ and foE minima after the maximum phase of the eclipse is just half of what is normally found in February for their diurnal maximum after noontime. This result is in conformity with the theoretical considerations (Rydbeck 1956) and the delay reported by Davies (1956) for an eclipse in the forenoon of summer at a place in auroral zone. But, it may vary from 0 to 23 minutes as reported in literature depending on the latitude of the place, the season and the time of maximum phase of the eclipse. The initial slow rate of decrease in $A_T(f)$ after the first contact may be due to obscuration of only a small amount of ionizing radiation flux coming from high latitudes of the solar disc, and in addition, to some effect of ionospheric sluggishness to the causal phenomena.

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Effects of linear trend and mean value on maximum entropy spectral analysis

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Abstract. Using artificial samples, it is shown that, even for the method of maximum entropy spectral analysis (MESA) evolved by Burg, a presence of a linear trend gives distortions (frequency shifts) in the low frequency (high periodicity) region and hence, it is advisable to eliminate linear trends before attempting spectral analysis.

The effect of the presence of a non-zero mean is not very definite; but, very large mean values seem to reduce the sharpness of the peaks.

The implications of these findings for the results by Jin and Thomas for some geomagnetic parameters are discussed.

Keywords. Linear trend; maximum entropy spectral analysis; high periodicity; low frequency; geomagnetic parameters.

1. Introduction

For the study of periodicities in time series, Burg (1972) evolved the method of MESA (maximum entropy spectral analysis) which is considered superior to earlier methods like those of Blackman and Tukey (1958). Ulrych and Bishop (1975) have given a critical appraisal of MESA.

There are some well-known drawbacks of MESA. Chen and Stegan (1974) showed that in cases where some large periodicities were involved, the spectral maxima may show frequency shifts (errors) as large as 20 %, depending upon the initial phase. Also, unlike the Blackman and Tukey method (1958) for which statistical significances can be estimated for the various observed peaks, there is no clear-cut method of estimating the statistical significance of MESA peaks.

In geophysical studies, spectral analysis is used frequently. Some geophysical parameters, like the geomagnetic field, have secular variations involving periodicities of several decades or sometimes just linear trends. Before subjecting such series to spectral analysis in search of smaller periodicities, it is recommended that long-term trends be removed (Bingham *et al* 1967) and many workers do so (e.g. Bhargava and Yacob 1970). However, recently, Jin and Thomas (1977) used MESA for studying the periodicities in the time-series of the geomagnetic dipole field H_0 (1901–1969) without removing the considerably large linear down-trend present in the data. We want to explore, with the help of artificial samples, whether the presence or absence of a non-zero mean and a linear trend affects the accuracy of the frequency determination. The drawbacks of MESA referred to above will be considered only to the extent that they affect conclusions about this specific aspect. Finally, the data used by Jin and Thomas (1977) will be

It must be borne in mind that MESA is a nonlinear method for the spectral estimate. Hence, the elimination of trends and the mean may affect the spectral estimate. We hope to get some clues for this effect from the study of the artificial samples. In physical series, much will depend upon the physical models for a given time series.

2. Analysis of artificial samples

We produced an artificial time series $f(t)$ composed as:

$$f(t) = a_0 - a_1 t + \sum_{k=1}^5 b_k \sin 2\pi (t/T_k) \quad (1)$$

where t ranged from 1 to 100, the five sinusoids had periods $T_k = 5, 10, 20, 40, 80$, the amplitudes b_k were set at 10 for all k , and the constants a_0 and a_1 were varied. First, a_0 was set at 1000 and the slope a_1 was given values $a_1 = 0, 12, 24$ in turn. Thus, for the largest slope of $a_1 = 24$, the series started with a value of about 1050 and ended with a value of about -1350. All the series were subjected to MESA, using the flow chart given by Anderson (1974).

The upper half of figure 1a shows the results for the length of the prediction error filter (LPEF) = 25, 50, 75, 85, 95. In each case, the full curve refers to $a_1 = 0$, i.e. the sample with no linear trend, and the dashed curve refers to $a_1 = 24$. Curves for $a_1 = 12$ are not shown; but the three numbers one below the other represent the periodicities observed for $a_1 = 0, 12, 24$. The ordinate is power spectral density on a log scale. Usually, the power is calculated at fixed frequency intervals Δf in the frequency range $f = 0$ to 0.5, the latter being the Nyquist (folding) frequency. However, in such a case, the fractional ratio $(\Delta f/f)$ becomes non-uniform and is large for small frequencies and gives large steps for large periodicities. Hence, we chose frequencies corresponding to periodicities T in steps of $\log_{10} T$ of 0.005 for the $\log_{10} T$ range of 0.300 to 2.300 corresponding to $T = 2$ to 200. This ensured that the steps were within about $\pm 1\%$ of periodicity T for all bands. In figure 1(a), the following may be noted:

- (1) For LPEF = 25 (i.e. 25% of the data length of 100), the resolution is poor. Only for LPEF of 50 or more, all the peaks are resolved, but only in the trendless sample (full lines). In the sample with linear trend (dashed lines), only the peaks at $T = 5, 10, 20$ are resolved while those at $T = 40$ and 80 are either missed or displaced, even at very high LPEF.
- (2) At higher LPEF, the low periodicity peaks show splitting. This is a well-known feature and can be corrected for by using the modification formulated by Fougere (1977).

From the above example, it seems that the presence of a linear trend may cause considerable distortions in the low frequency (high periodicity) region.

For the above sample, a_0 was 1000. To study the effect of large a_0 values, the value of a_0 was increased to 32000. The two curves in the lower half of figure 1a refer to spectra of the samples with $a_0 = 32000$ and, $a_1 = 0$ (full curve) and $a_1 = 24$ (dashed curve). The peaks are now less sharp. However, whereas for $a_0 = 1000$, $a_1 = 24$, LPEF = 95, the peaks at $T = 40$ and $T = 80$ were not resolved, for $a_0 = 32000$, $a_1 = 24$, LPEF = 95, the peaks are resolved. Thus, a very high initial value or mean seems to help in resolving the peaks in the presence of a linear trend. However, this needs further scrutiny.

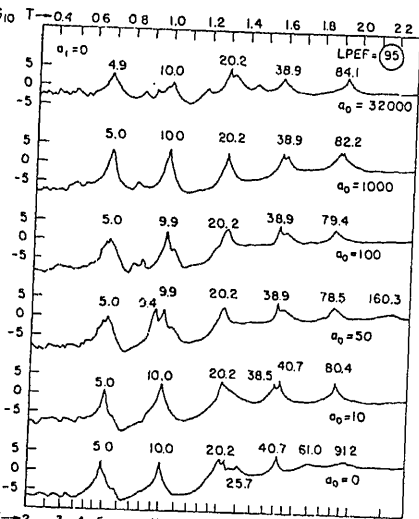
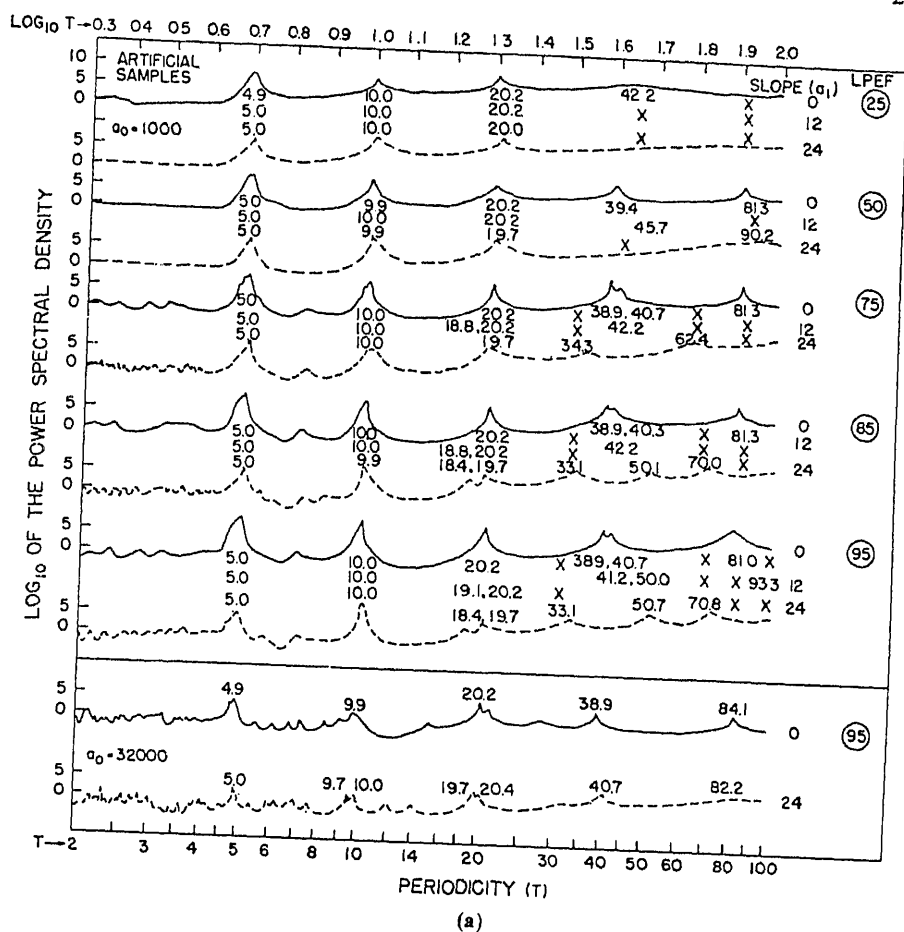


Figure 1. Burg spectra of artificial samples ($a_0 - a_1 t + \text{sinusoids}$) of 100 data points ($t = 1$ to 100) and five sinusoids of periods $T = 5, 10, 20, 40, 80$ with, (a) linear down-trends of slopes $a_1 = 0$ (full lines) and $a_1 = 24$ (dashed lines). The set of three numbers one below the other represent peak periodicities for $a_1 = 0, 12, 24$. Value of $a_0 = 1000$ for all curves except the bottom two curves for which $a_0 = 32000$, LPEF = 95 and, $a_1 = 0$ (full curve), $a_1 = 24$ (dashed curve); (b) no linear trends (i.e. $a_1 = 0$) but

To study the effect of a_0 values alone in greater detail and, in particular, to see whether a sample expressed as a deviation from the mean gave better results, we set $b_k = 10$ as before, set $a_1 = 0$ (i.e. no linear trend) and set $a_0 = 0, 10, 50, 100$ in turn. Figure 1b shows the spectra for LPEF = 95 only, for $a_1 = 0, b_k = 10$ and $a_0 = 0, 10, 50, 100, 1000$ and 32000, the latter two being the same as the full curves at the bottom of figure 1a. As can be seen, all the peaks are resolved reasonably well in all the curves (but for some peak splitting) except the bottom curve for $a_1 = 0$ where the high periodicity peak $T = 80$ is not resolved but splits into $T = 61.0$ and $T = 91.2$. Thus, spectra of a sample expressed as deviations from mean may not depict the very high periodicity peaks correctly. On the other hand, adding even a small number as 10, something of the same order as the amplitudes of the sinusoids, seems to help in resolving the very high periodicity peak, while adding too large a number (like $a_0 = 32000$ for the top curve) may distort the high periodicity peak by a few percent and reduce the sharpness of the peaks.

To test samples having linear trends only, i.e. having no sinusoids but only a down (or up) linear trend, we set $b_k = 0$ and $a_1 = 25$ and $a_0 = 1250$ and 32000. The former ($a_0 = 1250$) gives a sample having values ranging from +1250 to -1250 and thus having a mean zero. Figure 2 shows the spectra. Even though the sample contains no periodicities, the spectra show very low power for high frequencies only. For low periodicities, the power is comparatively large, indicating that a large linear trend could create distortions for periodicities in this region. The change from low power to high power is larger in the case of a lower mean ($a_0 = 1250$, lower curve) and occurs at lower periodicities. Thus, analysis of data which are expressed as deviations from mean will probably have larger distortions due to the presence of linear trends as compared to the data with the same linear trend but larger base value ($a_0 = 32000$, upper curve).

In the samples chosen so far, all sinusoids had the same amplitude ($b_k = 10$) and the periods formed a binary progression. To confirm that such a choice did not produce any special effect, we changed the amplitudes to $b_k = 10, 5, 10, 5, 10$ and chose various sets of T_k e.g. $T_k = 6, 10, 16, 36, 76$ and many similar ones where the successive T_k had no simple relationship. In all cases, results were similar to those mentioned above for $b_k = 10$ and $T_k = 5, 10, 20, 40, 80$.

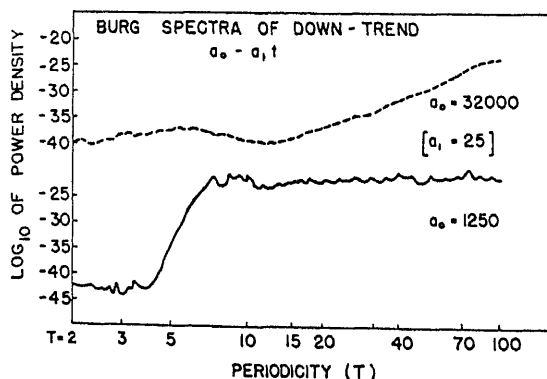


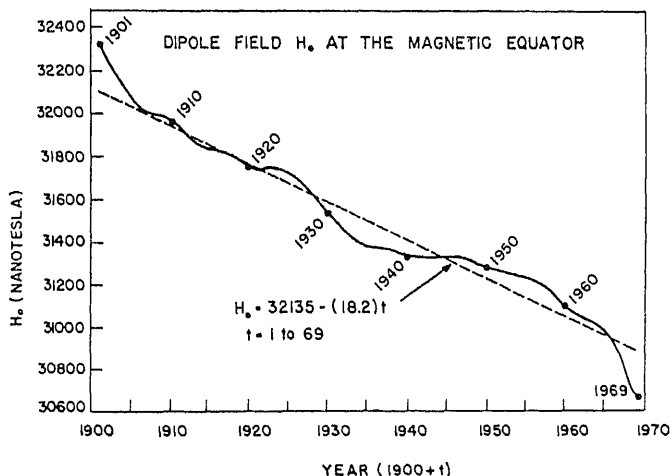
Figure 2. Burg spectra of pure linear trends ($a_0 - a_1 t$) for $a_1 = 25$ and $a_0 = 32000$ (upper, dashed curve) and $a_0 = 1250$ (lower, full curve).

We also tested whether, by removing some periods from the data by suitable filters, the residues showed the remaining periodicities in MESA. We found that all the periodicities which now existed were seen as sharp peaks in MESA. However, in all cases, the relative heights of the peaks were not in the same proportion as expected. This can be seen in figure 1 also where all peaks are not of the same height. But, there are no unwanted peaks either, at least in LPEF less than 50% of data length.

3. Analysis of the geomagnetic field and other parameters

From a spherical harmonic analysis of the annual mean magnetic data at various geomagnetic observatories round the globe, Jin and Thomas (1977) calculated the yearly Gauss-Schmidt coefficients and also H_0 , the dipole field at the magnetic equator. Figure 3 (full curve) shows a plot of the yearly values of H_0 for 1901–69. From about 32300 nT in 1901, the value of H_0 dropped to about 30600 nT in 1969, a drop of about 1700 nT in 70 years i.e. about 24 nT/year. However, the values near 1901 seem to be abnormally large and those near 1969 abnormally low. When a first-degree polynomial (straight line) is fitted statistically, the dashed line in figure 3 has a form $H_0 = 32135 - (18.2)t$, having a slope of only about 18 nT/year.

Figure 4 shows the spectra for H_0 (full lines) for LPEF = 25, 50, 60, 65, 68 which is 36%, 72%, 87%, 95%, 98% of the data length 69. For low LPEF, very few peaks are resolved. At very high LPEF, many peaks are resolved. The numbers represent periodicities in years. For LPEF = 68 the numbers in boxes are those reported by Jin and Thomas (1977) and tally with our numbers, indicating that our method of spectral analysis is the same as used by them. The periodicities $T = 66.8, 34.3, 22.1, 17.8, 13.3, 11.1, 9.4, 8.5, 7.0$ do seem to be harmonics of a fundamental period $T = 66.8$. However, when the linear trend is removed from the data, the Burg spectra are as shown by dashed lines in figure 4. For LPEF = 68, the peaks are now at $T = 47.3, 28.5, 20.2, 16.2$,



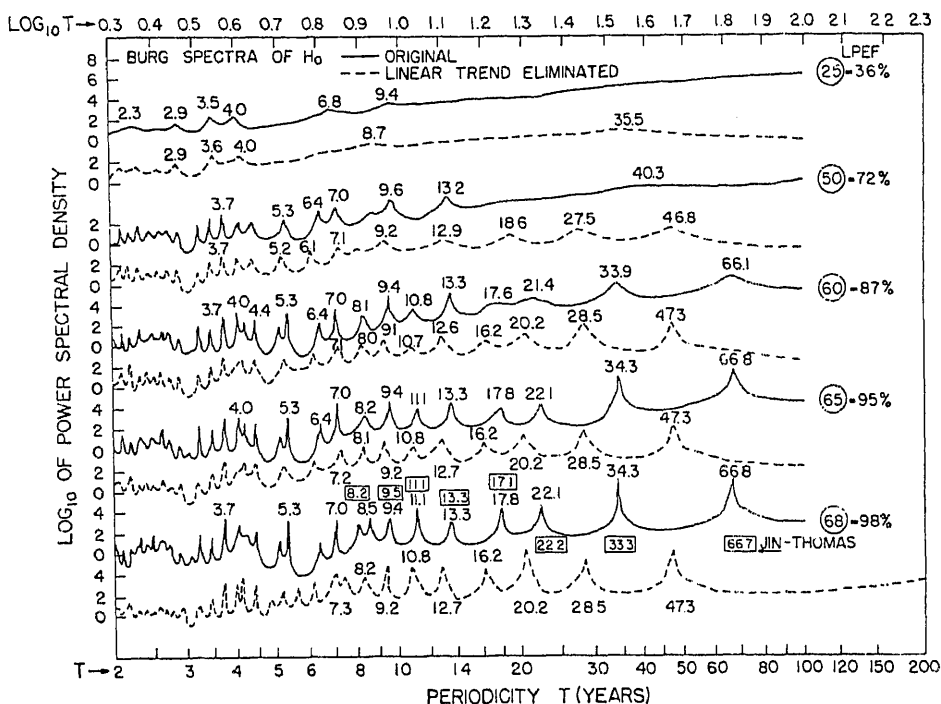


Figure 4. Burg spectra (full lines) of the 69 yearly values of geomagnetic dipole held H_0 for LPEF = 25, 50, 60, 65, 68 i.e. 36, 72, 87, 95, 98% of the data length. Numbers represent periodicities in years. For LPEF = 68, the numbers in boxes are the periodicities obtained by Jin and Thomas (1977). The dashed lines represent Burg spectra when a linear trend is statistically removed.

12.7, 10.8, 9.2, 8.2, 7.3 which seem to be odd harmonics (3rd, 5th, 7th etc.) of a fundamental period $T = 142$ (though such a peak was not actually observed when analysis was extended beyond $T = 100$ as shown in figure 4, bottom curve). We do not propose to discuss the physical cause of these periodicities, as our purpose is only to study the distortion effects due to linear trends. It would seem that the answers obtained by Jin and Thomas, viz, $T = 66.8, 34.3$ etc. may not be the correct ones, which seem to be $T = 47.3, 28.5$ etc.

Jin and Thomas (1977) have reported results for two more physical parameters. One is the time rate of change of the geomagnetic dipole field ($\dot{H}_0$) and the other is the fluctuations of the length of the day (l.o.d.). The plots of these parameters are shown in figures 5 and 6. Figure 5 refers to $\dot{H}_0$ for 1905–65 and the 61 values are in the range (0 to -50) nT/year with an average value of about -25 nT/year. Figure 6 refers to l.o.d. for 1865–1961 and the 97 values are in the range $+4$ to -4 milliseconds, with an average value of about zero but with a long-term up-trend shown by the dashed line. For both these samples, Jin and Thomas (1977) have reported peaks similar to those for H_0 , which led them to believe that all these have a common physical origin. We obtained spectra for the $\dot{H}_0$ and l.o.d. shown in figures 5 and 6 and obtained peaks similar to those reported by Jin and Thomas. However, when H_0 values were expressed as deviations from the mean of the sample, and l.o.d. values were corrected for the linear trend, the spectra looked as shown in figure 7. In the upper half, the full curve refers to

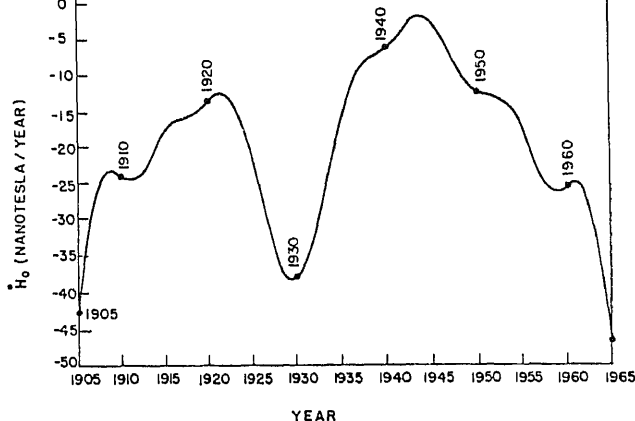


Figure 5. $\dot{H}_0$, the time rate of change of the geomagnetic dipole field H_0 , for 1905–65.

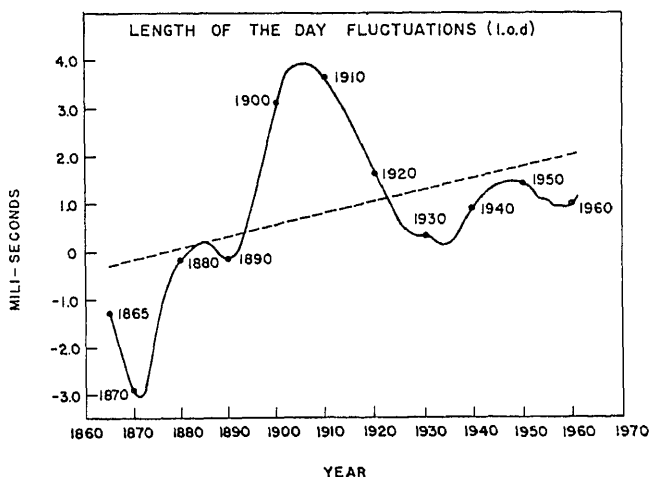


Figure 6. Fluctuations in the length of the day (l.o.d.) for 1865–1961. The dashed line is the statistical fit of a linear trend.

spectra of the original 61 values of $\dot{H}_0$, for an LPEF = 60 i.e. 98 %, and the dashed curve refers to the values of $\dot{H}_0$ expressed as deviations from mean. The high-periodicity peak is at $T = 65.3$ in one case and $T = 69.2$ in the other. In the lower half of figure 7, the full curve represents the spectra of the original 97 values of l.o.d. for an LPEF = 95 i.e. 98 % and the dashed is for l.o.d. values after eliminating the linear trend. The high-periodicity peak $T = 65.3$ for the original values has been replaced by two peaks at $T = 52.5$ and $T = 73.3$. Thus, removal of a linear trend and/or a change of the mean value alter the results considerably in the high periodicity region.

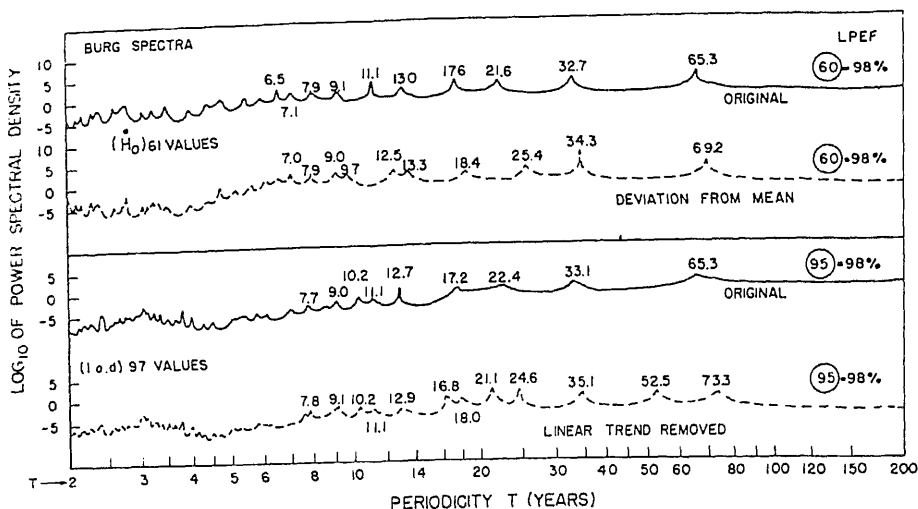


Figure 7. Burg spectra of $\dot{H}_0$, the time rate of change of H_0 (61 values, upper half) and of I.o.d. fluctuations (97 values, lower half), for LPEF of about 98%. For $\dot{H}_0$, the full curve is for original values and the dashed curve is for deviations from the mean of the whole sample. For I.o.d., the full curve is for original values and the dashed curve is for values after eliminating linear trend. Numbers represent periodicities in years.

4. Conclusion

From an analysis of artificial samples, it seems that the presence of linear trends distorts the high periodicity peaks and hence it is advisable to eliminate linear trends, even when the sophisticated maximum entropy spectral analysis (MESA, evolved by Burg) is used. The effect of a very large, non-zero mean is to reduce the sharpness of the peaks, and to distort the very high periodicity peaks by a few percent. However, a sample expressed as deviation from mean does not necessarily give the best resolution. A non-zero mean, of the same order as the amplitudes of the sinusoids involved, seems to give better resolution of high periodicity peaks.

Acknowledgements

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Physico-chemical conditions of ore deposition in Malanjhand copper sulphide deposit

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Abstract. Heating and freezing studies on fluid inclusions in quartz from mineralized quartz-feldspar reef reveal the presence of type A $\text{CO}_2 - \text{H}_2\text{O}$ ($\text{H}_2\text{O} > 50\%$ by volume), type B $\text{CO}_2 - \text{H}_2\text{O}$ ($\text{H}_2\text{O} < 50\%$ by volume), type C pure CO_2 and type D pure aqueous inclusions. Types A, B and C are primary and/or pseudo-secondary inclusions while type D are secondary. Types A and B homogenize on heating into different phases at similar temperatures ranging between 307 and 476 °C, indicating entrapment from boiling hydrothermal solutions. Type D inclusions homogenize into a liquid phase at temperatures between 88 and 196 °C. Boiling of hydrothermal solutions led to the formation of a CO_2 -rich phase of low density and salinity that coexisted with another dense and saline aqueous phase with very little CO_2 dissolved in it. Ore and gangue mineral assemblage of primary ores indicate that ore deposition was characterized by $\log f_{\text{O}_2} = -34.4$ to -30.2 atm, $\log f_{\text{S}_2} = -11.6$ to -8.8 atm and pH = 4.5 to 6.5.

Keywords. Fluid inclusion; homogenization; boiling; fugacity.

1. Introduction

The Malanjhand copper sulphide deposit is situated near Padritola, Baihar Tehsil, Balaghat district of Madhya Pradesh. Sharma and Kumar (1969) worked out the stratigraphic sequence of rocks of the area. Rao (1974) carried out ore microscopic studies and proposed a paragenetic sequence of ore-forming minerals. Narang *et al* (1979) discussed factors controlling ore localization while Tripathi (1979) found enough evidence to name Malanjhand a copper porphyry deposit. Earlier attempts to understand the physico-chemical conditions of ore deposition are few and controversial. Rao (1974), who studied ore paragenesis indicated very high temperatures (450 to 700 °C) of ore formation. Sharma (1982), based on the analysis of relevant $\log f_{\text{O}_2} - \log f_{\text{S}_2}$ diagrams for mineral assemblages in ores, indicated equally high temperatures (307 to 547 °C) of ore formation. In the present work, detailed fluid inclusion studies have been undertaken to establish physico-chemical (temperature, pressure, physical and chemical nature of mineral forming media, etc) conditions of ore deposition.

2. Geological setting

Archean granite-granodiorite are the main rocks of the area. These have been intruded by a number of basic dykes and are overlain unconformably by younger metasedimentaries of Chilpi Ghat series represented by conglomerates, arkosic grits, quartzites and phyllites. The metasedimentaries have a general ENE-WSW trend with dips of 30 to 50 ° due SSE (Rao 1974).

(see figure 1) with its convexity directed towards east. Strike direction of the reef changes from NNW-SSE in the northern to N-S in the central and NE-SW in the southern parts of the area. The reef is about 10 to 30 m thick and dips steeply (60° to 70°) due east in the central part.

According to Sharma (1982), within the suite of granite-granodiorite rocks, granodiorite is the most dominant type. Tonalite and quartz-monzonite are present in lesser amounts.

Copper mineralization is confined mainly to the quartz-feldspar reef and has been traced to a strike length of ≈ 1100 m. Mineralization persists to depths greater than 200 to 300 m. Within the reef, mineralization is irregular, distributed mainly as disseminations, stringers and pockets. Chalcopyrite is the main mineral of primary copper, followed by pyrite and magnetite. Molybdenite, sphalerite and cobaltite are present in minor amounts. At places, molybdenite forms relatively richer pockets within the reef. Primary sulphide ores are capped by zones of oxidation and secondary enrichment. Thickness of supergene zones varies from 100 to 110 m with maximum in the southern block of the ore deposit. Malachite, azurite, goethite, bornite, chalcocite, covellite and native copper are the main minerals of supergene zones.

Primary sulphide mineralization is accompanied by intensive wall rock alteration in granite-granodiorite rocks manifested in the development of pink feldspar, biotite, epidote, chlorite, sericite and kaolinite.

3. Types of fluid inclusions

Fifteen polished plates of quartz from the mineralized quartz-feldspar reef were studied using a normal polarized light microscope. Fluid inclusions in quartz are generally very small ($< 20\text{--}25\text{ }\mu\text{m}$) which makes their study rather difficult. Optical microscopy coupled with heating and freezing studies have helped to identify the following basic types of fluid inclusions in quartz: (i) Type A, $\text{CO}_2\text{--H}_2\text{O}$ inclusions ($\text{H}_2\text{O} > 50\%$ by volume); (ii) Type B, $\text{CO}_2\text{--H}_2\text{O}$ inclusions ($\text{H}_2\text{O} < 50\%$ by volume); (iii) Type C, CO_2 inclusions and (iv) Type D, aqueous inclusions.

Type A inclusions are characterized by sharp and regular cavity boundaries. Few of these have typical negative crystal shapes. These inclusions are distributed at random within one grain or form small bands cutting across one or more grains (see figure 2). Both the distribution pattern and the inclusion shape suggest that they are primary and/or pseudo-secondary inclusions.

At room temperature these inclusions contain a gas bubble (10–30% by volume of the cavity) but slight cooling below the room temperature nucleates a separate phase of liquid CO_2 occupying the interface between gaseous phase (CO_2) and aqueous phase. Some type A inclusions locally contain opaque and/or isotropic solid but such inclusions are extremely rare.

Type B inclusions at room temperature contain a gas bubble ($< 50\%$ by volume of the cavity), but slight cooling below room temperature, nucleates a separate phase of liquid CO_2 . In shape and pattern of distribution, these inclusions are similar to type A inclusions and occupy the same clusters of random inclusion or constitute the same bands of inclusions (see figure 2).

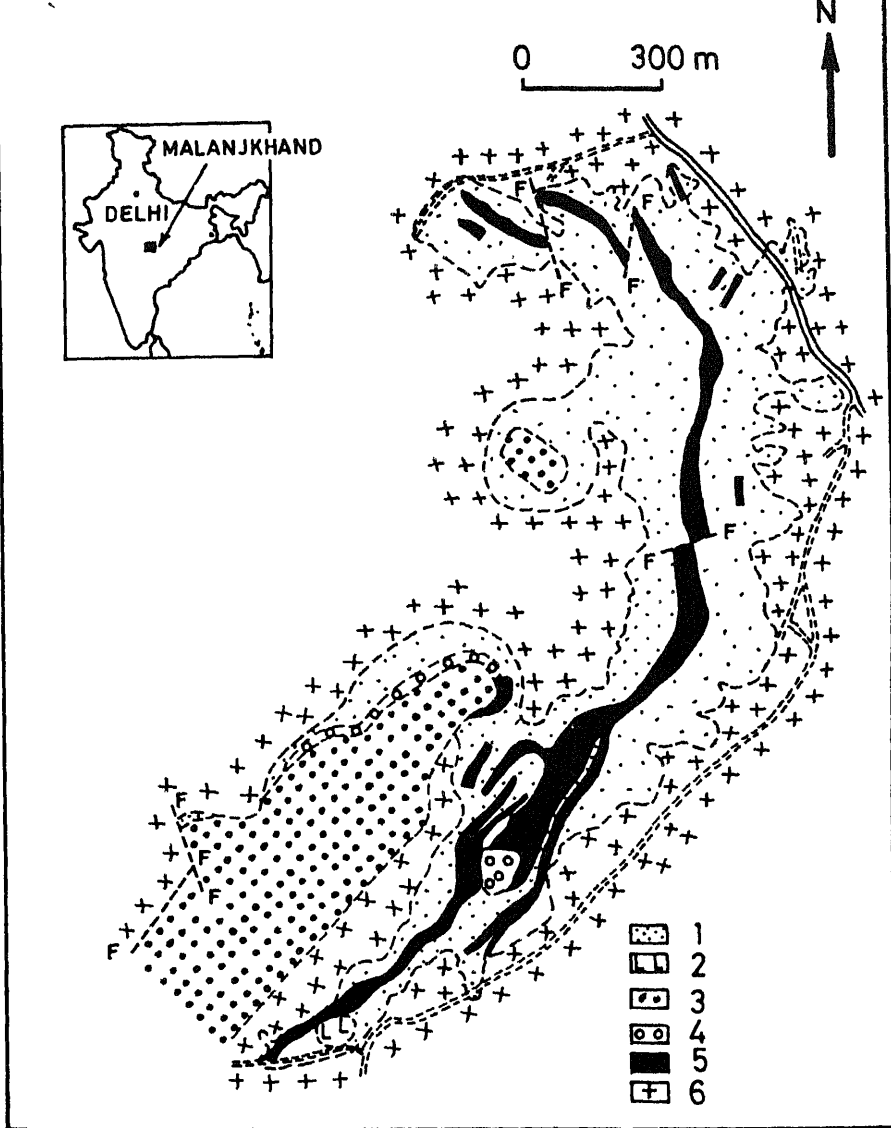


Figure 1. Geological map of Malanjkhanda copper deposit, modified after Narang *et al* (1979). 1. Talus 2. Laterite 3. Arkosic grit, quartzite and phyllite 4. Conglomerate 5. Quartz reef 6. Granite/granodiorite.

Type C inclusions at room temperature are similar to the so-called dry gaseous inclusions with CO_2 occupying the whole volume of the cavity. A thin film of aqueous phase might also be present in them. In some of these inclusions freezing nucleated a separate bubble representing gaseous CO_2 . Type C inclusions are closely associated with type A and type B inclusions (see figure 2). All the three types i.e. types A,B,C are thus primary and/or pseudo-secondary inclusions, syngenetic to each other.

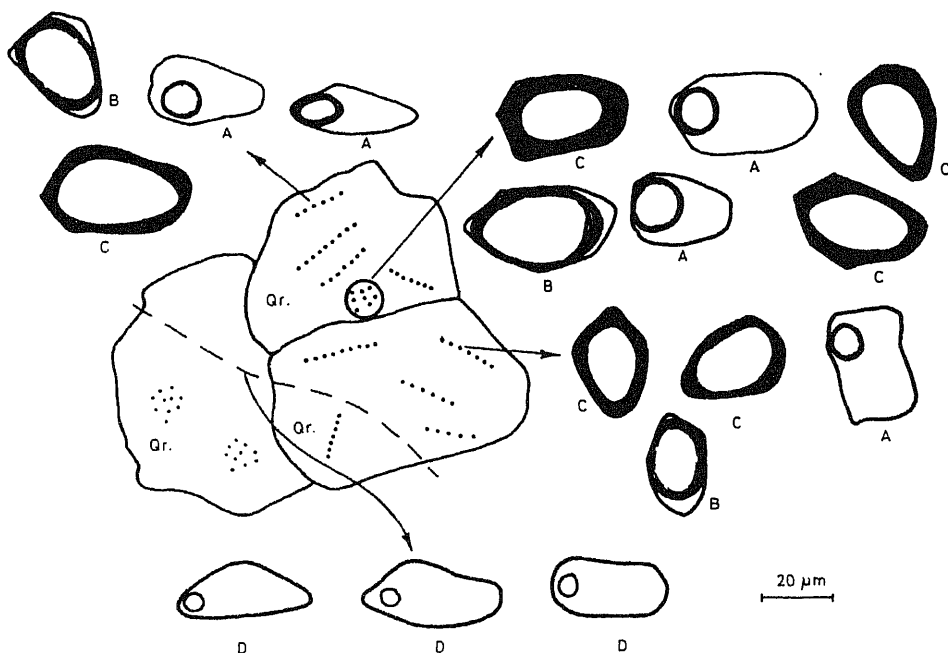


Figure 2. Schematic representation of quartz (Qr) grains showing distribution of types A, B, C and D inclusions. The dark bubble in all types A, B and C inclusions is CO_2 which is surrounded by an aqueous phase. In type D inclusions bubble is vapour surrounded by aqueous phase.

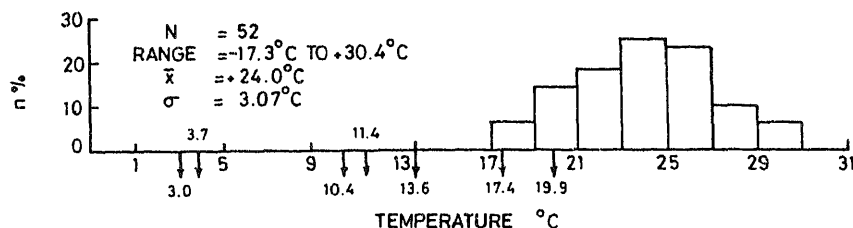
Type D inclusions contain at room temperature a gas bubble ($< 5\%$ by volume of the cavity) and are distributed as large bands cutting across large number of grains (see figure 2). They have highly irregular shapes and are thought to be formed due to healing of larger microfractures and hence are secondary. These inclusions are free of solids and do not contain CO_2 .

4. Heating studies

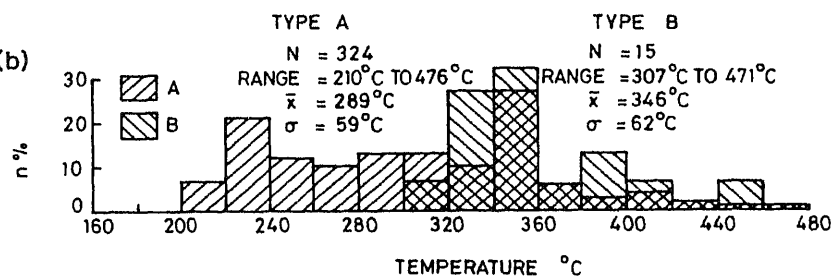
Selected fluid inclusions were heated in a temperature programmable Linkam TH 600 heating freezing system, calibrated carefully using calibration standards recommended by MacDonald and Spooner (1981). Inclusions having irregular cavity boundaries or showing signs of necking down were not heated. Heating was carried out at a constant rate of $10^\circ\text{C}/\text{min}$. At temperatures closer to temperatures of homogenization, rate of heating was further reduced to $5^\circ\text{C}/\text{min}$. Temperatures of homogenization of various types of inclusions are shown in figure 3.

All type A inclusions homogenized in two stages. In the first stage gaseous CO_2 homogenized into liquid CO_2 phase at temperatures between $+17.3^\circ\text{C}$ and $+30.4^\circ\text{C}$ (see figure 3a). It should be mentioned that heating at this stage was carried out at constant rates of $0.5^\circ\text{C}/\text{min}$ to $1^\circ\text{C}/\text{min}$. On further heating all these inclusions homogenized into a liquid phase with the disappearance of gas bubble (CO_2 phase) at temperatures between 210°C and 476°C with a mean value of 289°C obtained from

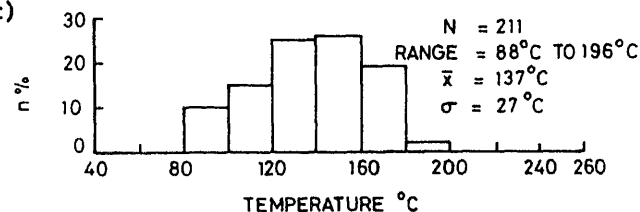
(a)



(b)



(c)



(d)

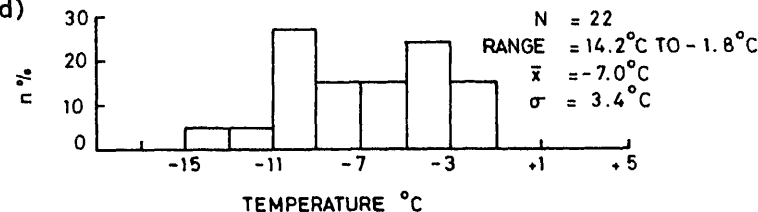


Figure 3. a. Temperature of homogenization of CO_2 phases of types A, B and C inclusions. Homogenization temperatures of a few type B and C inclusions are shown as inverted arrows at points corresponding to their temperatures of homogenization. In all inclusions, CO_2 phases homogenize by disappearance of gaseous CO_2 . b. Temperature of final homogenization of primary and/or pseudo-secondary types A and B inclusions. Type A inclusions homogenize into liquid phase while type B homogenize into supercritical CO_2 fluid; c. Temperature of final homogenization of secondary type D inclusions. d. Temperature of final melting of ice in type A inclusions.

heating of 324 such inclusions (see figure 3b). In type A inclusions with solids,

+ 3.0°C and + 19.9°C, CO₂ phase became homogeneous with the gradual disappearance of gaseous CO₂ (see figure 3a). On further heating, dark bubble (CO₂ phase) gradually expanded to fill up the whole volume of the cavity at temperatures of homogenization varying between 307 and 471°C, with a mean value of 360°C obtained from 15 such inclusions (see figure 3b).

Secondary type D inclusions homogenized into liquid phase at temperature between 83 and 196°C with a mean value of 137°C obtained from 211 such inclusions (see figure 3c).

5. Freezing studies

Freezing studies were carried out on a Linkam TH 600 heating-freezing system to determine the composition, and salinity of entrapped material. Before freezing, the stage was calibrated using distilled water (melting point = 0°C) and *n*-octane (melting point = -56.6°C) sealed in flat bottomed capillary tubes as standards. Inclusions were first frozen and phase changes were observed during gradual warming. Heating rate close to the expected points of phase changes was kept as low as 0.1°C/min and 0.5°C/min.

The liquid in type A inclusions usually froze at temperatures between -40 and -60°C. Accurate recording of the temperature of first melting (temperature of eutectic) is normally difficult and therefore repeated freezing runs were undertaken to determine this temperature as accurately as possible.

Liquid CO₂ in types A, B and C inclusions showed considerable resistance to solidification particularly in types B and C inclusions. Considerable overcooling (upto -130°C) was required to solidify liquid CO₂ in these inclusions. In types A, B and C inclusions, freezing first produced a small bubble separating gaseous CO₂ from liquid CO₂ followed by solidification of liquid CO₂ at considerably low temperatures. Formation of CO₂ hydrate (Clathrate) has been observed by many workers during freezing studies on CO₂ bearing inclusions (Roedder 1963; Kelly and Turneure 1970; Collins 1979 etc). Clathration is usually recorded with the help of double freezing of such inclusions (Collins 1979), where clathrate freezes out first followed by freezing of aqueous phase. In types A, B and C inclusions despite appreciable amount of detectable CO₂ phase, we have not been able to identify clathration. It might be possible that because of the small size of these inclusions (< 20–25 µm) it was difficult to observe double freezing. Further, in many type B inclusions, the amount of aqueous phase was probably too small to form detectable amounts of clathrate. It should be mentioned that clathration has not been detected in many other CO₂ bearing fluid inclusions from other localities (Luckscheiter and Morteani 1980). Heating and freezing data on all types of fluid inclusions are summarized in table 1.

6. Pressure and temperature conditions of ore formation

Primary sulphides, as mentioned earlier, are distributed as disseminations, small stringers and veinlets within quartz-feldspar reef. Textural relationship between primary sulphides and gangue (quartz and orthoclase) minerals indicates that sulphides are syngenetic to the bulk of the quartz and feldspar. Hence primary and/or pseudo-

Table 1. Heating and freezing data (in °C) for types A,B,C and D inclusions in quartz.

type of inclusions	T_m CO ₂	T_e	T_m ice	T_h CO ₂ V-L	T_{hV-L}
A	-56.0 to -56.4	-22.4 to -23.4	-14.2 to -1.8	+17.3 to +30.4	210 to 476 (L)
B	-56.0 to -56.4	nd	nd	+3.0 to +19.9	307 to 471 (CO ₂)
C	-56.0 to -56.4	nd	nd	+3.7 and +15.6	nd
D	—	nd	nd	—	88 to 196 (L)

T_m Temperature of melting; T_e Temperature of first melting i.e. temperature of eutectic; T_h CO₂V-L temperature of homogenization of CO₂ phases; T_{hV-L} Temperature of final homogenization. Letters in brackets represent final phase of homogenization; V-Vapour/gas; L-Liquid; nd-not determined

Secondary types A, B and C inclusions observed in quartz, represent mineral-forming solutions responsible for the main episode of ore formation. Secondary type D inclusions are either related to the same mineralization episode or represent any later hydrothermal event superimposed on it.

3.1 Boiling

As mentioned earlier types A, B and C inclusions which show highly variable CO₂/H₂O ratio occupy the same clusters of random inclusions or constitute the same bands of inclusions (see figure 2) indicating that these inclusions are syngenetic to each other. In addition, these inclusions homogenize into different phases (type A into liquid and type B into gaseous) at almost similar temperatures (see figure 3) suggesting that entrapment of these inclusions took place from heterogeneous mineral-forming solutions caused by boiling. Entrapment of different amounts of the immiscible fractions of the boiling solutions resulted in the formation of syngenetic inclusions showing large variation of CO₂/H₂O ratio. Large variation in the temperature of their final homogenization also supports this suggestion (Ramboz *et al* 1982). Heterogeneous entrapment i.e. boiling is also indicated by considerable variation of the homogenization temperatures of CO₂ phases in these inclusions (see figure 3a).

Experimental studies of H₂O-CO₂ system reveal an immiscibility solvus, with solvus crest at 275 °C (at 1 kbar) for pure CO₂ and H₂O mixture (Todheide and Frank 1963 in Hollister 1981). Addition of electrolytes to the mixtures greatly expands the region of immiscibility (Takenouchi and Kennedy 1965). For 6 wt % salt in the aqueous phase, solvus crest rises to about 420 °C at 1.5 kbar (Gehrig *et al* 1969 cited in Hollister 1981). In the case of Malanjhand ores, CO₂-rich saline solutions underwent immiscibility due to boiling, generating almost pure supercritical CO₂ coexisting with relatively CO₂-poor aqueous solution. Entrapment of mixed proportions of the two immiscible fluids generated inclusions showing large variation in temperatures of homogenization.

3.2 Composition and salinity of solutions

Freezing studies of type A inclusions show that solids within these inclusions start dissolving at temperatures between -22.9°C and -23.9°C (table 1). This temperature

Table 2. Concentration of CO₂ and Bulk molar volumes of entrapped media in Types A and B inclusions.

Type	Vol % H ₂ O at 40 °C*	Temperature of homogenization (Th CO ₂ v.-l) °C	Bulk molar volume (cm ³ /mole)**	Concentration of CO ₂ (mole %)**
A	90	+ 17.3	20.8	2
A	80	+ 30.4	22.2	5
B	40	+ 3.0	28.6	30

* Vol % of liquid phase estimated visually; ** Bulk molar volume of entrapped fluid and mole % of CO₂ determined using curves given by Burruss (1981)

which is known as temperature of eutectic, is sufficiently close to -22.9°C , the eutectic temperature in NaCl-KCl-H₂O system (Crawford 1981) which means that the liquid entrapped in these inclusions was sufficiently rich in chlorides of sodium and potassium.

Usually, salinities of aqueous phase in the fluid inclusions are determined using depression of the fusion temperature of ice recorded during freezing of these inclusions. But in all those inclusions, where CO₂ phase is also present, formation of clathrate makes residual aqueous phase more saline and depression of the fusion temperature of ice gives relatively higher estimates of salinities. For such inclusions Collins (1979) proposed use of the decomposition temperature of clathrate for the estimation of salinities. In type A inclusions though detectable amount of CO₂ is present (table 2), clathration could not be identified. On the other hand, temperature of fusion of ice in these inclusions varied between -14.2°C and -1.8°C with a mean value of -7.0°C obtained from freezing of 22 such inclusions (see figure 3d). This corresponds to a salinity varying from 18 to 2.3 wt % equivalent of NaCl with a mean value of 10.5 wt % equivalent of NaCl (Potter *et al* 1978 in Roedder 1984). These values represent maximum salinity assuming that clathration in type A inclusions did occur but could not be identified.

6.3 CO₂ in fluid inclusions

As mentioned earlier quartz from the mineralized reef contains large number of mixed CO₂-H₂O and pure CO₂ inclusions. Freezing studies of these mixed CO₂-H₂O (types A and B) inclusions give temperature of melting of CO₂ varying between -56.0°C and -56.4°C which is close to the temperature of melting of pure CO₂ (-56.6°C). Temperatures of homogenization of CO₂ phases in these inclusions have been used to estimate bulk molar volume and mol % of CO₂ in the entrapped fluid by taking the help of curves proposed by Burruss (1981). These data have been summarized in table 2. Thus, type B inclusions which homogenize with the disappearance of liquid phase at the final temperature of homogenization clearly have fluids of lower density, higher bulk molar volume and higher mol % of CO₂ in them. The coexisting type A inclusions that homogenize with the disappearance of gas bubble at the final

4.4 Temperature of formation

From the preceding discussion it is clear that the primary and/or pseudo-secondary inclusions of type A, type B and type C in quartz were formed from CO_2 -bearing aqueous solution of NaCl and KCl, boiling during entrapment, i.e. at the time of entrapment vapour pressure of the solution was higher than the confining pressure. As a result, the temperatures of homogenization of these inclusions do not need any pressure correction and these temperatures represent true temperatures of entrapment. Hence, it can be concluded that mineralized quartz-feldspar reef was formed at temperatures between 210°C and 476°C from hydrothermal solutions that underwent recurrent boiling between temperatures of 307°C and 476°C . More precisely, boiling occurred at around 307°C which is the lowest temperature of homogenization at which both types A and B inclusions homogenize into two different phases (type A into liquid and type B into gaseous).

4.5 Pressure during entrapment

Figure 4 shows a number of curves representing solubility of CO_2 in NaCl solution. These curves can be used to estimate vapour pressure of NaCl solution having different amounts of CO_2 dissolved in it.

As mentioned earlier, type A inclusions represent a fluid having a maximum CO_2 concentration of 2–5 mol % (i.e. 4.5 to 11.3 wt % CO_2) and an average salinity of

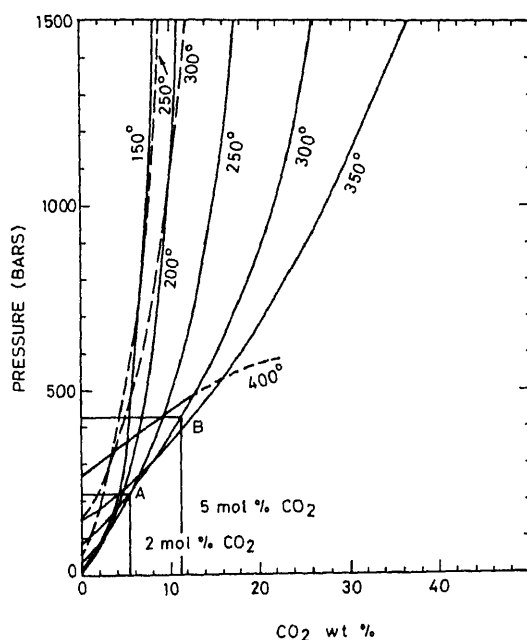


Figure 4. Diagram showing vapour pressure of CO_2 bearing NaCl solutions (modified after Takenouchi and Kennedy 1965). Thick lines are for 6 wt % NaCl solutions and dashed lines for 20 wt % NaCl solutions.

10.4 wt % equivalent of NaCl. The actual salinity, as discussed earlier, might be lower than this. Figure 4 shows that a 6 wt % NaCl solution having 2–5 mol % CO_2 and boiling at 300°C would generate a vapour pressure of 225 to 440 bars (points A and B) respectively. These values of pressure are good approximation of pressure during entrapment of type A and syngenetic to the type B and C inclusions from boiling hydrothermal solutions. These pressures correspond to hydrostatic column depths of about 2.25 to 4.4 km or rock depths of about 1 to 2 km respectively. It should be mentioned that these values of pressure are only approximate values. Increase in salinity of the solution will bring an appreciable change in these values. For comparison, two curves for CO_2 bearing solutions of 20 wt % NaCl have also been plotted on the same figure (see figure 4). A 6 wt % NaCl solution with 5 mol % of CO_2 generates a vapour pressure of about 400 bars at 350°C but increase in salinity upto 20 wt % NaCl enhances this pressure to about 1350 bars.

7. Geochemical parameters of ore deposition

It has been shown above that primary ore and gangue minerals were deposited from CO_2 -bearing NaCl–KCl aqueous solutions that were boiling at 307°C. Hence, to estimate basic geochemical parameters (f_{O_2} , f_{S_2} , pH etc) of ore deposition a $\log f_{\text{O}_2}$ – $\log f_{\text{S}_2}$ diagram at 300°C showing stability fields of main ore and gangue minerals has been calculated (see figure 5). First, a $\log f_{\text{O}_2}$ –pH diagram at 300°C was calculated using the method of Barnes and Kullerud (1961). This diagram had fields of dominant aqueous species of sulphur along with contours of $\log f_{\text{S}_2}$. The $\log f_{\text{O}_2}$ –pH diagram was then converted into a $\log f_{\text{O}_2}$ – $\log f_{\text{S}_2}$ diagram in which pH has been shown as contours of equal pH values. Stability fields of relevant ore and gangue minerals were then plotted on this diagram. All relevant reactions along with their equilibrium constants have been summarized in table 3.

The primary ores of Malanjkhand are made up of pyrite, chalcopyrite, magnetite and minor amounts of molybdenite with quartz and orthoclase as the dominant gangue minerals. Both gangue and ore minerals as discussed earlier are broadly syngenetic to each other. Sharma (1982), based on partitioning of trace elements between coexisting sulphides concluded that the ore minerals were formed in equilibrium with each other. Thus the syngenetic nature of ore and gangue mineral assemblage helps to put following limits to f_{O_2} and f_{S_2} conditions of ore deposition:

- Presence of molybdenite indicates that $\log f_{\text{S}_2}$ was > -20.65 atm.
- Absence of pyrrhotite in the ores indicates that $\log f_{\text{S}_2}$ was > -11.6 atm.
- Absence of bornite coexisting with pyrite in primary ores indicates that $\log f_{\text{S}_2}$ was < -6.06 atm.
- Presence of equilibrium association pyrite, chalcopyrite and magnetite in ore limits f_{S_2} – f_{O_2} conditions to boundary line AB in figure 5 which is the line of coexistence of these minerals.
- The boundary line between orthoclase and muscovite passing along $\text{pH} = 4.5$ contour further narrows down the conditions towards higher pH values along the

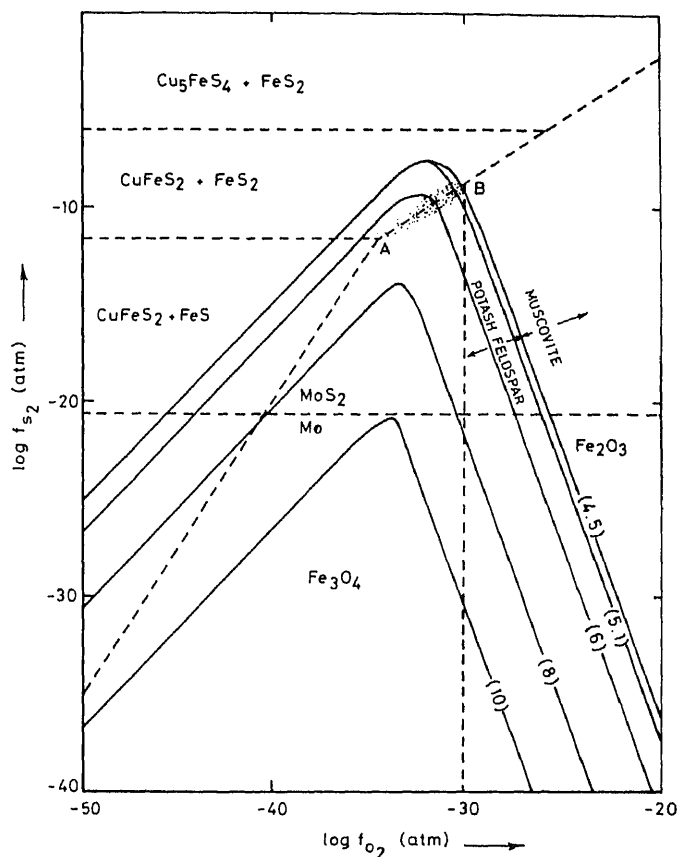


Figure 5. $\log f_{S_2}$ - $\log f_{O_2}$ diagram showing stability fields of main minerals of primary Malanjkhanda. Diagram constructed for $T = 300^\circ\text{C}$, $P = 1 \text{ atm}$, $\Sigma S = 0.01 \text{ m}$ boundaries between minerals. pH values are written in brackets. Boundary between feldspar and muscovite following a pH contour of 4.5 has been drawn for $K^+ = 0.5$. Area of boundary line AB represents domain of deposition of primary Malanjkhanda.

Table 3. Equilibrium constants of relevant reactions.

Reactions	$\log K_{300^\circ\text{C}}$
$3 \text{ FeS} + 2 \text{ O}_2 (\text{g}) = \text{Fe}_3\text{O}_4 + \frac{3}{2} \text{ S}_2 (\text{g})$	51.04 ^a
$3 \text{ FeS}_2 + 2 \text{ O}_2 (\text{g}) = \text{Fe}_3\text{O}_4 + 3 \text{ S}_2 (\text{g})$	33.60 ^a
$2 \text{ FeS}_2 + \frac{3}{2} \text{ O}_2 (\text{g}) = \text{Fe}_2\text{O}_3 + 2 \text{ S}_2 (\text{g})$	27.82 ^a
$4 \text{ Fe}_3\text{O}_4 + \text{O}_2 (\text{g}) = 6 \text{ Fe}_2\text{O}_3$	30.12 ^a
$\text{FeS} + \frac{1}{2} \text{ S}_2 (\text{g}) = \text{FeS}_2$	5.81 ^a
$5 \text{ CuFeS}_2 + \text{S}_2 (\text{g}) = \text{Cu}_5\text{FeS}_4 + 4 \text{ FeS}_2$	6.06 ^a
$\text{Mo} + \text{S}_2 (\text{g}) = \text{MoS}_2$	20.65 ^b
$3 \text{ KAlSi}_3\text{O}_8 + 2 \text{ H}^+ = \text{KAl}_3\text{Si}_3\text{O}_{10}(\text{OH})_2 + 6 \text{ SiO}_2 + 2 \text{ K}^+$	8.4 ^c

^aCalculated based on equations given in Ripley and Ohmoto (1977); ^cCalculated based on data from Crerar and Barnes (1976);

^bCalculated based on data in Vaughan and Craig (1978)

8. Modes of transportation and factors governing ore deposition

Freezing studies have shown that hydrothermal solutions entrapped in fluid inclusions were compositionally close to CO_2 -bearing aqueous solutions of NaCl and KCl. Hence, it is logical to conclude that basic ore-forming elements were transported mainly as chloride complexes. Such a mode of transportation of ore-forming elements has since long been proved viable both theoretically as well as experimentally (Barnes 1979).

Study of fluid inclusions also revealed that quartz along with primary ore minerals was deposited from hydrothermal solutions that were boiling, leading to separation of a CO_2 -rich, low-density, low-salinity fluid coexisting with a CO_2 -poor fluid of higher density and salinity. Boiling has been shown to be one of the main mechanisms of ore deposition in many ore deposits particularly in copper porphyry deposits (Roedder 1984). Boiling causes cooling and increases pH of solutions making chloride complexes of ore-forming elements unstable (Barnes 1979). According to Crerar and Barnes (1976) when Fe and Cu are transported as chloride complexes, an increase of pH by one unit leads to almost total precipitation of these elements.

9. Conclusions

Fluid inclusion and mineral equilibria studies in the Malanjkhand copper deposit have indicated the following:

- (i) Mineralized quartz-feldspar reef was formed from CO_2 -bearing hydrothermal solutions containing appreciable amount of chlorides of sodium and potassium at temperatures between 210 and 476 °C.
- (ii) Ore-forming hydrothermal solutions underwent boiling at about 307 °C, which led to the separation of a CO_2 -rich fluid of low density and salinity that coexisted with another comparatively more saline and dense fluid with less amount of CO_2 (2 to 5 mol %) dissolved in it.
- (iii) Pressure during entrapment from boiling hydrothermal solutions varied between 225 and 440 bars which is equivalent to a hydrostatic column depth of about 2.25 to 4.45 km or a rock depth of about 1 to 2 km respectively.
- (iv) Ore-forming elements (copper, iron, molybdenum) were transported mainly in the form of their chloride complexes and boiling was the main factor which controlled ore deposition.
- (v) Primary ore deposition in the reef was characterized by the following range of $\log f_{\text{O}_2} = -30.2$ to -34.4 atm., $\log f_{\text{S}_2} = -8.8$ to -11.6 atm and $\text{pH} = 4.5$ to 6.5 .

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Inverse problem for the determination of the bounds on the mantle heat flow

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Abstract. This paper presents a method for determining the nature of the spatial structure of the heat flow anomaly at depth given a few observations of surface heat flow. We use the constraint of the positivity of the heat flow throughout the domain under consideration to arrive at the solution. Both Parker's ideal body and linear programming approaches have been considered. We find that a few measurements are able to give a reasonable good estimation of the heat flow anomaly at depth. The method has been applied to a real case in the French Massif Central. Results point to the presence of enhanced basal heat flow anomaly.

Keywords. Inverse problem; linear programming; mantle heat flow.

1. Introduction

Heat flow measurements and their interpretations are of primary importance for the knowledge of earth's internal state and processes. In last few decades numerous heat flow measurements have been made in various tectonic provinces of both continental and oceanic regions. These observations have led to the widely accepted idea that below the lithosphere heat is brought up as a result of (global and local) convective systems and that, within the lithosphere and in particular in the crust, this heat, supplemented by heat due to radioactive decay, is brought to the surface by conductive processes (Sclater *et al* 1980). In the continental area, where the average heat flow is about 65 mWm^{-2} , there exist numerous areas where the observed heat flow values are anomalous showing high values and short space scales. For instance, a recent Sino-French expedition in Southern Tibet obtained a high heat flow value of 146 mWm^{-2} , within a short distance of 25 km (Francheteau *et al* 1984), reminiscent of similar observations in trench areas (Blackwell *et al* 1982). These anomalies can be explained either by enhanced radiogenic or magmatic heat in the crust or/and enhanced regions of heat flow/temperature below the crust. It is crucial to derive the nature of these perturbing processes from the few available heat flow data as they are responsible for a variety of geological, geochemical and geophysical phenomena. These problems involving inferences from the finite number of noisy data are ideally suited for the applications of inverse methodologies developed in the recent literature.

Geophysical inverse problems have a long history but their resolutions proliferated after the development of the Backus-Gilbert method (Backus and Gilbert 1967, 1968, 1970) and its various simplifications/generalizations/sophistications (Backus 1972; Tarantola and Valette 1982; Oldenburg 1983; Jackson and Matsu'ura 1985; Spensor 1985). Parker (1977 a, b) has provided a lucid review of these problems and techniques. The problems here essentially reduce to determining the distribution of the physical properties (represented by piecewise continuous functions) from a finite set of data

unstable, in the sense that a small error in the initial/boundary data would lead to unacceptable large values of the fields. These problems have been sought to be resolved by prescribing a class of models which are constrained by the data and by showing how the input of additional data would further constrain the model. Often, the solutions are given with associated resolution widths, the bounds on the model parameters are prescribed, or unique properties are chosen which are common to all solutions.

Parker (1974, 1975) has pioneered an ideal body solution approach to geophysical inverse problems. This approach seeks, in general, the lower bound on the maximum absolute value of piecewise-continuous functions representing the class of all solutions. Given this solution, it can be confidently asserted that the true solution would reach or exceed this value somewhere in the region under consideration. Huestis (1982) has given a geometrical interpretation of the Parker's ideal body solution by reducing the infinite-dimensional problem to a finite-dimensional one and using the Fenchel Duality Theorem. For few data cases the solution can be obtained analytically or graphically. However for a large data set, recourse has to be made to linear programming (LP)/methods pioneered by Safon *et al* (1977) following the formulation of Sabatier (1977 a, b), Huestis and Parker (1977), Huestis (1979) and Garmany *et al* (1979). If we are seeking an answer to the question of existence of the high heat flow region at depth, such an approach would be highly useful. Our work presented here is directed towards this goal. Some of the recent LP applications are by Bayer and Lesquer (1978), Bayer and Cuen (1981), Grasso *et al* (1983) and Richards *et al* (1984).

The problem of obtaining the ideal body solution i.e. minimising α ,

$$\alpha = \max |x(t)|,$$

given the constraints connecting y data with x parameters (to be determined) as

$$\begin{matrix} A & x & = & y \\ M \times N & N \times 1 & & M \times 1 \end{matrix}$$

(or more generally

$$y - \delta \leq Ax \leq y + \delta$$

where δ is error) and

$$x \geq 0,$$

can be converted into a linear programming problem using $(N + 1)$ slack variables (τ_n , Sabatier 1977 a, b; Safon *et al* 1977) as

$$\sum_{j=1}^N A_{ij} x_j = y_i \quad (i = 1, 2, \dots, M),$$

$$x_n + \tau_n - \alpha = 0 \quad (n = 1, 2, \dots, N),$$

$$x_n \geq 0,$$

$$\tau_n \geq 0,$$

$$\alpha \geq 0.$$

Various versions of linear programming computer programs using the Simplex

algorithm of Dantzig (1963) originally proposed in 1947 are available such as IBM supplied MPS 360 program. We shall use the programs given by Cuen and Bayer (1980) who have provided the following innovations (using recent numerical results) in their formulations:

- (i) To invert matrices for calculating bases, the Bartels Golub decomposition of matrices (Bartels and Golub 1969; Bartels 1971) is used. The problem due to the addition or deletion of a column can be handled easily. Stability in the computation is increased by scaling the variables.
- (ii) Choice of the incoming nonbasic variable to the basic variable set is done by taking the steepest edge (Kuhn and Quandt 1963; Reid 1975; Goldfarb 1976) in searching the vertices of the polyhedron on which the solution could lie.

In the present paper we shall first present the mathematical solution for the basal heat problem without the radiogenic heat source and next consider the problem with the heat source. We then obtain an ideal body solution analytically for a small number of observed surface heat flow values using Parker's formulation for a synthetic example. For larger data sets, the ideal body solution has been obtained using a linear programming method as developed by Cuen and Bayer (1980), again on the synthetic example. The method has been further used on a synthetic example wherein radiogenic heat sources have been included. After showing its usefulness for the synthetic case, a real example of inferring high Moho heat flow in the French Massif Central has been considered. The presence of a high heat flow zone at the Moho depth has been confirmed. We should add that the inverse problem of the estimation of basal temperature variations has already been considered by Huestis (1980, 1981) and applied over Sierra Nevada and the Basin and Range provinces. We have taken a different boundary condition, because in certain cases, such as the French Massif Central, it would be more realistic to prescribe heat flux boundary condition rather than temperature boundary condition. Nature of the basal boundary condition on the surface heat flow has recently been discussed by Nakiboglu and Lambeck (1985).

2. Inverse basal anomalous heat flow problem without radiogenic heat sources

The problem can be written as one of determining $q_L(X)$ at some depth $Z = L$, given the knowledge of the surface heat flow, $q_s(X)$. We shall first derive this relationship. The differential equation governing the temperature distribution in the layer $Z \in (0, L)$ is

$$(\partial^2/\partial X^2) + (\partial^2/\partial Z^2)T = 0 \quad (6)$$

where $X(Z)$ is the horizontal (vertical, positive downward) coordinate. The boundary conditions are

$$T = 0, \quad \text{at } Z = 0, \quad (7)$$

$$\frac{\partial T}{\partial Z}$$

in (6) to (8) to get

$$\left(\frac{\partial^2}{\partial X^2} + \frac{\partial^2}{\partial Z^2} \right) \theta = 0,$$

$$\theta = 0 \text{ at } Z = 0, \quad (11)$$

$$K \frac{\partial T}{\partial Z} = q_L(X) \text{ at } Z = L. \quad (12)$$

Using the Fourier transformation, we obtain the following relationship between enhanced surface heat flow, $q_s(x)$, and $q_L(x)$:

$$Q_s(k) = Q_L(k) \operatorname{sech}(kL), \quad (13)$$

where $Q_s(k)$ and $Q_L(k)$ are the Fourier transforms of $q_s(x)$ and $q_L(x)$ respectively.

This equation reduces in space domain as

$$q_s(X) = \int_{-\infty}^{\infty} \frac{q_L(X')}{2L} \operatorname{sech}[\pi(X - X')/2L] dX' \quad (14)$$

using the inverse Fourier transform of $\operatorname{sech}(kL)$ as $\operatorname{sech}[\pi x/2L]$ (from Campbell and Foster 1957). Let us non-dimensionalise this equation by substituting

$$(X, X') = (x, x') \cdot 2L \quad (15)$$

We thus get

$$q_s(x) = \int_{-\infty}^{\infty} q_L(x') \operatorname{sech}[\pi(x - x')] dx'. \quad (16)$$

This is the relationship for deriving $q_L(x)$ given $q_s(x)$. We shall first calculate Parker's ideal body solution following the formulation by Huestis (1982). Suppose we have two data, one at $x = 0$ and another at $x = x_1$. We thus have from (16)

$$q_s(0) = \int_a^b q_L(x') \operatorname{sech}(\pi x') dx', \quad (17)$$

$$q_s(x_1) = \int_a^b q_L(x') \operatorname{sech}(\pi(x_1 - x')) dx'. \quad (18)$$

Here we have assumed that

$$q_L(x) = 0 \text{ for } x \notin (a, b) \quad (19)$$

Equations (17) and (18) can be recast, by using (so that the range of integration becomes 0 to 1):

$$x' = (b - a)y + a, \quad (20)$$

as

$$q_0 = q_s(y_1) = (a - b) \int_0^1 q_L(y) \operatorname{sech}[\pi[(b - a)y + a]] dy \quad (21)$$

$$q_1 = q_s(y_2) = (a - b) \int_0^1 q_L(y) \operatorname{sech}[\pi[(b - a)(y_2 - y)]] dy,$$

Equations (21) and (22) can be further changed so that the integral of data kernels (H_1 and H_2 in eq. (23) and (24)) be 1:

$$C_1 = q_0/N_1 = \int_0^1 q_L(y)H_1(y)dy, \quad (24)$$

$$C_2 = q_1/N_2 = \int_0^1 q_L(y)H_2(y)dy, \quad (25)$$

where

$$H_1(y) = (b-a)\operatorname{sech}[\pi[(b-a)y+a]]/N_1 \quad (26)$$

$$H_2(y) = (b-a)\operatorname{sech}[\pi[(b-a)(y_2-y)]]/N_2 \quad (27)$$

$$N_1 = [\tan^{-1}(\operatorname{sech}(\pi b)) - \tan^{-1}(\sinh(\pi a))]/\pi \quad (28)$$

$$N_2 = [\tan^{-1}(\operatorname{sech}(\pi(b-a)y_2)) - \tan^{-1}(\sinh(\pi(b-a)(y_2-1)))]/\pi \quad (29)$$

If $C_1 > C_2$, the ideal body of Parker (1974) (see Huestis 1982) is specified by

$$q_L(y) = \begin{cases} q_0; a_1H_1 + a_2H_2 > 0 \\ 0; a_1H_1 + a_2H_2 < 0, \end{cases} \quad (30)$$

where a_1 and a_2 are constants and q_0 is the desired bound which gives a minimum of $\max|q_L(y)|$. (31)

Zero-crossing of $a_1H_1 + a_2H_2$ within $y \in (0, 1)$ occurs at a point, say y_0 . In that case the ideal body can be given as

$$q_L(y) = \begin{cases} q_0; 0 \leq y < y_0, \\ 0; y_0 \leq y \leq 1. \end{cases} \quad (32)$$

Thus the problem reduces to the determination of q_0 and y_0 which gives the strength and the geometry of the 'ideal body' anomalous zone. This can be easily obtained using (23) and (24). We thus get

$$q_0 \int_0^{y_0} H_1(y)dy = C_1 \quad (33)$$

$$q_0 \int_0^{y_0} H_2(y)dy = C_2 \quad (34)$$

Given the data C_1 and C_2 , we can find q_0 and y_0 .

We have performed the above calculations over a synthetic example. We take $a = -b = 1$. We further take for $q_L(\text{mWm}^{-2})$ (see figure 1)

$$q_L = 20.9 [H(x+.3) - H(x-.3)]. \quad (35)$$

Suppose we are given two observations at $x = 0$ and $x = x_1$. We first solve the direct problem to calculate data C_1 and C_2 . We then use this data to obtain the nature of the ideal body solution. We illustrate the use of these calculations in figure 2. Here we show the estimation of q_0 and y_0 as a function of x_1 which indicates the separation of the observation points.

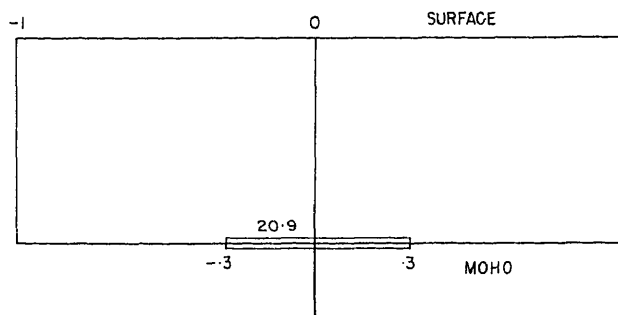


Figure 1. Geometrical configuration of the synthetic crustal model without heat source.

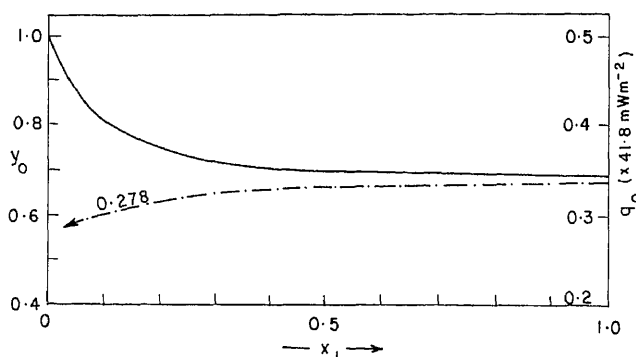


Figure 2. Analytical ideal body solution of the crustal model without heat source.

The curve reveals that for $x_1 = 1$, the ideal body is given by:

$$q_L(y) = \begin{cases} 14.21, & 0 \leq y \leq 0.7, \\ 0, & 0.7 < y \leq 1, \end{cases} \quad (36)$$

$$q_L(y) = \begin{cases} 12.25; & 0 \leq y \leq 0.8, \\ 0; & 0.8 < y \leq 1. \end{cases} \quad (37)$$

Thus the size of the ideal body is smaller and the value of the anomaly is higher if both heat flow sites are well separated. Given these two observations, one can assert that the true maximum of heat flow anomaly at depth would exceed these 'ideal body' values.

When more than 2 data are considered, this approach cannot be used. However, ideal body solutions can still be obtained numerically using the approach of Cuen and Bayer (1980).

We take the model as:

$$q_L = 41.8 [H(x + .3) - H(x - .3)], \quad (38)$$

and discretize the domain $x \in (-1, +1)$ into 20 blocks of equal size. Taking successively 2, 3 and 5 data we obtain the solution giving the smallest supremum of q_L on the whole range. These solutions are plotted in figure 3 which indicate that with

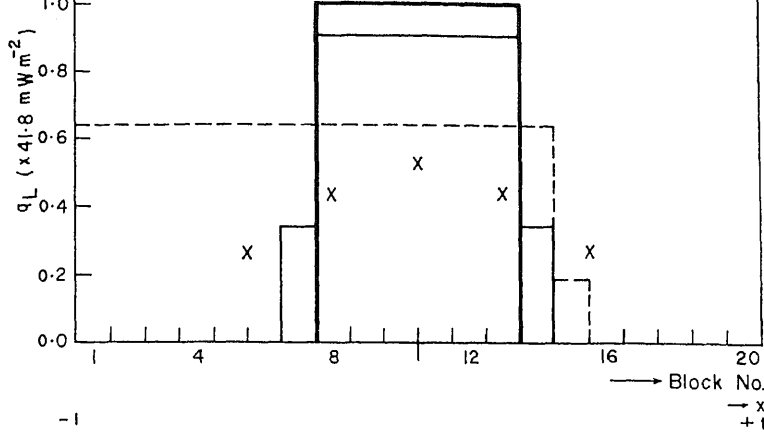


Figure 3. Estimates of the basal heat flux for the crustal model without heat source (x-input heat flow data—model, --- using two data points, — using three data points).

a few observations (3 or 5 which give the same result) one can resolve the heat flow anomaly quite well.

3. Inverse basal anomalous heat flow problem with radiogenic heat production

We have in this case the following mathematical model:

$$K \left(\frac{\partial^2}{\partial X^2} + \frac{\partial^2}{\partial Z^2} \right) T = -A_1 \quad (39)$$

$$T = 0, \quad \text{at } Z = 0 \quad (40)$$

$$K \frac{\partial T}{\partial Z} = q_L(X), \quad \text{at } Z = L \quad (41)$$

$$K \frac{\partial T}{\partial Z} = 0, \quad \text{at } X = 0 \text{ and}$$

$$X = L_1. \quad (42)$$

We have assumed (41) to limit the size of the model. We can non-dimensionalise the problem by assuming

$$(x^*, z^*) = (X, Z)/L, \quad (43)$$

$$T = q_0 L \theta / K. \quad (44)$$

We then get after dropping *s,

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2} \right) \theta = A \quad (45)$$

$$\theta = 0 \quad \text{at } z = 0 \quad (46)$$

$$\frac{\partial \theta}{\partial z} = q(x) \quad \text{at } z = 1 \quad (47)$$

$$\frac{\partial \theta}{\partial x} = 0 \quad \text{at } x = 0 \text{ and } 1 \quad (48)$$

where

$$A = A_1 L / q_0 \quad q = q_L(x) / q_0, \quad l = L_1 / L. \quad (49)$$

We can use Fourier cosine transformation in x . We then get

$$\begin{aligned} \theta_c = M \exp(n\pi z/l) + N \exp(-n\pi z/l) \\ - \frac{l}{n\pi} \int_0^1 A_c(z', n) \sinh\left(\frac{n\pi}{l}(z-z')\right) dz' \end{aligned} \quad (50)$$

where M and N are constants and

$$\theta_c[A_c] = \int_0^1 \theta[A] \cos\left(\frac{n\pi x}{l}\right) dx \quad (51)$$

Applying the boundary conditions, we get the transform of the surface heat flow as

$$q_{sc} = q_c \operatorname{sech}\left(\frac{n\pi}{l}\right) + \int_0^1 A_c(z', n) \operatorname{Cosh}\left[(1-z')\frac{n\pi}{l}\right] \operatorname{sech}(n\pi/l) dz'. \quad (52)$$

Using inverse Fourier cosine transformation, we get

$$\begin{aligned} q_s(x) = \int_0^1 q(x') M(x, x') dx' \\ + \int_0^1 dz' \int_0^1 dx' A(x', z') \cdot N(x, x', z'). \end{aligned} \quad (53)$$

where

$$M = \left(\frac{1}{l} \delta_{n0} + \frac{2}{l} \sum_{n=1}^{\infty} \frac{\cos(n\pi x/l)}{\cosh(n\pi/l)} \cos(n\pi x'/l) \right) \quad (54)$$

$$N = \left[\frac{1}{l} \delta_{n0} + \frac{2}{l} \sum_{n=1}^{\infty} \frac{\cos(n\pi x/l)}{\cosh(n\pi/l)} \operatorname{Cosh}[(1-z)n\pi/l] \times \cos(n\pi x'/l) \right]. \quad (55)$$

Now we can pose the inverse problem as one of determining $q(x)$ and $A(x, z)$ knowing the values of q_s . We shall use the approach of Safon *et al* (1977) and Cuen and Bayer (1980).

We discretize the problem by approximating q and A by piecewise constant functions. In that case we have to obtain the following integrals:

$$\bar{I}_1 = \int_{x_i}^{x_{i+1}} q(x') M(x, x') dx' \quad (56)$$

$$\bar{I}_{ji} = \frac{lq_i}{n\pi} \left[\frac{1}{l} \delta_{n0} + \frac{2}{l} \sum_{n=1}^{\infty} \frac{\cos(n\pi x_j/l)}{\cosh(n\pi/l)} \right] [\sin(n\pi x_{i+1}/l) - \sin(n\pi x_i/l)] \quad (58)$$

$$\begin{aligned} \bar{I}_{ji} = A_i \left(\frac{l}{n\pi} \right)^2 & \left[\frac{1}{l} \delta_{n0} + \frac{2}{l} \sum_{n=1}^{\infty} \frac{\cos(n\pi x_j/l)}{\cosh(n\pi/l)} \right] \\ & \times [\sin(n\pi x_{i+1}/l) - \sin(n\pi x_i/l)] \cdot \left[\sinh(n\pi(1-z_i)/l) \right. \\ & \left. - \sinh \frac{n\pi(1-z_{i+1})}{l} \right]. \end{aligned} \quad (59)$$

Here we have taken

$$q_i = q(x) [H(x - x_i) - H(x - x_{i+1})], \quad (60)$$

$$A_i = A(x, z) [H(x - x_i) - H(x - x_{i+1})]. \quad (61)$$

We thus have a discrete diversion of (52) as:

$$q_{sj} = \bar{I}_{ji} q_i + \bar{I}_{jm} A_m, \quad j = 1, \dots, r,$$

$$i = 1, \dots, p,$$

$$m = 1, \dots, s.$$

$$Id = \mathbf{q}_s,$$

where

$$I = [\bar{I}, \bar{I}] \text{ and } \mathbf{d} = [\mathbf{q}, \mathbf{A}]^T. \quad (62)$$

The system of (62) for estimating $\mathbf{d}$ is under-determined. One way to proceed further is to fix bounds on various variables and to look at the external solution for every unknown, q . This can be achieved by the use of the linear programming technique. In this direction, an interesting approach is the ideal body solution and we would again use the numerical scheme of Cuen and Bayer (1980) for this determination. We shall first use it over a synthetic example and then take up a real case.

3.1 Synthetic example

The model used is shown in figure 4; it consists of a $100 \text{ km} \times 30 \text{ km}$ domain. The surface heat flow results from a layered heat production and of a bottom heat flow of 30 mWm^{-2} for $X \geq 70 \text{ km}$ and $X \leq 30 \text{ km}$ and of 60 mWm^{-2} for $30 < X < 70 \text{ km}$. Five heat flow data with an error at $\pm 2 \text{ mWm}^{-2}$ are given at $X = 0, 25, 50, 75, 100 \text{ km}$. The discretization of the problem involves 50 prismatic blocks for the heat production and 10 segments (10 km long) for the bottom heat flow. In each block the heat production is assumed to be known within some bounds which will be discussed later. A priori bounds at 30 to 100 mWm^{-2} for q_L are chosen.

Let us consider the case where the heat production model is very well defined (i.e. bounds at $\pm 0.05 \mu\text{Wm}^{-3}$). Taking these bounds into account we computed the minimum and maximum values at q_L in each segment using the Cuen and Bayer (1980) procedure. These min/max can be displayed as a range where all compatible solutions

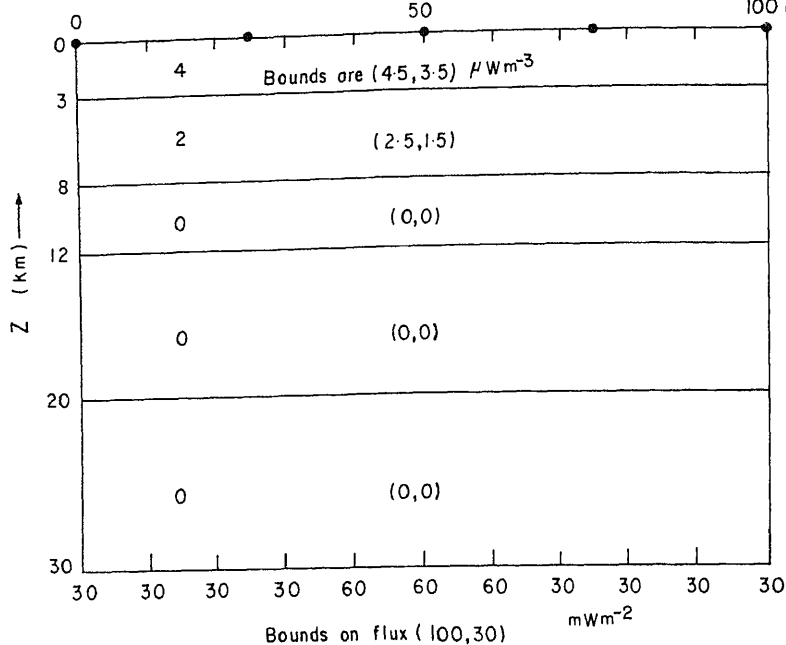


Figure 4. Geometrical configuration of the synthetic crustal model with heat source.

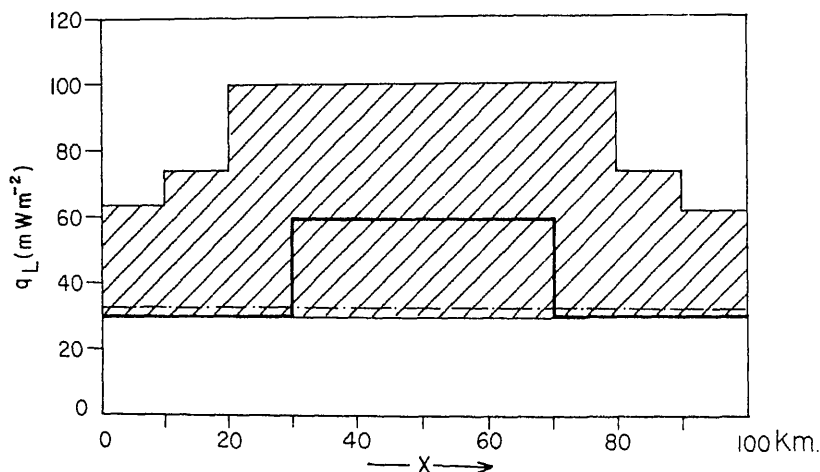


Figure 5. Estimates of the basal heat flux from a synthetic model (--- ideal body, — model, the hatched area covers the range of possible solutions).

of the problem lie. This is shown in figure 5. The range of possible solutions is very wide; however the maximum of the envelope reproduces a maximum similar to the actual variation of q_L . The ideal body solution corresponds to a constant value of 30.9 mWm^{-2} i.e. close to the assumed lower bound. It is therefore, clear that uncertainties in heat production result in a degradation of the resolution of the basal heat flow.

The proposed technique can now be applied to a real case where the presence of a mantle heat flow anomaly has already been proposed. The area under study is the French Massif Central which is outlined by a positive heat flow anomaly of broad wavelength (Vasseur and Lucazeau 1982). This heat flow anomaly was interpreted as partly due to the presence of rocks with enhanced radioactivity and also to the existence of a mantle heat flow anomaly related to the Cenozoic volcanism, whose manifestations have been recognized (Lucazeau and Vasseur 1981). In order to evaluate the crustal contribution to the surface heat flow many geochemical and geophysical data were used in order to propose values of heat production in various segments of the crust. The basic data are then 5 heat flow data on an E. W. profile, estimates of heat production in the various layer of the crust.

The a priori information for heat production is synthesized in table 1 giving a vertical cross section of the earth crust along the E-W direction. The structure of the earth crust is deduced from deep seismic soundings (Perrier and Ruegg 1977) and numerous heat production measurements have been obtained on various geologic units (Lucazeau and Vasseur 1981). Between 0 and 32 km depth the crust is divided into 210 rectangular blocks (figure 6); in each block an upper and a lower bound are set, taking into account the various uncertainties on the heat production measurements and on the geological structure. For the unknown bottom heat flow q_L , the horizontal distance is discretized into 5 steps; a priori bounds of 20 to 100 mWm^{-2} are assumed for each unknown (figure 6).

Using a Linear programming approach we can compute various solutions of the system, i.e. models of q_L and A which are consistent with the data and with the a priori

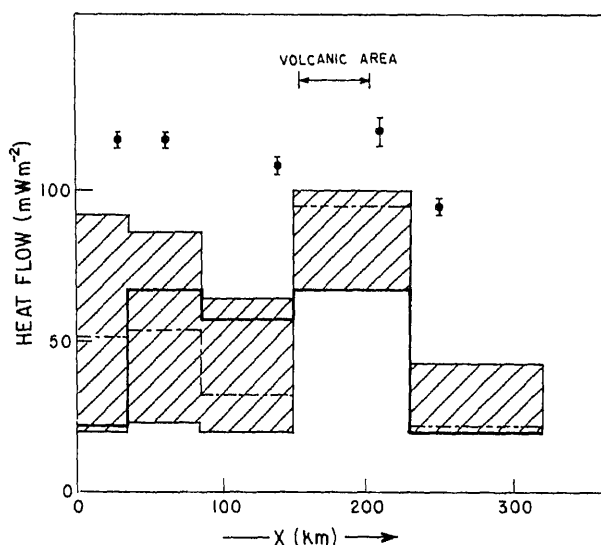


Figure 6. As a function of horizontal distance are plotted: the surface heat flow data with error bars as dots (in mWm^{-2}); the range of possible solutions for the bottom heat flow (hatched area); the solution which gives $\min \text{Sup } |q_L|$ as heavy line; the solution which gives $\min \text{Sup } |A_i - A_{0i}|$ as broken line.

Table 1. East-west vertical cross section of the earth's crust. This section is divided into rectangular blocks (irregular in size due to surface geology). In each block the upper and lower figures give the upper and lower bounds of heat production (in μWm^{-3}).

W

E

0-0	6.5 5.5	2.5 2.0	8.0 5.0	4.0 3.0	8.0 5.0	2.5 2.0	5.5 4.5	2.5 2.0	4.0 3.0	2.5 2.0	5.5 4.5	1.5 0.5	5.5 4.5	1.5 0.5	5.5 4.5
1-0	6.5 2.0	2.5 2.0	4.0 2.0	4.0 2.0	7.0 2.0	2.5 2.0	5.5 2.0	2.5 2.0	4.0 2.0	2.5 2.0	5.5 2.0	1.5 0.5	5.5 2.0	2.5 2.0	5.5 2.0
3-0	2.5 2.0	2.5 2.0	2.5 2.0	2.5 2.0	2.5 2.0	2.5 2.0	2.5 2.0	2.5 2.0	2.5 2.0	2.5 2.0	2.5 2.0	2.5 2.0	2.5 2.0	2.5 2.0	2.5 2.0
5-0	2.5 2.0	2.5 2.0	2.5 2.0	2.5 2.0	2.5 2.0	2.5 2.0	2.5 2.0	2.5 2.0	2.5 2.0	2.5 2.0	2.5 2.0	2.5 2.0	2.5 2.0	2.5 2.0	2.5 2.0
7-5	2.5 2.0	2.5 2.0	2.5 2.0	2.5 2.0	2.5 2.0	2.5 2.0	2.5 2.0	2.5 2.0	2.5 2.0	2.5 2.0	2.5 2.0	2.5 2.0	2.5 2.0	2.5 2.0	2.5 2.0
10-0	2.5 2.0	2.5 2.0	2.5 2.0	2.5 2.0	2.5 2.0	2.5 2.0	2.5 2.0	2.5 2.0	2.5 2.0	2.5 2.0	2.5 2.0	2.5 2.0	2.5 2.0	2.5 2.0	2.5 2.0
12-5	2.5 2.0	2.5 2.0	2.5 2.0	2.5 2.0	2.5 2.0	2.5 2.0	2.5 2.0	2.5 2.0	2.5 2.0	2.5 2.0	2.5 2.0	2.5 2.0	2.5 2.0	2.5 2.0	2.5 2.0
16-0	2.5 2.0	2.5 2.0	2.5 2.0	2.5 2.0	2.5 2.0	2.5 2.0	2.1 1.5	2.1 1.5	2.1 1.5	2.1 1.5	2.1 1.5	2.1 1.5	2.1 1.5	2.5 2.0	2.5 2.0
18-0	2.5 2.0	2.5 2.0	2.5 2.0	2.5 2.0	2.5 2.0	2.1 1.5	2.1 1.5	2.1 1.5	2.1 1.5	2.1 1.5	2.1 1.5	2.1 1.5	2.1 1.5	2.1 1.5	2.5 2.0
20-0	2.1 1.5	2.1 1.5	2.1 1.5	2.1 1.5	2.1 1.5	2.1 1.5	2.1 1.5	2.1 1.5	2.1 1.5	2.1 1.5	2.1 1.5	2.1 1.5	2.1 1.5	2.1 1.5	2.1 1.5
24-0	2.1 1.5	2.1 1.5	2.1 1.5	2.1 1.5	2.1 1.5	0.5 0.2	0.5 0.2	0.5 0.2	0.5 0.2	0.1 0.0	0.1 0.0	0.1 0.0	0.5 0.2	0.5 0.2	0.5 0.2
26-0	2.1 1.5	2.1 1.5	2.1 1.5	2.1 1.5	0.5 0.2	0.5 0.2	0.5 0.2	0.5 0.2	0.5 0.2	0.1 0.0	0.1 0.0	0.1 0.0	0.1 0.0	0.5 0.2	0.5 0.2
28-0	0.5 0.2	0.5 0.2	0.5 0.2	0.5 0.2	0.5 0.2	0.5 0.2	0.5 0.2	0.5 0.2	0.5 0.2	0.1 0.0	0.1 0.0	0.1 0.0	0.1 0.0	0.1 0.0	0.1 0.0
30-0	0.5 0.2	0.5 0.2	0.5 0.2	0.5 0.2	0.5 0.2	0.1 0.0	0.1 0.0	0.1 0.0	0.1 0.0	0.1 0.0	0.1 0.0	0.1 0.0	0.1 0.0	0.1 0.0	0.1 0.0
32-0															

Z_{km}

X_{km}

bounds. Various solutions are presented in figure 6; only those components of the solution corresponding to the bottom heat flow are displayed.

First we obtain solutions giving maximum and minimum values of q_L in each step. The corresponding range of solutions (hatched area) is quite large, mainly in the

western part; however under the "volcanic area" (figure 6), the minimum q_L is clearly enhanced.

Two particular solutions are also shown in figure 6: the solution which gives the smallest supremum of q_L (ideal body on q_L) and the solution which gives the minimum of:

$$\text{Sup } |A_i - A_{0i}|$$

where A_{0i} is the average estimated heat production in each block (table 1). The first solution corresponds to the "ideal body" for q_L : it yields a smallest supremum of 67 mWm^{-2} .

These results compare well with the model presented by Lucazeau and Vasseur (1981) and the presence of a strong perturbation of bottom heat flow around the volcanic area seems to be ascertained. However, in the western part the existing data cannot bring out any firm conclusion for the origin of high heat flow in this part of the studied area.

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Effect of waves in the redistribution of sediments along the Konkan coast

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Abstract. From the wave refraction diagrams it is delineated that the Jaigad Head and Warori Bluff are the zones of wave energy convergence and the Narvan and Ambwah bays the areas of wave energy divergence. The presence of two distinct mineral phases noticed at the Jaigad, Ambwah and Varvade bays shows that there are two different circulations of sediment movements. The presence of natural barriers restricts the movement of sediment along the coast. The sand bar at the mouth of the Jaigad Bay has different orientations during the monsoon and non-monsoon periods causing obstruction to navigation during the former period.

Keywords. Konkan coast; erosion; accretion; sediment movement; natural barriers; sand bar.

1. Introduction

Wave action is the primary source of energy available in the nearshore zones for various processes responsible for sediment motion along any open coast. Depending upon the configuration of the coastal line, nature of the wind, tide and several other factors, there is a possibility of waves being modified to increase or to dissipate their energy, in any particular coastal region. The area under study is affected by waves with short periods and high amplitudes during the monsoon season from June to October, whereas the low swells prevail from October to May. Further, these waves break offshore, widening the surf zone and setting more sand in motion. The wave velocity depends on the depth, and as the wave approaches the coast, it undergoes 'wave refraction'. As a result of refraction the waves tend to become parallel with the coast where they break at a slight angle to the coast. The long shore compound of wave energy gives rise to littoral currents in the surf zone contributing dominantly to the movement of the nearshore sediments. The angle between the crest of the breaking wave and the shoreline determines the direction of the long-shore component of water motion in the surf zone and also the long-shore transport direction. One of the important effects of littoral currents is the movement of sands as littoral drift along the coast. Moni (1972) calculated the quantum of the gross littoral drift along the west coast of India to be about 1.25 million cubic meters per annum.

Knowledge about the wave climate (wave height, period and direction) is necessary for better comprehension of wave action on or near the shore. Shrivastava *et al* (1968), Moni (1972) and Reddy (1976) indicated from their studies of the west coast of India that the predominant direction of waves in this area is from SW, WSW, W and WNW as in Reddy (1976) with periods ranging from 5 to 14 seconds. The convergence and

divergence of the littoral currents in several places along the west coast of India were also noted under the action of the WSW and W waves. Wave refraction diagrams indicate the zones of divergence and convergence of waves, which are directly related to the sediment motion in the nearshore area. The relationship between wave characteristics and sediment movement in various localities has been studied by several workers including Sastry (1958) and Murthy (1977).

2. Area of study

The few bays that have been selected for this study are bounded by a submergent type of coast line on the east, wide Arabian Sea on the west and bounded by cliffs or headlands to the south and north. Within the investigated area (figure 1) are three prominent bays: namely Jaigad, Ambwah and Varvade in which a total area of about 36 km² was chosen for detailed sampling in connection with the placer mineral exploration. The bays have a crenellated coastline with narrow sandy beaches. Sometimes along the headlands there are rocky boulders and pockets of coarse sandy beaches (figure 2). Although there are a number of small rivers/creeks draining into these bays, nowhere is formation of estuaries noticed. Instead, small tidal inlets are seen. Small bars and spits are present in almost all the bays, obstructing navigation during low tide periods and completely closing the seaward opening of the bays during the monsoon months.

2.1 *Description of the bays*

During field work, a perceptible shift in the coastline, as compared to the one shown in the Admiralty chart in 1950, was noted and hence the re-surveyed coast line is presented here (figure 1). Inasmuch as the coastline from Jaigad to Varvade is very irregular and indented, it has become quite long. The presence of rocky terraces, headlands, numerous cliffs and rias made Ahmed (1972) classify this coastline as of the submergent type. Barring the northern part of the Jaigad Bay and also the Varvade Bay, this coastline is completely devoid of straight sandy beaches. The existing few short seasonal streams and creeks originating in the nearby basaltic cliffs, are characterised by their steep gradient, deep cut V-shaped valleys, and bed load of pebbly and bouldery material. Well formed wave cut terraces are exposed during low tide in some of the head lands.

Although quite a large number of rivers and streams drain the area, the Shastri and Varvade rivers are worth mentioning and they join the sea obliquely to the coastline without affording any opportunity for estuarine formation. However, they cause the development of tidal inlets, whose presence can be marked by sandbars in the Shastri river confluence and also by a small shoal in the Varvade creek mouth. Despite the hilly terrain with very few pockets of coastal plain and beach, many tidal inlets do exist. These inlets get flooded during high tides resulting in the formation of swamps and mangroves, particularly in the Nandivade, Ambwah and Varvade areas. Besides the above, littoral concrete, which is a rocky beach, also occurs in a few places along this coast.

The coastline, which is flanked by flat topped hills and headlands, is indented by a number of arcuate bays. For the purpose of presenting the physiographic and

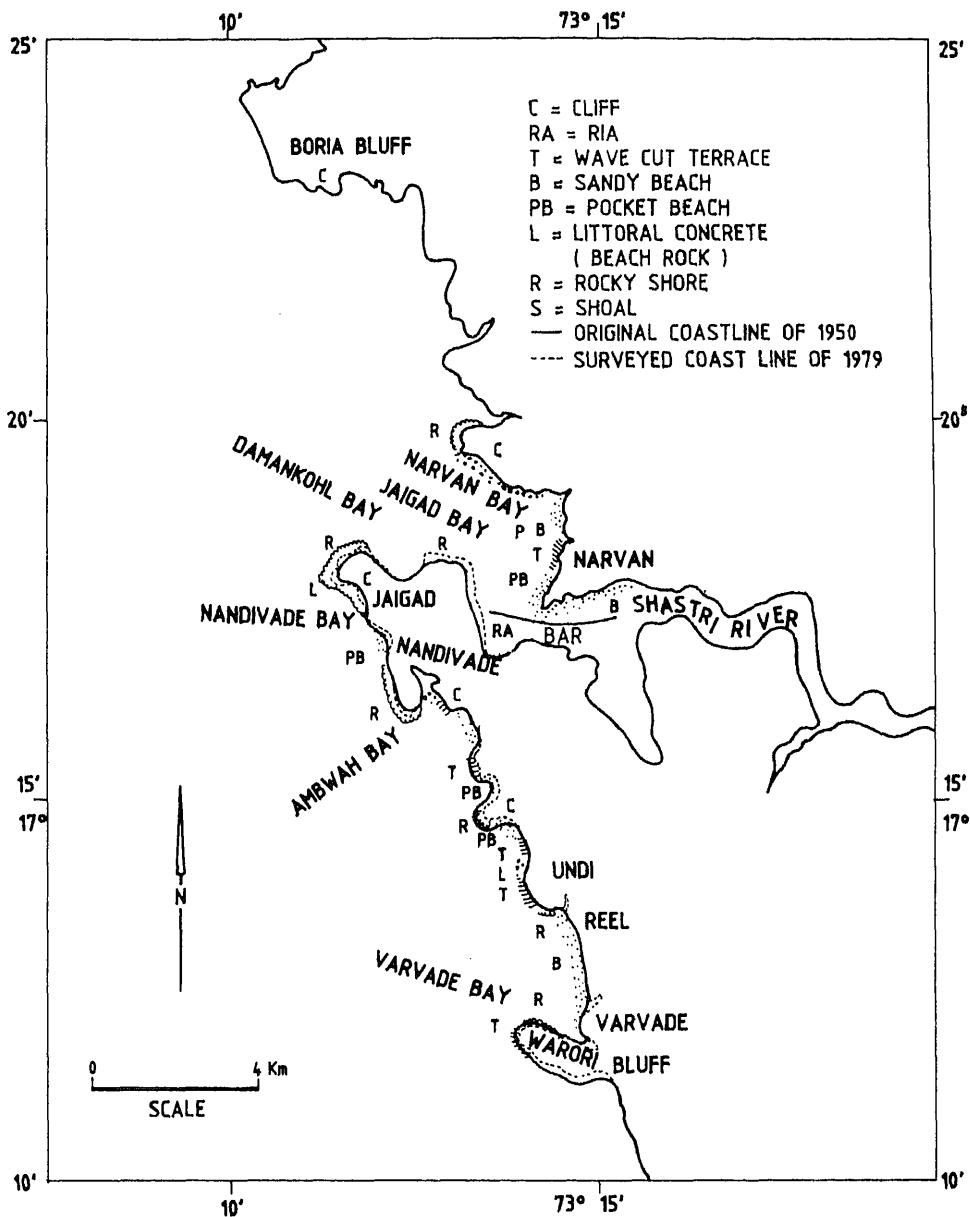
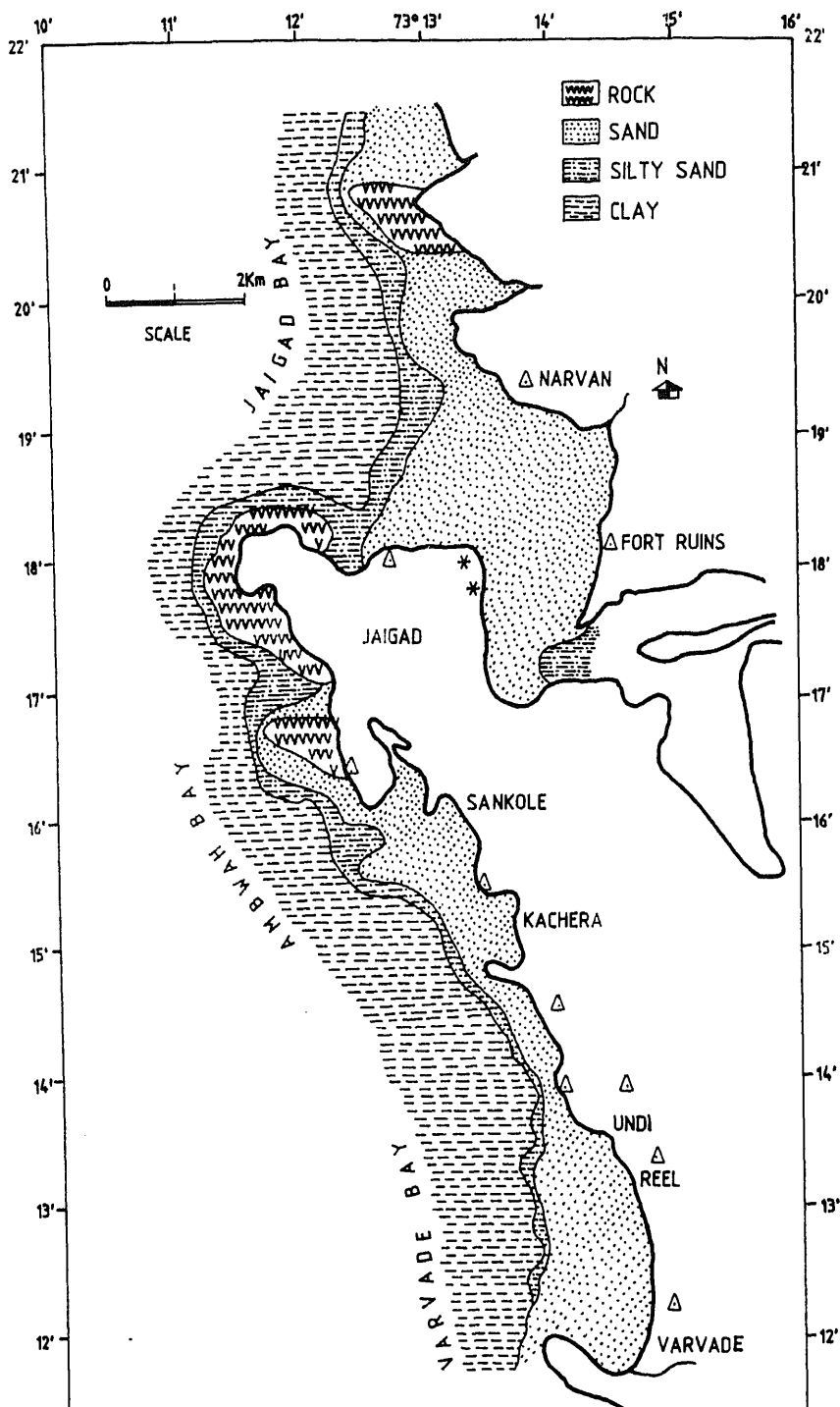


Figure 1. Coastline features (modified after Ahmed, 1972).

oceanographic data, these bays are grouped under three heads: viz., (1) the Jaigad Bay, which is a protected bay, is flanked by the Jaigad headland and the Boria bluff and it receives a large quantity of sediment influx from the Shastri river and is subject to the effect of waves. (2) The Varvade Bay having limited protection by the Warori Bluff, is exposed only to a weak tidal inlet and receives very limited sediment supply, while (3) the Ambwah Bay, which has primarily an open sea coast, is neither influenced by tidal inlets nor by sediment supply.



3. Method of study

Wave refraction studies can be done either by graphical or numerical methods. Harrison and Wilson (1964) have opined that the numerical method is comparatively more accurate and less time consuming and it has been employed in the present study, for which a programme developed by Mahadevan and Radhiya (1983) and applied at the National Institute of Oceanography, has been used. Common assumptions as included in the Shore Protection Manual (Anon 1977) for wave refraction calculations, have been adopted. Water depth, wave period, and deep water wave direction are the three major input parameters. The offshore area, starting from 200 m water depth is divided into 1.25 inch square grids. The interpolated depths from the hydrographic chart at the intersection points of the grid lines are taken for the purpose of input depth. The starting depth of each orthogonal (wave ray) is always maintained at half the wave length. Based on the report of Reddy (1976), the wave periods 6, 8, 10 and 12 seconds, and wave directions SW, WSW, W and WNW have been taken into consideration for computations on the Institute's computer TDC 316. Over and above, the effects of the presence of natural barriers in the form of headlands, and the observed sand bar and the shoal in front of the river confluences have also been taken note of in the interpretation of sediment movement.

4. Results and discussion

An examination of the wave refraction diagrams (figure 3) shows that the waves approaching this part of the coastline from WNW to SW in deep water converge in the neighbourhood of Jaigad. The inner portion of the Jaigad Bay forms a wave shadow zone for waves coming from SW to W which are the dominant directions in the SW monsoon period. The overall results indicate that: (1) the Jaigad Head and the Warori Bluff are wave converging points while the northern part of Jaigad or Narvan and Ambwah bays are the places of wave divergence. Because of the convergence of wave energy near the Jaigad Head and the Warori Bluff, removal of sediments, especially lighter materials takes place. This conclusion is also supported by the presence of a rocky beach, as well as a rocky seabed, even upto a depth of 10 m in front of these promontories. In the clay whatever silt remains is made up of only high specific gravity materials like magnetite and ilmenite; (2) On the contrary, along the coasts of Ambwah and Narvan, the divergence of wave rays indicate reduced wave activity and, consequently, deposition. The deposition is also substantiated by the nature of sediments i.e. the sediments containing rich plant materials and light fractions like quartz and feldspars.

In addition, it is also observed that there is no sediment transport either from the Varvade to the Jaigad Bay or vice-versa because the heavy mineral assemblages reported by Rajamanickam (1983) indicate enrichment in two distinct mineral phases of olivine and pyroxene in the Jaigad Bay, and tourmaline-zircon-kyanite in the Ambwah and Varvade bays. It is observed that the Jaigad Bay sediments have red-coloured ferruginous coating, while the sediments of the other two bays are made up of crystal clear grains. It is a case of clear zonation of two different circulations of sediment movements within the studied area, viz., one circulation from the Shastri river confluence at the Narvan Bay to the Boria Bluff and back to the Jaigad Bay, and the

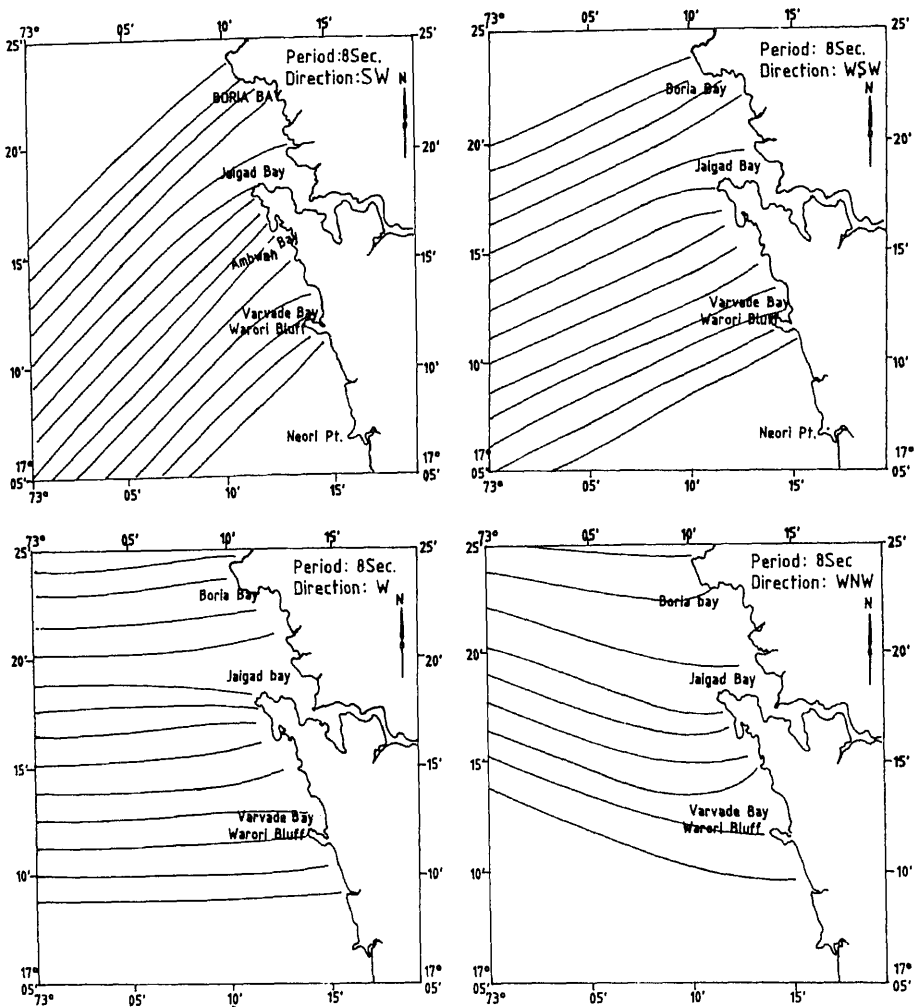


Figure 3. Wave refraction diagrams for different wave directions.

other from Varvade to Ambwah, and to some extent to Nandivade, and back. There is no sedimentation in front of the Jaigad Head but for fine silty-clay adjacent to the rocky seabed. Such circulations have also been observed between the promontories in other parts of the west coast of India by Veerayya *et al* (1981).

Nevertheless, the presence of distinct promontories might have also aided the restrictions caused by the convergence because if an artificial or a natural barrier is present perpendicular to a straight beach, contrary effects will be observed even with the same direction of incident wave according to the size of the material, wave energy, length of the barrier etc (LeMahaute and Brebner 1961). In such a case, there will be accretion on the updrift side and erosion on the downdrift side. In the present case, the Jaigad Head and the Warori and Boria Bluffs are natural barriers affecting the littoral drift considerably.

The Warori Bluff extends upto a water depth of 9 m while the Jaigad Head projects

upto 11m water depth. Generally, 9 m is considered to be the optimum depth of water for the conduction of coastal processes (LeMahaute and Soldate 1978). Due to the presence of natural barriers the coast immediately to the south of the Jaigad Head (Ambwah Bay) and south of the Boria Bluff (Narvan Bay) experiences deposition of sediments, whereas the coast on the northern side undergoes erosion. Similarly, Varvade Bay, situated in the northern part of the Warori Bluff shows erosion. Such erosions are found to remove mainly the light minerals and flaky constituents. Hence, the presence of enriched heavy minerals in the Varvade Bay and finer heavy minerals together with coarse lighter minerals in the Jaigad Bay might have occurred due to such erosion. The lighter material removed from the Varvade Bay is deposited in the Ambwah bay (south of the Jaigad Head), which is relatively enriched in this type of sediment. Likewise, the north of the Jaigad head suffers erosion by losing light minerals along with fine heavier material, but both of similar hydraulic equivalent, and the same is deposited in the northern part of the Narvan Bay due to the presence of the Boria Bluff in the north. There are two inlets in the area, one in the mouth of the Varvade Creek and the other in the Shastri River mouth, which has a sand bar constituted of coarse black sand formed parallel to the coast of the Narvan Headland. During the southwest monsoon months (June-October) the bar shifts slowly and attains a change in its orientation becoming almost perpendicular to the coast. At this stage, the bar is fully submerged whereas once it shifts and becomes parallel to the shore, it is clearly exposed during low tide. During monsoon/non-monsoon seasons the orientation of the bar shifts in accordance with the relative higher effectiveness of either wave activity or river runoff. On the other hand, the other inlet at the Varvade Creek mouth is associated with a shoal made of medium grained shelly sand at 10 m water depth extending from the northern bank of the channel to about 300 m parallel to the shore and rising to about 3-4 m.

When an offshore bar has been built up sufficiently high, breakers sweeping over the crest of the bar, transfer bar material from the seaward face causing the migration of the bar landwards. If the influence of waves is reduced, bar deformations are determined by means of river run-off. The Jaigad Head is a zone of erosion. The formation of a bar (if any) at the inlet and its orientation are also important parameters to consider wherever erosion occurs. The inlet at the Shastri river mouth acts as the sand sink to absorb sand coming from the Jaigad Head and to a certain extent from the Narvan Coast due to WNW waves.

If the natural barrier were not formed, the northerly longshore current would have fed sediment to the bay. Since only sand is observed in the bay as well as in the bar, there is every possibility of its derivation from the adjacent coast. Part of the material which is fed to the inlet is taken away by the ebb flow causing the bar to grow (Walton and Adams 1976). The river discharge is unable to wash out the sand accumulation, which slowly moves to block the navigational channel, specially during the monsoon months in which wave action is considerable.

Dean and Walton (1975) stated that the seaward transport of sand is due to tidal currents, whereas the landward movement of sand is probably due to wave forces. Since the channel is close to the natural barrier, no material is transported inside due to the strong erosion taking place, which is substantiated by the presence of a non-scouring hard rock bottom at the channel. Since the bar is formed in the wave divergence zones, it becomes a permanent feature and acts as a barrier for the mixing of river sediments with the bay

5. Conclusions

This study has revealed the presence of two strong wave convergent points at the Jaigad Head and the Warori Bluff, wherein the bays north of the above convergent zones, i.e. the northern parts of the Jaigad and Ambwah bays, are seen as wave divergence zones. This is also supplemented by the deposition of higher amounts of lighter minerals. In addition to the above wave refraction effects, the presence of natural barriers like the Jaigad Head, and the Warori and Boria Bluffs, contribute appreciably to restricting the littoral sediment movement among the bays. The presence of a sand bar and a shoal in front of the Shastri River and the Varvade Creek mouth, respectively, are considered to be the result of obstruction to the free mixing of the incoming sediments with the bay sediments. The overall heavy mineral percentage distribution within the three bays are the result of wave refraction, physical barriers and the presence of a sand bar shoal.

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Preferential solution of ^{234}U from recoil tracks and $^{234}\text{U}/^{238}\text{U}$ radioactive disequilibrium in natural waters

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Abstract. A mathematical model to calculate the $^{234}\text{U}/^{238}\text{U}$ activity ratio (AR) in an aqueous phase in contact with rock/soil is presented. The model relies on the supply of ^{238}U by dissolution and that of ^{234}U by dissolution and preferential release from radiation damaged regions (recoil tracks). The model predicts that values of $^{234}\text{U}/^{238}\text{U}$ AR > 1 in the aqueous phase can be obtained only from weathering "virgin" surfaces. Thus, to account for the observed steady-state supply of ^{234}U excess to the oceans by the preferential leaching model, 'virgin' rock/soil surfaces would have to be continually exposed and weathered. The ^{238}U concentration and $^{234}\text{U}/^{238}\text{U}$ AR in continental waters allow us to estimate the exposure rates of "virgin" rock/soil surfaces.

Keywords. Uranium isotopes; radioactive disequilibrium; alpha-recoil; preferential leaching.

1. Introduction

In most natural waters, ^{234}U activity exceeds that of its grandparent ^{238}U . This excess of ^{234}U serves as a geochemical tool in the study of the behaviour of uranium in rocks, soils and natural waters and also as a geochronometer for earth surface processes (Ivanovich and Harmon 1982). Three basic mechanisms have been suggested (see for example, Gascoyne 1982) for the commonly observed excess ^{234}U in natural waters, (i) Change of the chemical valence state of ^{234}U atom as a result of the 'hot atom effect' during its radioactive production from ^{238}U ; it was proposed that ^{234}U is oxidised to a +6 valence state during this process and is preferentially mobilised during its weathering (Dooley *et al* 1966), (ii) release of ^{234}Th from soil/rock surface into the pore fluids by recoil during alpha decay of ^{238}U (Kigoshi 1971) and subsequent decay of ^{234}Th to ^{234}U , and (iii) preferential leaching of ^{234}U contained in the tracks produced as a result of radiation damage during the alpha decay of ^{238}U (Cherdynstsev 1971). More recently, Fleischer and Raabe (1978) and Fleischer (1982a, b) have demonstrated, based on laboratory experiments, the importance of alpha recoil damage on the preferential release of recoil fragments during etching and its implications on uranium isotopic disequilibrium in natural waters.

In this paper we present a mathematical model to predict the $^{234}\text{U}/^{238}\text{U}$ activity ratio in the aqueous phase, based on the preferential release of ^{234}U from recoil tracks during natural weathering processes. While this work was in progress, Fleischer (1983) suggested a model to estimate the release of alpha recoil nuclei from solid surfaces. This model is based mainly on the equations of Flugge and Zimens (1939) which relate the fractional escape of recoils from the surface of a given grain to its size. On a broader frame work, the model presented in this paper and that of Fleischer (1983) pertain to the

same problem, viz. alpha recoil effects on the $^{234}\text{U}/^{238}\text{U}$ radioactive disequilibrium in nature. However, the models differ greatly both in concepts and details. Fleischer's model calculates the fraction of recoiling nuclei stopped in pore spaces between grains and those implanted onto adjoining grains. These calculations in conjunction with a knowledge of other parameters such as grain size, porosity etc., can be used to derive an estimate of the $^{234}\text{U}/^{238}\text{U}$ activity ratio in the solids (rocks/soil) and the ^{234}U supply to pore fluids via recoil. Our model is to predict the $^{234}\text{U}/^{238}\text{U}$ activity ratio in the aqueous phase resulting from natural weathering of rocks/soils. It considers the release of ^{238}U and ^{234}U by dissolution of rocks and the preferential release of ^{234}U from recoil tracks.

2. Model concepts

In rocks/soils, ^{238}U and its daughter products occupy different sites. The daughter nuclides of ^{238}U are displaced from the site of their production because of recoil effects following alpha decay and would be located in radiation damaged regions (recoil tracks) of the rocks/soil; whereas ^{238}U would be occupying relatively healthier sites. It is well known that radiation damaged areas are etchable relatively faster than the surrounding healthy material (Huang and Walker 1967; Fleischer *et al* 1975; Petit *et al* 1985). This difference in the etching rate between the radiation damaged and healthy regions of rocks/soils results in the preferential mobilization of daughter nuclides relative to the progenitor ^{238}U into the aqueous phase. Thus ^{234}U would be preferentially leached into the aqueous phase following its production from ^{238}U . However, the activity ratio (AR) of $^{234}\text{U}/^{238}\text{U}$ in the aqueous phase resulting from the weathering of rock/soil would depend on several parameters such as the nature of the rock, its surface texture, its geometry, its size in relation to alpha recoil range etc. In the model which is discussed below, calculations are made for the leachable amounts of ^{234}U and ^{238}U for two rock geometries, slab and sphere. For each of these geometries, three calculations are made corresponding to (i) the zeroth etch, (ii) continuous etching of the surface to a depth x , after the zeroth etch and (iii) singularity.

The zeroth etch in this discussion refers to the first etch of an unweathered (virgin) rock surface. In this etch, the tracks having an intersection at the virgin surface would be leached out whereas the surface as a whole would be dissolved to an infinitesimally small depth, ϵ (ϵ is assumed to be much smaller relative to the alpha recoil range L). However this assumption is not critical; the calculations can readily be adjusted to any value of ϵ . After this etch, the rock surface reduces to a weathered surface which can no more offer the zeroth etch contribution. As the rock gets continually weathered, its size becomes comparable to the α -recoil range, which is termed as singularity in our model.

The model is based on the following assumptions: (i) ^{238}U and ^{234}U are in radioactive equilibrium (i.e. $^{234}\text{U}/^{238}\text{U}$ AR = 1.0) in unweathered rocks. Both ^{238}U and ^{234}U are uniformly distributed within the rock body. (ii) The radioactive mean life of ^{234}U is considerably shorter than the annealing time of alpha recoil tracks. This assumption appears to be valid from the data of Huang and Walker (1967) who have shown that the annealing characteristics of alpha recoil tracks and fission tracks are similar. They have also shown that the fission track ages of different micas and their ages based on radioactive decay, upto about 400 million years, are in excellent agreement. However, the recent results of Eyal and Fleischer (1985), indicate that the annealing time of alpha recoil tracks in minerals is $< 10^5$ yr, questioning the validity of

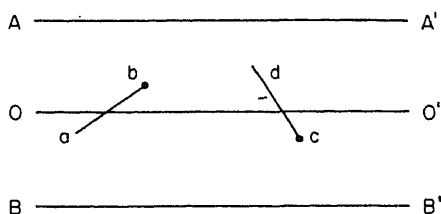


Figure 1. The rock body $AA'B'B$ is broken at OO' to yield a virgin surface OO' . ^{234}U atoms lie at the positions b and c . Leaching of the parts of the tracks ab and cd in $OO'B'B$ and $AA'O'O$ respectively will not lead to the recovery of the ^{234}U atom.

the above assumption. If future experiments certify the recent findings, the calculations given below can be adjusted to include this factor. (iii) The etching rate within the radiation damaged area (recoil track) is much greater than the rate at which the healthy regions of the rock are attacked. However, ^{234}U would be supplied to the leach only if the surface of the grain being weathered, has the stopping end of the recoil track (where ^{234}U resides, figure 1). The number of recoil tracks containing ^{234}U atoms is Q/λ_4 , where Q is the activity of ^{238}U per unit volume (dpm cm^{-3}) and λ_4 the decay constant of ^{234}U (min^{-1}). This number would be considerably less than the total number of recoil tracks per unit volume of rock which is approximately $\{8(^{238}\text{U}) + 7(^{235}\text{U}) + 6(^{232}\text{Th})\} \tau$ where (^{238}U) , (^{235}U) and (^{232}Th) are the activities of these nuclides per unit volume of rock (dpm cm^{-3}) and τ , the storage time of alpha recoil tracks or the age of the rock, if less than τ . (iv) Track leaching is isotropic (Fleischer and Price 1963, 1964).

In the zeroth etch, each track containing a ^{234}U atom and having an opening at the surface will release its ^{234}U atom to the solution. Since the dissolution of the healthy region of the grains is assumed to be much smaller relative to that of radiation damaged zones a large excess of ^{234}U relative to ^{238}U (in activity units) would be supplied to the solution in the zeroth etch. As dissolution/weathering proceeds, new recoil tracks would be revealed and ^{234}U atoms would be continually supplied to the solution along with ^{238}U atoms. The $^{234}\text{U}/^{238}\text{U}$ AR of the solution after the zeroth etch would depend on the geometry of the grain. The surface area of spherical grains decreases as weathering proceeds radially inwards, because of this, the supply of ^{234}U to solution and $^{234}\text{U}/^{238}\text{U}$ AR in solution also decreases. When the grain dimension approximates alpha recoil range, no ^{234}U would be left in the grain and hence $^{234}\text{U}/^{238}\text{U}$ AR in the grain would be zero. In this way, the initial excess of ^{234}U obtained in the zeroth etch of a rock body is compensated and the total ^{234}U and ^{238}U activities leached out from a total rock piece remain equal. For a slab, the leached out activity of ^{234}U does not fall as gradually as in the case of a sphere. Instead, it decreases sharply only after the slab has attained a thickness equal to that of alpha recoil range. In the following, we quantitatively estimate the amounts of ^{234}U and ^{238}U leached out from a unit area of a rock surface as it passes through the above mentioned stages of weathering.

3. Etching of slab

3.1 Zeroth etch

The rock body $AA'B'B$ was broken/cleaved at OO' to yield a freshly exposed virgin

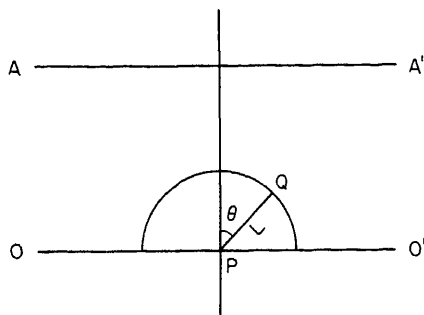


Figure 2. Zeroth etch of a virgin slab surface. AA'O'O is a slab type rock piece with a freshly opened up virgin surface at OO'. P is the test point. Recoils crossing P towards AA' would deposit their ^{234}U atom within a range $PQ = L$. (It is assumed that the actually recoiled ^{234}Th atom does not move from its emplacement site while it decays to ^{234}U .)

surface OO' corresponding to an infinitesimal etching of the surface ($x = \varepsilon$) of the piece, OO'A'A, at the test point P (figure 2) would come from atoms stopping between P and Q ($PQ = L$, the recoil range), but only those originating from the part of the rock OO'B'B (like track ab, figure 1)

$$\begin{aligned} Z_4(s, x = \varepsilon) &= \int_0^{\pi/2} 2\pi J \sin \theta \cos \theta L d\theta \\ &= \pi J L, \end{aligned} \quad (1)$$

where $Z_4(s, x = \varepsilon)$ is the ^{234}U activity (dpm cm^{-2}) leached out of a unit virgin surface area of a slab of thickness s ; and J , the source strength $= Q/4\pi$, for ^{234}Th recoils ($\text{dpm cm}^{-3} \text{ sterad}^{-1}$).

It must be noted that the ^{234}U atoms originating as a result of recoils within OO'A'A and crossing the surface OO' are lost as far as Z_4 is concerned. This is to say that only the recoils going upwards and stopping in the rock piece OO'A'A contribute to the zeroth etch.

Since it is assumed that no appreciable surface dissolution takes place during the zeroth etch, the amount of ^{238}U leached out in zeroth etch is almost negligible and hence

$$Z_8(s, x = \varepsilon) \simeq 0, \quad (2)$$

where $Z_8(s, x = \varepsilon)$ is the ^{238}U activity (dpm cm^{-2}) leached out of a unit virgin surface area of a slab of thickness s , in the zeroth etch.

3.2 Continuous etching (after zeroth etch)

After the zeroth etch, the new ^{234}U atoms recovered in etching a layer at depth $x + dx$ would be due to tracks where the recoiled atom stops anywhere in the channel QPQ' (figure 3). For tracks with zenith angle from 0 to θ , where $\theta = \cos^{-1}(dx/L)$, contributions to ^{234}U activity will come from the recoils travelling upwards as well as due to recoils travelling downwards (i.e. from both the hemispheres) corresponding to a

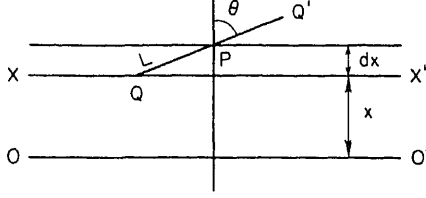


Figure 3. Continuous etching of a slab after the zeroth etch. dx is the infinitesimal thickness. The contribution to ^{234}U comes from a range $dx \sec \theta$, in both the directions, for the zenith angle θ such that $PQ = L$. For $PQ > L$, the range remains fixed at L .

range of $dx \sec \theta$. At zenith angles exceeding θ , the contribution will be from the whole range L from both the hemispheres. The activity of ^{234}U recovered per unit area by leaching between x and $x + dx$ would then be given by

$$q_4(s, x) dx = 2 \int_0^{\cos^{-1}(dx/L)} 2\pi J \sin \theta \cos \theta dx \sec \theta d\theta + 2 \int_{\cos^{-1}(dx/L)}^{\pi/2} 2\pi J \sin \theta \cos \theta L d\theta$$

$$q_4(s, x) = 4\pi J, \quad (3)$$

where $q_4(s, x) dx$ is the total uranium activity leached out per unit area (after the zeroth etch) from a slab of thickness s , in etching from a depth x to $x + dx$.

Since the contribution to ^{238}U in any dissolution is due to the net volume of the rock dissolved, for the case of a slab it can be written as

$$q_8(s, x) = 4\pi J, \quad (4)$$

where $q_8(s, x) dx$ is the total uranium activity leached out per unit area (after the zeroth etch) from a slab of thickness s , in etching from a depth x to $x + dx$.

3.3 Singularity

Equation (3) is valid as long as the thickness of the slab exceeds L . For slab thickness $l \leq L$ (XA in figure 4), it can be shown, based on similar arguments as in the above section, that

$$q_4(l \leq L, x) dx = 2 \int_{\cos^{-1}(l/L)}^{\cos^{-1}(dx/L)} 2\pi J \sin \theta \cos \theta dx \sec \theta d\theta + 2 \int_{\cos^{-1}(dx/L)}^{\pi/2} 2\pi J \sin \theta \cos \theta L d\theta,$$

$$q_4(l \leq L, x) = 4\pi J (l/L). \quad (5)$$

There will be no contribution to ^{234}U activity from the zenith angle 0 to $\theta_1 \{= \cos^{-1}(l/L)\}$ since recoils with $PQ' \leq L$, will leave the slab (figure 4).

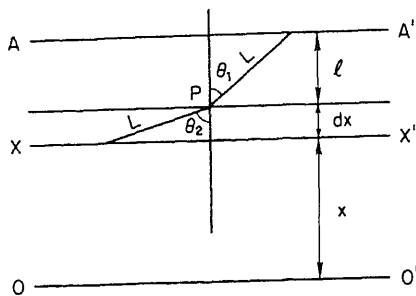


Figure 4. Singularity in slab. The thickness of the slab AA'XX' is less than the recoil range L . Recoils with zenith angle $\theta \leq \theta_1$ will be lost. Contribution to ^{234}U atom would come from the range $dx \sec \theta$ for zenith angles between θ_1 and θ_2 and from the range L for angles $\geq \theta_2$.

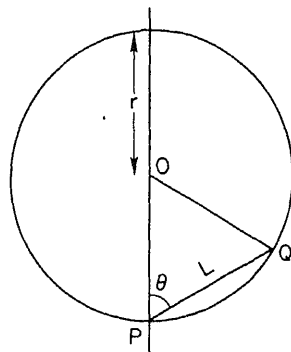


Figure 5. Zeroth etch of a virgin spherical grain. Recoils crossing the test point P into the grain would deposit their ^{234}U atom within the grain for all zenith angles $\leq \theta$ from range L . For $\theta > \cos^{-1}(L/2r)$, the range would be limited by $PQ = 2r \cos \theta$.

4. Etching of spherical grain

4.1 Zeroth etch

Following earlier lines of reasoning, the contribution to ^{234}U activity results from an infinitesimal etching of a unit surface, at the test point P (figure 5) from range L for values of PQ exceeding L $\{0 \leq \theta \leq \cos^{-1}(L/2r)\}$ and from range $2r \cos \theta$ for $PQ \leq L$ $\{\cos^{-1}(L/2r) \leq \theta \leq \pi/2\}$. Recoils travelling downwards do not contribute to ^{234}U atoms in leaching the grain.

$$Z_4(r, x = \epsilon) = \int_0^{\cos^{-1}(L/2r)} 2\pi J \sin \theta \cos \theta L d\theta + \int_{\cos^{-1}(L/2r)}^{\pi/2} 2\pi J \sin \theta \cos \theta 2r \cos \theta d\theta,$$

$$Z_4(r, x = \epsilon) = \pi J L (1 - L^2/12 r^2). \quad (6)$$

And as before, for the case of ^{238}U

$$Z_8(r, x = \epsilon) \simeq 0. \quad (7)$$

4.2 Continuous etching (after zeroth etch)

For continuous etching after zeroth etch between x and $x + dx$, the activity of ^{234}U recovered per unit area would be only from twice the range $PQ (\simeq dx \sec \theta)$ until $PQ' = L$ (figure 6).

$$q_4(r, x) dx = 2 \int_0^{\cos^{-1}\{L/2(r-x)\}} 2\pi J \sin \theta \cos \theta \sec \theta d\theta,$$

$$q_4(r, x) = 4\pi J \{1 - L/2(r-x)\}; 2(r-x) > L, \quad (8)$$

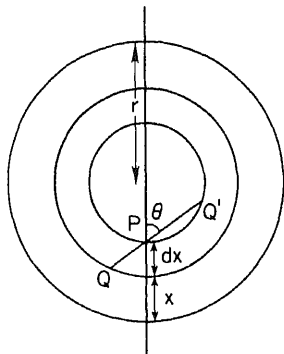


Figure 6. Continuous etching after zeroth etch of a spherical grain. The only contribution to ^{234}U atom arises due to recoils in both directions from a range PQ for all zenith angles θ such that $PQ' = L$. For larger θ the atom is lost.

And for the case of ^{238}U as before,

$$q_8(r, x) = 4\pi J, \quad (9)$$

4.3 Singularity

Equation (8) is valid only as long as $2(r-x) > L$. When $2(r-x) \leq L$, the atom recoils out of the rock section (figure 6) and hence,

$$q_4(r, x) = 0; \quad 2(r-x) \leq L. \quad (10)$$

But in the case of ^{238}U ,

$$q_8(r, x) = 4\pi J. \quad (11)$$

The above calculations, (1) to (11), are summarised in table 1.

5. $^{234}\text{U}/^{238}\text{U}$ in leaching rock/soil

In the following, based on the previous calculations (table 1), we shall calculate the total amounts of leachable ^{234}U and ^{238}U and the $^{234}\text{U}/^{238}\text{U}$ AR, resulting from leaching a rock with $^{234}\text{U}/^{238}\text{U}$ AR = 1.0 to a depth x . Both the geometries are considered to be with initial virgin surface area. We shall then consider a case of the population of grains also. The total activities of ^{234}U and ^{238}U per unit area leached out are designated as a_4 and a_8 respectively.

5.1 Slab

When a rock of slab geometry of thickness s is leached to a depth x ,

$$a_4(s, x) = Z_4(s, x = \varepsilon) + \int_0^x q_4(s, x) dx \quad (12)$$

$$a_8(s, x) = \int_0^x q_8(s, x) dx \quad (13)$$

Table 1. Theoretically expected amounts of ^{234}U and ^{238}U on leaching virgin surface of a rock containing these isotopes in equilibrium.

Geometry	Zeroth etch (dpm cm ⁻²)	Continuous etching after zeroth etch (dpm cm ⁻³)
Slab	$Z_4(s, x = \varepsilon)^a = \pi J L$	$q_4(s, x)^b = 4\pi J$
	$Z_8(s, x = \varepsilon) = 0$	$q_8(s, x) = 4\pi J$
Sphere	$Z_4(r, x = \varepsilon)^c = \pi J L 1 - L^2/12r^2$	$q_4(r, x)^c = 4\pi J 1 - L/2(r - x)$
	$Z_8(r, x = \varepsilon) = 0$	$q_8(r, x) = 4\pi J$

See text for details of symbols.

^a Valid if the initial slab thickness, s exceeds L . For $s = l \leq L$, the value of $Z_4(l, x = \varepsilon) = \pi J l/L$.

^b Valid if the final slab thickness s (i.e. at x), exceeds L . For $s = l \leq L$, the value of $q_4(l, x) = 4\pi J l/L$.

^c Valid for all values of grain radius, $r > L/2$. For $r \leq L/2$ (3)^{1/2}, $Z_4(r \leq L/2(3)^{1/2}, x = \varepsilon) = 0$ and $q_4(r \leq L/2, x) = 0$.

Substituting the values of $Z_4(s, x = \varepsilon)$, $q_4(s, x)$ and $q_8(s, x)$ from table 1 in (12) and (13), we obtain $R = a_4/a_8$.

$$R = \frac{\pi J L + 4\pi J x}{4\pi J x} = 1 + L/4x \quad (14)$$

where R is the $^{234}\text{U}/^{238}\text{U}$ activity ratio. Equation (14) is subject to the condition that the thickness of the slab s exceeds L .

5.2 Spherical grain

Following the previous arguments, let the total amount of uranium activity released from a virgin grain of radius r in leaching it from its present surface along the radial direction to a depth of x (cm) be represented by $a(r, x)$. Then we obtain

$$a_4(r, x) = 4\pi r^2 Z_4(r, \varepsilon) + \int_0^x 4\pi (r-x)^2 q_4(r, x) dx, \quad (15)$$

$$a_8(r, x) = \int_0^x 4\pi (r-x)^2 q_8(r, x) dx, \quad (16)$$

and

$$R = 1 - L/2r + L/4x \{1 - (L/2r)^2\} \text{ for } x \ll r. \quad (17)$$

If the grain is much larger compared to the recoil range (i.e. $2r \gg L$), (17) would reduce to (14).

distribution (Lal and Lerman *et al* 1977), given by $dN/dr = n(r) = Cr^{-b}$, where C and b are constants, the amounts of ^{234}U and ^{238}U leached in etching each grain to a depth x are

$$a_4 \{n(r); r_1, r_2\} = \int_{r_1}^{r_2} 4\pi Cr^{2-b} Z_4(r, \varepsilon) dr + \int_{r_1}^{r_2} \int_0^x 4\pi Cr^{-b} (r-x)^2 q_4(r, x) dx dr, \quad (18)$$

$$a_8 \{n(r); r_1, r_2\} = \int_{r_1}^{r_2} \int_0^x 4\pi Cr^{-b} (r-x)^2 q_8(r, x) dx dr,$$

and

$$R = a_4 \{n(r); r_1, r_2\} / a_8 \{n(r); r_1, r_2\}. \quad (19)$$

Substituting from (18) and (19), the ratio equals

$$R = 1 + L/4x. \quad (20)$$

Equation (20) is identical to (14). This is expected since as long as the grain size is much larger compared to the recoil range ($2r \gg L$), the effect due to curvature is not prominent. For $2r \sim L$, subtractive terms of $(L/2r)$ and its higher powers, as in (17) would arise.

Now if ε is not infinitesimal as assumed (say 0.1 % of the recoil range L), but is a finite fraction α of L . In this case all the tracks with length l ($l \leq L/1000\alpha$) would be still etched for infinitesimal surface dissolution ε . ($L/1000\alpha$) can be viewed as a new recoil range L' , that would replace L in all expressions for zeroth etch contribution of ^{234}U (i.e. equations 1, 6, 14, 17 and 20). Singularity would also shift to L' instead of L . Calculations for dissolved contributions will be unaffected. Figure 7 shows the variation of R as a function of the thickness of the rock dissolved, starting from a virgin surface, for different assumed values of α .

6. Experimental evidences for preferential leaching of alpha recoil tracks

To check on the validity of the assumptions of preferential leaching of alpha recoil tracks and its role in producing radioactive disequilibrium among members of the natural radioactive decay chains, two sets of laboratory experiments were carried out. These are: (i) leaching of implanted recoils from annealed and unannealed mica, and (ii) granite leaching experiments.

6.1 Leaching of implanted recoils from mica

Mica discs (2.5 cm dia) were irradiated with recoiling ^{224}Ra and ^{212}Pb nuclei emanating from a weightless ^{232}U - ^{228}Th electrodeposited source ($\sim 10^4$ dpm). The irradiation was carried out for about four hours in an evacuated chamber. For each experiment two discs were irradiated, of which one was heated after irradiation in a furnace at $\sim 350^\circ\text{C}$ for 3 to 4 hours to anneal the recoil tracks. The annealed and un-annealed discs were assayed for their alpha activity (using a solid state detector coupled to MCA). After their

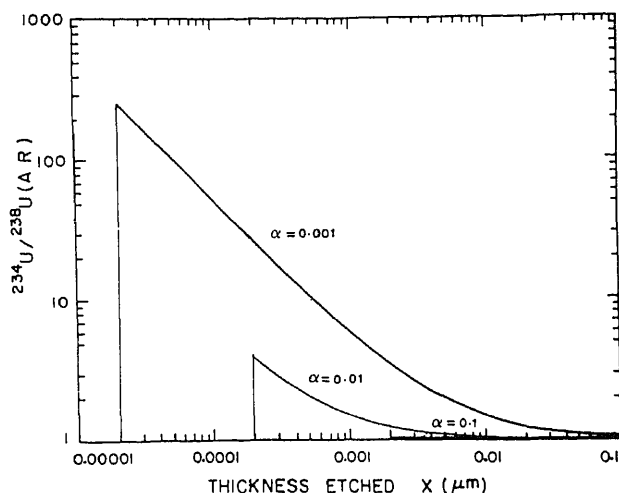


Figure 7. Theoretically estimated values of $^{234}\text{U}/^{238}\text{U}$ AR from a virgin rock surface as a function of the thickness of the rock dissolved, for different assumed values of α , the fraction of the recoil range which must be dissolved from the healthy surface in order to etch the complete recoil track. For $\alpha = 0.001$, the total range of the curve shown is available. It decreases for increasing α .

alpha assays, the samples were etched in 48% HF for 0.5 min, washed, dried and recounted. The samples were re-etched for periods of 1–5 min, counted and the process of re-etching and counting repeated for a total etch period of about 20 min. The ^{224}Ra and ^{212}Pb (counted via ^{212}Bi alphas) remaining in the discs after various etch periods are given in table 2 and figure 8 (Hussain 1983).

6.2 Granite leaching experiment

Granite of 4 ppm ^{238}U concentration was powdered to $< 150\ \mu\text{m}$ size and was dispersed in 1 litre of artificial river water (Strickland and Parsons 1972). It was stirred continuously with a magnetic stirrer for a period of 5 hours and then the slurry was filtered through a millipore. The filtrate was acidified with nitric acid and processed for uranium after adding ^{232}U as the yield tracer (Hussain 1983). The granite residue after the first leach was redispersed in a fresh aliquot of 1 litre of artificial river water. After another 5 hours stirring, the solid and solution phases were separated using a millipore and the whole procedure repeated. The results are given in table 3. The errors quoted are one sigma uncertainties due to counting statistics.

7. Discussion of results

The results on the leaching of ^{224}Ra and ^{212}Pb from annealed and unannealed mica (table 2, figure 8) show that these nuclides are released at a faster rate from the unannealed mica (figure 8). This result is attributable to preferential etching of recoil

Table 2. Results of leaching recoil implanted ^{224}Ra and ^{212}Pb from mica

Code	Leaching time	% initial activity remaining on mica (counts per minute)			
		Unannealed		Annealed	
		^{224}Ra	^{212}Pb	^{224}Ra	^{212}Pb
20, 21	0-0	100	100	100	100
	5-0	29 ± 2	62 ± 3	70 ± 3	91 ± 4
	10-0	12 ± 1	45 ± 2	60 ± 3	83 ± 4
22, 24	0-0	100	100	100	100
	0-5	65 ± 3	94 ± 3	83 ± 4	100 ± 4
	2-0	54 ± 2	87 ± 3	77 ± 4	100 ± 4
	5-0	34 ± 1	80 ± 2	67 ± 3	102 ± 5
	10-0	17 ± 1	58 ± 2	61 ± 4	100 ± 6
	17-0	7 ± 1	37 ± 2	36 ± 3	89 ± 8
25, 26	0-0	100	100	100	100
	0-5	58 ± 2	90 ± 2	80 ± 3	96 ± 4
	2-0	48 ± 2	84 ± 2	69 ± 3	91 ± 4
	5-0	33 ± 1	68 ± 2	65 ± 3	95 ± 4
	10-0	17 ± 1	55 ± 2	58 ± 3	81 ± 4
	17-0	7 ± 1	40 ± 2	49 ± 3	84 ± 4
	22-0	—	—	40 ± 2	74 ± 4

Table 3. Results of the granite leaching experiments

Leach number	Period of contact (hrs)	^{238}U activity in leach (dpm)	$^{234}\text{U}/^{238}\text{U}$ (AR)
1	5	1.03 ± 0.05	1.45 ± 0.09
2	5	0.77 ± 0.02	1.5 ± 0.09
3	5	0.91 ± 0.04	1.46 ± 0.07
4	20	0.74 ± 0.04	1.52 ± 0.09
5	39	0.32 ± 0.04	1.35 ± 0.08

in the annealed mica (table 2), most probably results from mica dissolution. 48 % HF is a strong etchant and therefore the dissolution effect could be quite significant. However, the dissolution through recoil tracks is significantly faster and is almost complete over a period of about 20 to 30 min in 48 % HF.

The results of the granite leaching experiment (table 3) show the presence of excess ^{234}U in the solution phase in all the leaches. The short period of contact between the granite and the water precludes the source of this ^{234}U excess being the *in situ* recoil of ^{234}Th and its decay to ^{234}U (Kigoshi 1971) and shows that it could only be due to the

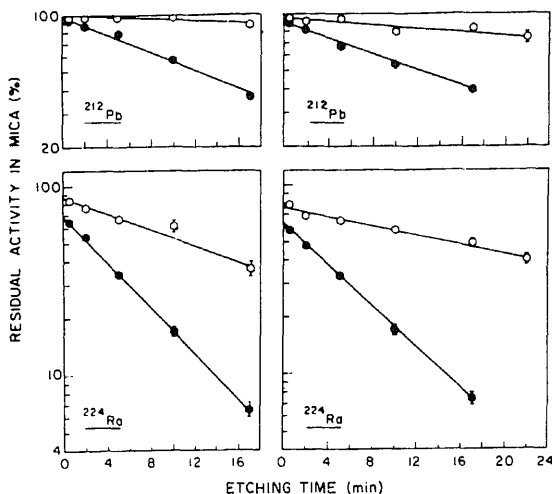


Figure 8. The percent activity of ^{212}Pb and ^{224}Ra remaining in the mica disc for various etch periods. Solid hexagons represent the activity in the unannealed samples and open ones that in the annealed samples. The sharper decrease in the residual activity in the unannealed mica shows the effect of preferential solution through alpha recoil tracks.

7.1 ^{234}U leaching dynamics in natural waters

The radioactive disequilibrium between ^{234}U and ^{238}U in natural waters is intimately related to the mechanism of rock weathering. In the previous section it was shown that excess $^{234}\text{U}/^{238}\text{U}$ AR was expected to arise by preferential track leaching only when a 'virgin' surface is leached by an aqueous medium.

Rivers, lakes and groundwaters, by and large, are characterised by $^{234}\text{U}/^{238}\text{U}$ AR > 1 (Scott 1982; Osmond and Cowart 1982). It can be inferred from coral data that the ^{234}U excess in the oceans is in a steady state relative to its supply and removal. Such a continuous supply of ^{234}U excess from the same drainage area is difficult for explanation by the above track leaching model, unless virgin surfaces are made available for preferential supply of ^{234}U .

Almost all known crystalline rocks like granite, dunite or quartzite contain abundant microcracks and cavities known as flaws (Sprunt and Brace 1974; Montgomery and Brace 1975). These are produced mainly as a result of the stress and temperature changes to which an ordinary rock is subjected after its formation. Large differences are expected between rock types since the elastic constants and thermal expansion coefficients of minerals vary appreciably (Nur and Simmons 1970).

Weathering results in a continuous fragmentation of the rock (figure 9). The character of fissuring will depend on the rock type, the greater the density of flaws in a rock body (Nur and Simmons 1970; Tapponier and Brace 1976; Logan 1979), the greater would be the fissuring per unit rock dissolution. It is thus expected that as the exposed surface is leached, simultaneously connecting virgin (fresh) surface will be exposed. With the progression of weathering as exposed surfaces are leached, further exposure would occur. The fragmentation of a rock into rocklets proceeds at all levels, larger grains breaking into smaller grains and so on.

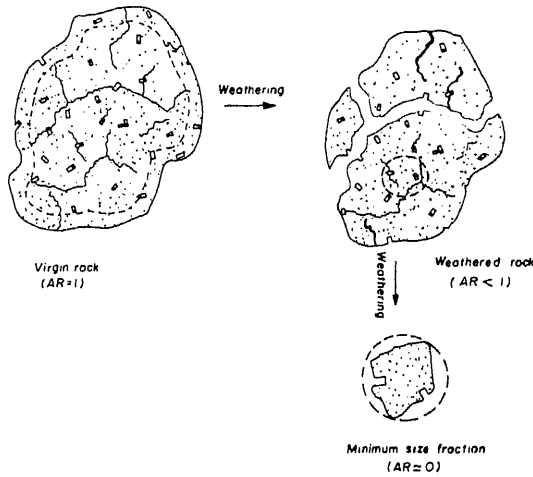


Figure 9. Cartoon showing the breaking up of grains in yielding virgin surfaces and the excess ^{234}U in a weathering process. Dots represent ^{238}U atoms (shown exaggerated), small rectangles, the alpha recoil tracks, and the lines, the flaws or the micro-cracks in the body of the grain (the dotted line shows the extent of dissolution). Only those tracks with a dot inside have a ^{234}U atom within. ^{234}U atom is preferentially extracted from an exposed track to the surface in weathering. AR stands for $^{234}\text{U}/^{238}\text{U}$ activity ratio.

The excess of ^{234}U in leach waters would depend on the rate at which new fissures open up during weathering. Since the rate of leaching would depend on the chemical composition of the waters leaching the rock, and also the chemical composition of the rock itself, we may quantify the weathering process relevant to the supply of excess ^{234}U activity and solution as K , the ratio of two parameters, viz, the rate at which virgin surface is exposed during weathering, $\dot{A}$ ($\text{cm}^2 \text{y}^{-1}$) and the rate of dissolution of an exposed surface during weathering, ξ (g y^{-1}). $K = \dot{A}/\xi$ has the dimension $\text{cm}^2 \text{g}^{-1}$ and signifies the mean area residing in microcracks per unit rock mass. It is this area which opens up during the mass dissolution of a rock.

In the following we present a model relating the $^{234}\text{U}/^{238}\text{U}$ AR in the aqueous phase resulting from leaching a rock body with a characteristic ratio K . For simplicity we shall consider a spherical grain of a unit surface area and density ρ (g cm^{-3}). The time variation in the ^{234}U and ^{238}U activities in the aqueous phase are given by:

$$da_4/dt = \dot{A} Z_4(r, x = \epsilon) + q_4(r, x) (\xi/\rho) + \lambda_4 a_8 - \lambda_4 a_4, \quad (21)$$

and

$$da_8/dt = q_8(r, x) (\xi/\rho) - \lambda_8 a_8, \quad (22)$$

t denotes the time parameter, other symbols are as stated earlier.

In general, magnitude of the radioactive production and decay terms in (21) and (22) are negligible relative to leaching and dissolution supply rates. Further for $t \ll \lambda_4^{-1}$, the production of ^{234}U in the leach from a_8 and its decay, will be insignificant and hence the $\lambda_8 a_8$ and $\lambda_4 a_4$ terms in (21) and (22) can be deleted. Substituting $Z_4(r, x = \epsilon)$, $q_4(r, x)$ and $q_8(r, x)$ from table 1 and solving (21) and (22) we get

$$a_4 = (\dot{A} L + 4 \xi/\rho) \pi J t + a_4^0, \quad (23)$$

and

$$a_8 = 4\pi J \xi t / \rho + a_8^0,$$

where a_4^0 and a_8^0 are the values of a_4 and a_8 at $t = 0$.

At $t = 0$ when the grain comes in contact with the etchant, there is no dissolution hence $\hat{a}_8 = 0$. However, if the grain had a virgin surface, $\hat{a}_4$ would be the ^{234}U brought into solution in the zeroth etch because the etching through

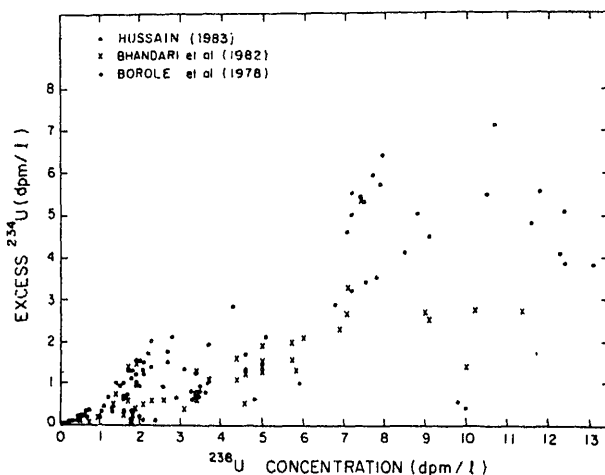


Figure 10. Scattergram of ^{234}U excess vs ^{238}U for Indian groundwaters. The coefficient for the least squares fit is about 0.8–0.9 and the slope between 0.3 to

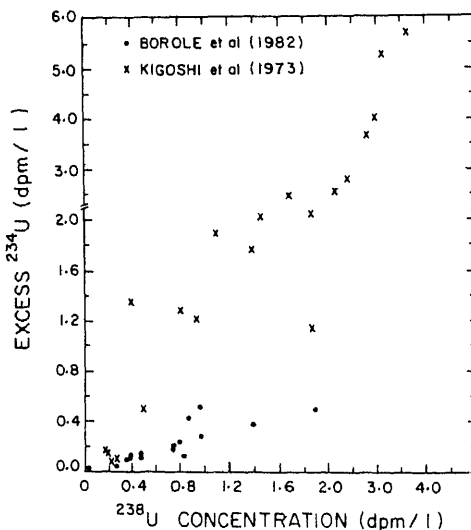


Figure 11. Scattergram of ^{234}U excess vs ^{238}U for Indian river waters (Borole et al) and groundwaters from Tokyo (Kigoshi 1973). The correlation coefficients are 0.97 and 0.99, respectively. The slopes are 0.7 and 1.5, respectively.

assumed to be instantaneous. For a weathered grain, $\hat{a}_4 = 0$. From (23) and (24), we obtain the excess ^{234}U activity $a_4(\text{exc}) (= a_4 - a_8)$ for a virgin grain as:

$$a_4(\text{exc}) = (KL\rho/4)a_8 + \hat{a}_4. \quad (25)$$

Equation (25) is a linear equation and shows that the excess activity of ^{234}U should be linearly dependant on the ^{238}U activity with a slope of $(KL\rho/4)$.

Linear plots of the available data on groundwaters as well as river waters (figures 10 and 11) fit (25). The slope values $(KL\rho/4)$ as estimated through least squares regression analysis for Indian waters range between 0.3 to 0.8 (table 4). Similar analysis for uranium isotopic data for the granite leaching experiments in the laboratory (table 3) also yields the value of the slope as 0.3 ± 0.2 . For typical values of ρ and L as 2 g cm^{-3} and $200 \text{ \AA}$ respectively, the range of the slope corresponds to a range of K , the characteristic ratio of the rock, present in the Indian zone referred here, between 3×10^5 to $8 \times 10^5 \text{ cm}^2 \text{ g}^{-1}$.

^{234}U and ^{222}Rn are produced in a rock at similar sites and at equal rates and thus whatever be the mechanisms of their introduction from rocks into the solution phase,

Table 4. Results of the least squares regression analysis for uranium isotopic data in natural waters to (25) from text.

Water type	Data points	Correlation coefficient	Slope ($\rho LK/4$)	Intercept	References
Groundwaters	43	0.88	0.36 ± 0.03	-0.09 ± 0.23	Hussain (1983)
Groundwaters	53	0.81	0.3 ± 0.03	0.22 ± 0.26	Bhandari <i>et al</i> (1982)
River waters	15	0.97	0.7 ± 0.05	-0.21 ± 0.05	Borole <i>et al</i> (1982)
River waters	6	0.92	0.55 ± 0.11	-0.1 ± 0.28	Bhat and Krishnaswami (1969)
River waters	4	1.0	0.29 ± 0.02	0.002 ± 0.007	
Groundwaters	25	0.91	0.83 ± 0.08	0.35 ± 0.14	
Groundwaters	24	0.92	0.7 ± 0.06	-0.61 ± 0.3	Borole <i>et al</i> (1979)
River waters	13	0.93	0.83 ± 0.01	-1.64 ± 0.54	
Groundwaters	19	0.94	1.46 ± 0.13	-0.14 ± 0.22	Kigoshi (1973)
Groundwaters	4	0.95	7.99 ± 1.93	-1.17 ± 0.42	Gilkeson (1982)
Groundwaters	11	0.85	16.38 ± 3.37	0.65 ± 0.55	

the extent of the rock surface area associated with a unit dissolved activity of either ^{234}U or ^{222}Rn would be the same, provided that both these nuclides are transported from rock into solution quantitatively. Hussain (1983) estimated the specific surface area of the solids in Gujarat aquifers, based on ^{222}Rn measurements, to be $c. 1.6 \times 10^5 \text{ cm}^2 \text{ g}^{-1}$. Accounting for the ^{222}Rn escape factor of 0.235 (Andrews and Wood 1972) the specific surface area of the Gujarat aquifer solids would become $c. 7 \times 10^5 \text{ cm}^2 \text{ g}^{-1}$, which is in excellent agreement with the value of K derived above, based on ^{234}U excess, for similar rocks.

It should be noted that Kulp and Carr (1952) have reported the average specific surface area of the deep sea sediments to be $c. 2.5 \times 10^5 \text{ cm}^2 \text{ g}^{-1}$.

The value of K between $3-8 \times 10^5 \text{ cm}^2 \text{ g}^{-1}$ is an upper limit, for there could be a finite contribution to the excess ^{234}U used in the calculation, from the direct recoil of ^{234}Th followed by its decay. The importance of recoil becomes particularly significant when the intensity of weathering is low and the time of soil-water contact high. The higher values of the slopes in the case of the Japanese groundwaters (Kigoshi 1973) and the Ordovician aquifer (Gilkeson 1982) (table 4, figure 11) could be due to this effect.

For a given soil-water system, (K_e) is a measure of the rate of opening of new surface area ($\text{cm}^2 \text{ y}^{-1}$). It is possible to estimate this rate on the continents from a knowledge of the continental weathering rate. Using the value of ϵ as the flux of total dissolved solids, $4 \times 10^{14} \text{ g y}^{-1}$ (Garrels and Mackenzie 1971) and K as the average specific surface area of the deep sea sediments, $2.5 \times 10^5 \text{ cm}^2 \text{ g}^{-1}$ (Kulp and Carr 1952), the rate of production of the surface area on the continents can be calculated as $10^{20} \text{ cm}^2 \text{ y}^{-1}$. Implicit in this calculation is an assumption that the surface area produced on the continents through rock weathering is lost to the oceans by the suspended load of the continental run-offs. Based on the total suspended load, $2.5 \times 10^{16} \text{ g y}^{-1}$ (Garrels and Mackenzie 1971), the average specific surface area of the suspended load of rivers works out to be $4 \times 10^3 \text{ cm}^2 \text{ g}^{-1}$. This is the area from which direct recoils are ejected into the waters.

7.2 $^{234}\text{U}/^{238}\text{U}$ activity ratio and rock characteristics

In the previous discussion we have shown that there exists a linear relation between the leachable excess of ^{234}U and total ^{238}U from rocks (25). In terms of $^{234}\text{U}/^{238}\text{U}$ AR leached out from weathered rock ($a_4^0 = 0$), (25) simplifies to:

$$R = 1 + (KL\rho/4). \quad (26)$$

Equation (26) suggests that for a given type of rock ($K = \text{constant}$), the AR obtainable in leaching is fixed and is independent of the rock dissolution. In other words, the leachable AR from a rock characterizes it.

Some results of the laboratory leaching experiments on zircon, granite and limestone, by mild leachants are presented in table 5. In all these, a single respective rock aliquot was leached repeatedly after discarding the zeroth etch. The resistant mineral zircon yields AR close to unity at various stages of its dissolution (varying dissolved ^{238}U), whereas carboniferous limestone yields higher AR. The mean AR for zircon, granite and limestone (table 5) correspond to their respective K values (26) of $0.5 \pm 5 \times 10^4$, $4 \pm 0.8 \times 10^5$ and $9 \pm 1 \times 10^5 \text{ cm}^2 \text{ g}^{-1}$ ($L = 0.02 \mu\text{m}$ and $\rho = 4, 2.5$ and 2.0 g cm^{-3} , respectively). Andrews *et al* (1982) have reported $^{234}\text{U}/^{238}\text{U}$ AR centered around 3 in shallow Stripa groundwaters for dissolved ^{238}U concentrations ranging from 1 to about 90 ppb. This

Table 5. Dissolved $^{234}\text{U}/^{238}\text{U}$ activity ratios in continuous leaches of three rock minerals, zircon, granite and carboniferous limestone.

Rock type	Dissolved activity in solution		Reference
	^{238}U (dpm)	$^{234}\text{U}/^{238}\text{U}$ (AR)	
Zircon	1.0 ± 0.1	0.94 ± 0.1	Hussain (1983)
	3.0 ± 0.3	0.98 ± 0.1	
	5.9 ± 0.5	0.95 ± 0.1	
	6.4 ± 0.6	0.94 ± 0.1	
Granite	0.7 ± 0.1	1.52 ± 0.09	Present Work
	1.0 ± 0.1	1.45 ± 0.09	
Carboniferous* limestone	1.1 ± 0.1	1.81 ± 0.1	Present Work
	1.3 ± 0.1	1.98 ± 0.07	

*From Cornwall, England

constancy of AR in shallow and young groundwaters for their increasing dissolved ^{238}U concentrations, is in an excellent agreement with our model and suggests that Stripa granites from the shallow groundwater zone (Andrews *et al* 1982) can be characterized with their leachable AR of *c.* 3. One of us (NH) is working on the distribution of uranium isotopes in the laboratory leaches of Stripa granites and the preliminary results do seem to confirm the view.

A rock in equilibrium ($^{234}\text{U}/^{238}\text{U}$ AR = 1.0) on leaching cannot yield $R > 1$ ad infinitum as suggested by (26). This apparently perpetual generation of ^{234}U in (26) is an artifact of neglecting the contribution to the excess ^{238}U from the dissolution of singular particles as and when they are produced in the process of rock dissolution-cum-fragmentation. To make an appropriate provision for the excess ^{238}U from singular particles, information about their production rate is essential. For simplicity from the requirement of mass balance of ^{234}U and ^{238}U in the rock, certain deductions can be made. Figure 12 shows qualitatively the variations of the total dissolved ^{234}U and the AR as a function of rock dissolution in a closed system. Curve a is an ideal line for a case where rock dissolution does not produce fragmentation ($K=0$) and the dissolved concentrations of ^{234}U and ^{238}U remain almost in the same proportion as they exist in the parent rock (AR = 1.0) at all stages of its dissolution. When dissolution is followed by fragmentation, the line becomes steeper (curve b), the slope depending on the specific fragmentation per unit dissolution. It is assumed here that the particles with singular dimensions are isolated from dissolution as and when they are produced. Discontinuity in curve b occurs when the singular particles alone are left in the system. Any further dissolution does not add ^{234}U but only ^{238}U and yields curve c. Curve c joins curve a when all the singular particles are dissolved, thus leaving the dissolved concentrations of ^{234}U and ^{238}U the same as existed in the rock. In case the singular particles are not isolated from dissolution but dissolve as and when produced, curve d

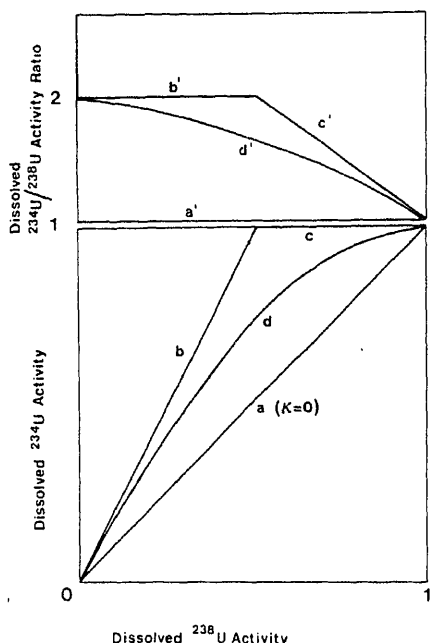


Figure 12. Variations of cumulative activities of ^{234}U and ^{238}U , and the $^{234}\text{U}/^{238}\text{U}$ activity ratios in leaching a rock, are shown qualitatively. See text for a discussion of the curves a, b, c, d and a', b', c', and d'.

infinitesimal dissolution, AR tends to be very high and comes down to a quasi-steady state value until singular particles start dissolving. $\text{AR} < 1$ in solution is possible when the majority of the particles being leached are of singularity dimensions.

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The effect of whitecaps and foam on wind speed extraction with a pulse limited radar altimeter

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Abstract. The effect of whitecaps and foam on wind speed extraction with a pulse limited radar altimeter has been studied using a specular point model. By modelling foam and water as a two-layer media, we have investigated the changes in reflectivity as a function of thickness using electro-magnetic field theory. Our analysis indicated a change of reflectivity from a value of 0.617 at 13.9 GHz normal incidence to a value of 0.10 for a foam thickness of less than 1 cm. The values of reflectivity computed from a two-layer model compared fairly well with these derived using an emissivity model.

The modified specular point model gave an improved relationship between σ^0 and wind speed in comparison to Brown's model. The Seasat altimeter's data analysis over the Arabian sea and the Bay of Bengal gave an rms difference of 2.2 m/sec in wind speed retrieval using the present modified model and Brown's model.

Keywords. Radar altimeter; Seasat; wind speed; specular point model; two-layer media.

Introduction

Measurement of the wind field is important for several meteorological and oceanographic studies, e.g. Moisture transport, upwelling phenomena, sea-state and air-sea interaction study etc. Major oceanic currents are, in fact, driven by wind stress. For a tropical country like India, where geostrophic assumption breaks down, measurement of wind speed assumes even greater significance. *In situ* ship observations are inadequate and can take several days to cover a tiny portion of the ocean. Satellites, therefore, offer the best means of covering the vast data sparse oceanic regions.

Both active and passive microwave sensors have been used for wind speed determination. The Seasat satellite launched by NASA in June 1978 and managed by the Jet Propulsion Laboratory, USA, carried onboard an SMMR (scanning multichannel microwave radiometer), a radar altimeter, a scatterometer and a synthetic aperture radar. The SMMR, altimeter and scatterometer data have been used to determine wind speeds and a global comparison of wind speeds have been made by Pandey (1983). The scatterometer, in addition, provides wind direction. It was specifically designed for wind vector measurement with a wide swath coverage (500 km). The altimeter, on the other hand, provides along the track wind field data most useful for climatological studies. The primary objective of the radar altimeter is to provide significant wave height and sea surface topography. Also, the nadir wind speed together with significant wave height could be used for estimating swell amplitude (or energy) from the radar

For a radar altimeter, the backscattering coefficient, σ° is dominated by specular reflection and is proportional to the Fresnel reflection coefficient and inversely proportional to the mean square slope (Brown 1978). For a high sea state, foam and whitecaps appear due to breaking of waves. The waves break when the condition, that water particles speed exceed the average speed of the waves in the ocean, is satisfied. Foam and whitecaps in the ocean surface have electromagnetic properties different from the background sea water. Air, foam and sea water form a layered media, distort the fine structure of the surface and alter reflectivity due to resonant phenomenon. We present the investigation of this phenomena, following Wait (1962) and Zheng *et al* (1983), using electromagnetic field theory and its impact on wind speed extraction from a radar altimeter. In our study we have used a frequency dependent fractional foam coverage and an empirical emissivity model proposed by Pandey and Kakar (1982), and Hollinger's (1973) formulations for the Debye parameters.

2. Specular point model

In the interaction of microwaves with a target such as the surface of the sea in which a large area is illuminated, the average returned power $P_r(t)$ can be expressed by the radar equation

$$P_r(t) = \frac{\lambda^2}{(4\pi)^3} \int_{\text{illuminated area}} \frac{P_t(t) G_t^2 \sigma^\circ}{R^4} dA, \quad (1)$$

where

P_r = the average received power (in watts), P_t = the transmitted power (in Watts), G_t = the antenna gain, λ = the operating wavelength, R = the range to the target and σ° = the scattering cross-section per unit area (W/m^2).

The average normalised backscattering cross-section is a convenient measure used to describe the reflectivity of an extended target or surface. For a radar altimeter, at a frequency of about 14 GHz and 0° incidence angle, the average power scattered by the sea surface is proportional to the average number of specular points per unit area and to the average Gaussian and isotropic curvature of the specular points. These two parameters have been related to surface statistics through geometric optics considerations and result in the following expression for average scattering cross-section (Barrick 1974).

$$\sigma^\circ(\theta) = \pi |R(\theta)|^2 \sec^4 \theta p(\zeta_{xsp}, \zeta_{ysp}), \quad (2)$$

where

θ = incidence angle,

$p(\zeta_{xsp}, \zeta_{ysp})$ = surface slope probability density function, (p.d.f.)

$$\zeta_x = \frac{\partial n}{\partial x}, \zeta_y = \frac{\partial n}{\partial y}, n = \text{surface elevation}, \quad (3)$$

$R(\theta)$ = the Fresnel reflection coefficient of air to surface interface at incidence angle (θ) .

Under the classical assumption when the sea surface slopes are nearly Gaussian and isotropic in their distribution, the p.d.f. is given by

$$p(\zeta_x, \zeta_y) = \frac{1}{\pi \bar{S}^2} \exp(-\tan^2(\theta)/\bar{S}^2). \quad (4)$$

$\bar{S}^2$, the mean square slope, is given by

$$\bar{S}^2 = \langle \zeta_x^2 + \zeta_y^2 \rangle \quad (5)$$

and the scattering coefficient is given by

$$\sigma^\circ(\theta) = \frac{|R(\theta)|^2}{\bar{S}^2} \sec(\theta) \exp(-\tan^2(\theta)/\bar{S}^2). \quad (6)$$

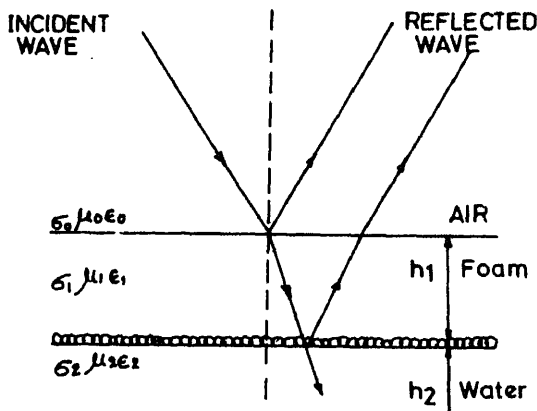
The expression (6) for $\sigma^\circ(\theta)$ is valid for $(\theta) \leq 15^\circ$ and under the condition that the curvature of the surface waves is much larger than the wavelength of the incident electromagnetic wave (Valenzuela 1978). Different forms of dependence of $\bar{S}^2$ on the wind speed are available, the classical one being the work of Cox and Munk (1954).

3. Reflection from two-layer foam/water media

The general analysis of reflection of plane waves from a parallel stratified medium consisting of M homogeneous slabs has been described by Wait (1962) and a two-layer case by Zheng *et al* (1983). Considering the case of a two-layer medium (figure 1) and normal incidence with the interface of air, foam and water where the conductivity, permeability and dielectric constant of each layer is denoted by $\sigma_i, \mu_i, \epsilon_i$ ($i = 0, 1, 2$), the reflection coefficient is given by:

$$\text{total reflection coefficient} = \frac{b_0}{a_0} = \frac{k_0 - k_1 Q}{k_0 + k_1 Q} \quad (7)$$

and reflectivity, $rf = \left| \frac{b_0}{a_0} \right|^2$ under the condition $r_1/r_0 \gg 0$, $r_2/r_0 \gg 0$, $h_2 \sim \infty$,



account for the presence of the lower layer. The quantities k_0 , k_2 and Q are given by

$$k_0 = (\mu_0/\varepsilon_0)^{1/2}, \quad (8)$$

$$k_1 = r_1/(\sigma_1 + i\varepsilon_1\omega), \quad (9)$$

$$Q = \frac{r_1/r_2 + \tanh(r_1 h_1)}{[1 + (r_1/r_2) \tanh(r_1 h_1)]}. \quad (10)$$

The propagation constants r_1 , r_2 are given by

$$r_m = [i\sigma_m \mu_m \omega - \varepsilon_m \mu_m \omega^2]^{1/2}, \quad m = 1, 2 \quad (11)$$

and

$$\begin{aligned} \varepsilon_i &= \varepsilon_0 \varepsilon'_i, \\ \sigma_i &= \omega \varepsilon_0 \varepsilon''_i, \end{aligned} \quad (12)$$

$$\varepsilon'_i = \varepsilon'_i - i\varepsilon''_i,$$

and

$$C = (\mu_0 \varepsilon_0)^{1/2},$$

where ε'_i is called the complex dielectric constant of the medium and ε_0 is the dielectric constant of the vacuum rewriting (9) and (11).

$$k_1 = r_1/\omega \varepsilon_0 (\varepsilon'_1 + i\varepsilon''_1), \quad (13)$$

$$r_1 = (2\pi/\lambda) (i\varepsilon''_1 - \varepsilon'_1)^{1/2}, \quad (14)$$

$$r_2 = (2\pi/\lambda) (i\varepsilon''_2 - \varepsilon'_2)^{1/2}, \quad (15)$$

where λ is the free space wavelength

$$\frac{r_1}{r_2} = \left(\frac{\varepsilon'_1 - i\varepsilon''_1}{\varepsilon'_2 - i\varepsilon''_2} \right)^{1/2} = \left(\frac{\varepsilon'_1}{\varepsilon'_2} \right)^{1/2}. \quad (16)$$

Following Lane and Saxton (1952), the dielectric constant of the sea water, ε'_2 is given by

$$\varepsilon'_2 = \varepsilon'_2 - i\varepsilon''_2, \quad (17)$$

where

$$\varepsilon'_2 = [\varepsilon_\infty + (\varepsilon_s - \varepsilon_\infty)/(1 + \omega^2 \tau^2)], \quad (18)$$

$$\varepsilon''_2 = [(\varepsilon_s - \varepsilon_\infty) \omega \tau / (1 + \omega^2 \tau^2) + \sigma_i / \omega \varepsilon_0]. \quad (19)$$

The different terms in (19) are given by Hollinger (1973). Here T is the temperature of the sea water (25°C) and S the salinity (35‰). Troitsky (1962) has given the following expression for the complex dielectric constant of foam water, consisting of air and sea water,

$$\varepsilon'_1 = \varepsilon'_2 \left[1 - \frac{3R}{(2\varepsilon'_2 + 1)/(\varepsilon'_2 - 1) + R} \right], \quad (20)$$

where R ($= 0.97$), the volume ratio, is the ratio of volume of air in foam to the total volume of foam.

Using the above formulations, at 13.9 GHz, the frequency of radar altimeter, the plot

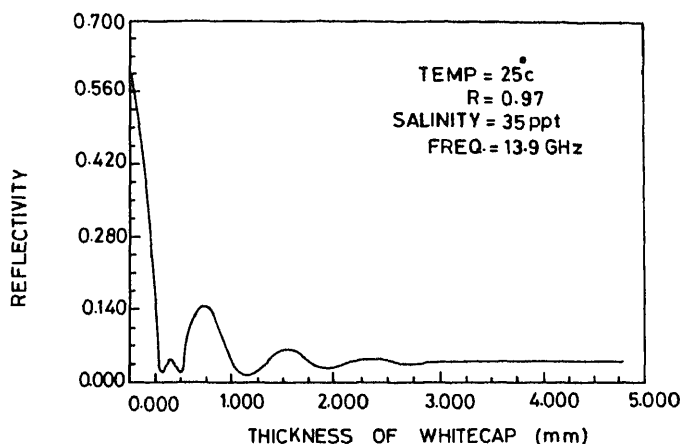


Figure 2. Microwave reflectivity versus thickness of whitecaps at 13.9 GHz.

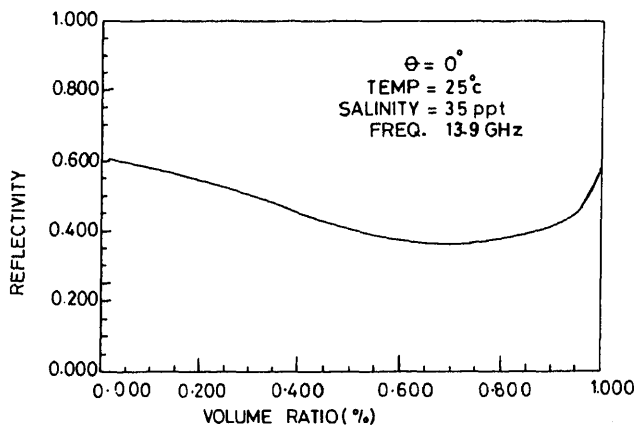


Figure 3. Effect of volume ratio (R) on reflectivity.

of rf versus hl , the thickness of the foam layer is given in figure 2. A change in reflectivity from a value of 0.6 to 0.1 is noticed for a foam thickness of ~ 0.1 cm. The oscillation in the curve is due to resonant absorption phenomenon in the foam layer.

Variation of reflectivity versus volume ratio has also been studied and depicted in figure 3.

4. Reflectivity calculations with an emissivity model and its comparison with the two-layer model

Several microwave emissivity models have recently been proposed by different workers (Wilheit 1979; Pandey and Kakar 1982) for interpreting data using passive microwave radiometers. Microwave emissivity is related to the reflectivity by the following

relation,

$$\text{emissivity} = 1 - \text{reflectivity}.$$

These relations were used to check the calculations presented in the preceding section using an empirical emissivity model proposed by Pandey and Kakar (1982). The emissivity is expressed as

$$\varepsilon_p(\theta) = [\varepsilon_{sp} + \Delta\varepsilon_{rp}] (1 - F) + \varepsilon_{fp} F, \quad (21)$$

where p denotes polarization, which may be horizontal or vertical, (θ) is the incidence angle, ε_{sp} , specular emissivity and ε_{rp} and ε_{fp} are contribution to emissivity due to roughness and foam for a case appearing to be the nadir,

$$\varepsilon_{rp} = \frac{U_{10}}{T_s} (1.59) - 0.00065\nu, \quad (22)$$

$$\varepsilon_{fp} = 0.005\nu + \frac{208 + 1.29\nu}{T_s}. \quad (23)$$

Where ν is the frequency in GHz, the fractional foam coverage is given in terms of wind speed U_{10} , by the following expression

$$F = a_0 + a_1 U_{10} + a_2 U_{10}^2. \quad (24)$$

The coefficients of (24) are given by

$$a_0 = 1.707 \times 10^{-2} + 8.560 \times 10^{-4}\nu + 1.120 \times 10^{-5}\nu^2,$$

$$a_1 = 1.501 \times 10^{-2} + 1.821 \times 10^{-3}\nu - 4.634 \times 10^{-5}\nu^2,$$

and
$$a_2 = 2.442 \times 10^{-4} - 2.282 \times 10^{-6}\nu + 4.194 \times 10^{-7}\nu^2.$$

We have compared reflectivity calculated through layered media model and through emissivity model calculations using a foam thickness of 0.1 cm. The results for few cases are given in table 1.

5. The effect of foam on wind speed estimation

From (6), the backscattering coefficient for normal incidence angle is given by

$$\sigma^0(0) = |R(0)|^2 / S^2. \quad (25)$$

Table 1. Reflectivity through layered model vs. emissivity model.

Wind speed (m/sec)	Reflectivity through a layered model ($h = 0.1$ cm)	Reflectivity through an emissivity model
0.0	0.5916	0.5942
5.0	0.5730	0.5773
10.0	0.5669	0.5636
15.0	0.5510	0.5499
20.0	0.5171	0.5205
25.0	0.4529	0.4507

Following Wu (1972), $\bar{S}^2$, the mean square slope, is given by

$$\bar{S}^2 = a \ln U_{10} + b,$$

$$a = 0.010, \quad b = 0.012 \dots \dots \dots U_{10} \leq 7 \text{ m/sec},$$

$$a = 0.850, \quad b = -0.145 \dots \dots \dots U_{10} \geq 7 \text{ m/sec}.$$

Equating (25) and (26) we have

$$\sigma^\circ(\theta) = |R(0)|^2 / (a \ln U_{10} + b).$$

Modifying the reflectivity in the presence of spray and foam, based on the analysis (§4), (27) becomes

$$\sigma^\circ = [|R(0)|^2 (1 - F) + r_1 F] / [a \ln U_{10} + b].$$

The model by Brown (1979) for wind speed measurement is given by

$$\sigma^\circ(\text{dB}) = -2.1 - 10 \log (A \ln U_{10} + B)$$

$$A = 0.02098, \quad B = 0.01075 \dots \dots \dots U_{10} \leq 9.2 \text{ m/sec},$$

$$A = 0.08289, \quad B = -0.12664 \dots \dots \dots U_{10} \geq 9.2 \text{ m/sec}.$$

The σ° calculated from the modified expression (28) and Brown's model show agreement at higher wind speed showing the importance of foam in the wind retrieval (figure 4).

A comparison of the wind speed derived using the present modified model of Brown's empirical model is shown in the figure (5) using Seasat-radar altimetry in the Arabian sea and the Bay of Bengal. The data spanned the period 27-28 April-11 August 1978 and covered the latitudes -10° to $+20^\circ$ and longitudes 60° to 100° . The data over land was excluded from the analysis. The comparison gave a difference of 2.2 m/sec. This was done since no concurrent ship observations were available for that period and region.

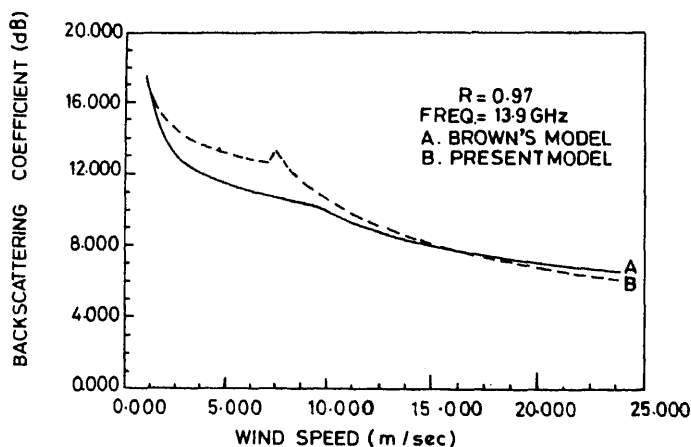


Figure 4. Comparison of $\sigma^\circ(\text{dB})$ versus wind speed (U_{10})

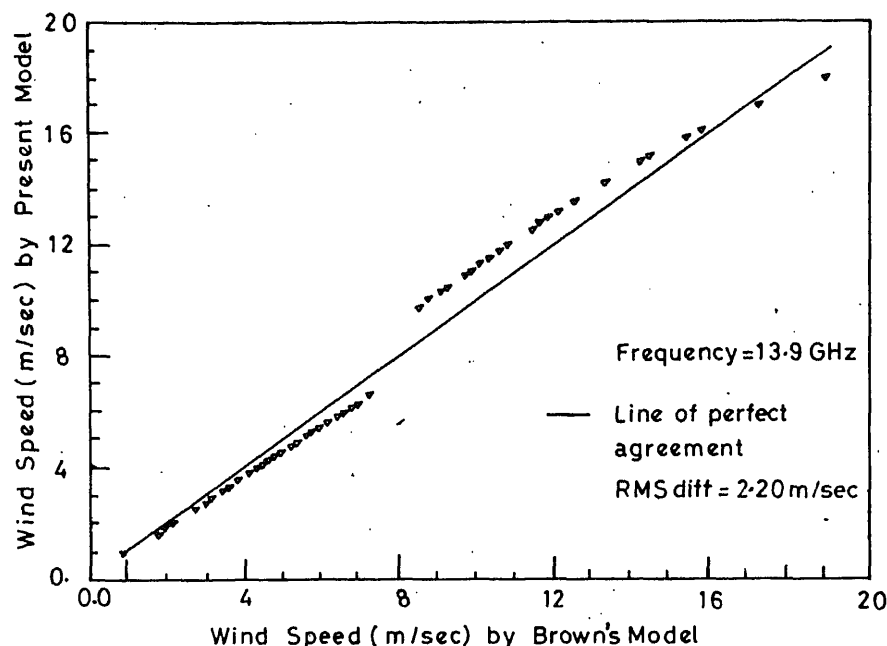


Figure 5. Comparison of altimeter inferred wind speeds by Brown's model and the present model.

6. Conclusions

The modified model proposed in the paper provides a better agreement with Brown's empirical model for wind speed retrieval. The agreement is more pronounced at higher wind speed. Seasat-radar altimeter data over the Arabian sea and the Bay of Bengal were used for wind speed extraction and a comparison with Brown's model gave an rms difference of 2.20 m/sec.

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Gravity anomalies of a trapezoidal model with quadratic density function

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Abstract. The decrease of density contrast with depth in sedimentary strata is approximated by a quadratic function. The anomaly equation of a trapezoidal model with the quadratic density function is derived. Nonlinear optimization technique using the Marquardt algorithm has been developed and used to interpret a synthetic anomaly profile of the trapezoidal model. The exact values of the coefficients of the quadratic density function are assumed to be known. The convergence of the method is shown by plotting the values of the objective function λ and the various parameters with respect to iteration number. θ and half-width of the trapezoidal model are found to be correlated. The method is also applied to interpret the gravity anomalies over San Jacinto graben, California. Finally, the use of modelling with quadratic density function is discussed.

Keywords. Marquardt algorithm; quadratic density function; gravity anomalies; trapezoidal model.

1. Introduction

Density data obtained from bore-holes and the results of seismic surveys in sedimentary basins show that the density of the sedimentary rocks increases with depth. Athy (1930) observed that the density of sediments increases exponentially with depth. Cordell (1973) derived the anomaly equation for a prismatic model with exponential density function. The representation of density contrast by an exponential function, however, is not convenient in the analysis of gravity anomalies, as the anomaly equations are not obtained in closed form. The exponential decrease of density contrast over a small interval of depth is approximated by a linear function by Murthy and Rao (1979). They have divided the side of a polygon body into several segments and calculated the contribution of each segment with linear decrease of density contrast. It was earlier shown that the decrease of density contrast in many sedimentary basins could be approximated to a fair degree of accuracy by a quadratic function (Bhaskara Rao 1986).

Modelling with variable density contrast has recently gained importance because of the ready availability of density-depth data from borewells drilled for oil and other investigations. We felt the necessity of deriving the anomaly equations with quadratic density function for various structural models usually associated with sedimentary basins. In this connection, the gravity anomaly over a fault model was recently analyzed with a quadratic density function (Bhaskara Rao 1985). Anticlines and synclines are two important structures associated with sedimentary basins and are usually approximated by two-dimensional isosceles triangular prisms (Rao and Avasthi 1973). However, it is believed that a trapezoidal prism would be a better approximation to an anticline (Bhimasankaram *et al* 1977). Similarly, a better approximation to syncline would be a trapezoidal model. In this paper we have derived the anomaly equation for a

trapezoidal model approximating a syncline structure with quadratic density function and developed the methods of interpretation.

2. Anomaly equation

It was found that the decrease of density contrast with depth in many sedimentary strata could be approximated by a quadratic function (Bhaskara Rao 1986), as

$$\Delta\rho(Z) = a_0 + a_1 Z + a_2 Z^2, \quad (1)$$

where a_0 , a_1 and a_2 are constants of the quadratic function and a_0 represents the extrapolated value of density contrast at the surface.

The equation for gravity anomaly of a trapezoidal model is derived by integrating the gravity effect of an elementary mass (figure 1). Thus,

$$\Delta g(x) = 2\gamma \int_{Z=Z_1}^{Z_2} \int_{u=-T-(Z_1-Z)\cot\theta}^{T+(Z_2-Z)\cot\theta} \frac{\Delta\rho(Z)Z \, dZ \, du}{(u-x)^2 + Z^2}, \quad (2)$$

where Z_1 and Z_2 are the depths to top and bottom, T is the half-width of the lower surface, and θ is the inclination with the horizontal of the sides. Substituting the value of $\Delta\rho(Z)$ in (2), we have

$$\begin{aligned} \Delta g(x) = & 2\gamma a_0 \int_{Z=Z_1}^{Z_2} \int_{u=-T-(Z_1-Z)\cot\theta}^{T+(Z_2-Z)\cot\theta} \frac{Z \, dZ \, du}{(u-x)^2 + Z^2} \\ & + 2\gamma a_1 \int_{Z=Z_1}^{Z_2} \int_{u=-T-(Z_1-Z)\cot\theta}^{T+(Z_2-Z)\cot\theta} \frac{Z^2 \, dZ \, du}{(u-x)^2 + Z^2} \\ & + 2\gamma a_2 \int_{Z=Z_1}^{Z_2} \int_{u=-T-(Z_1-Z)\cot\theta}^{T+(Z_2-Z)\cot\theta} \frac{Z^3 \, dZ \, du}{(u-x)^2 + Z^2}. \end{aligned}$$

Thus, the equation for the gravity anomaly of a trapezoidal model is given by

$$\begin{aligned} \Delta g(x) = & 2\gamma a_0 [Z_2(\phi_4 - \phi_3) - Z_1(\phi_1 - \phi_2) + F_1 \{ \sin \theta \ln (r_4/r_1) \\ & + \cos \theta (\phi_1 - \phi_4) \} - F_2 \{ \sin \theta \ln (r_3/r_2) - \cos \theta (\phi_2 - \phi_3) \}] \\ & + \frac{2\gamma a_1}{2} [Z_2^2(\phi_4 - \phi_3) - Z_1^2(\phi_1 - \phi_2) + F_1 \{ (Z_2 - Z_1) \sin \theta \\ & + F_1 (\sin 2\theta \ln (r_4/r_1) + \cos 2\theta (\phi_1 - \phi_4)) \\ & - F_2 \{ (Z_2 - Z_1) \sin \theta - F_2 (\sin 2\theta \ln (r_3/r_2) - \cos 2\theta (\phi_2 - \phi_3)) \}] \\ & + \frac{2\gamma a_2}{3} \left[Z_2^3(\phi_4 - \phi_3) - Z_1^3(\phi_1 - \phi_2) + F_1 \left\{ \frac{(Z_2^2 - Z_1^2)}{2} \sin \theta \right. \right. \\ & + F_1 (Z_2 - Z_1) \sin 2\theta + F_1^2 (\sin 3\theta \ln (r_4/r_1) + \cos 3\theta (\phi_1 - \phi_4)) \left. \right\} \\ & - F_2 \left\{ \frac{(Z_2^2 - Z_1^2)}{2} \sin \theta - F_2 (Z_2 - Z_1) \sin 2\theta \right. \end{aligned}$$

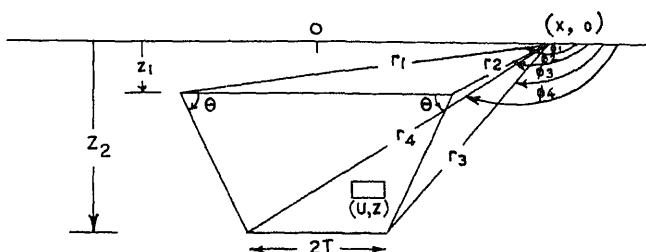


Figure 1. Trapezoidal model.

where

$$F_1 = (x + T) \sin \theta + Z_2 \cos \theta,$$

$$F_2 = (x - T) \sin \theta - Z_2 \cos \theta,$$

$$\phi_1 = \pi/2 + \arctan [x + T + (Z_2 - Z_1) \cot \theta] / Z_1,$$

$$\phi_2 = \pi/2 + \arctan [x - T - (Z_2 - Z_1) \cot \theta] / Z_1,$$

$$\phi_3 = \pi/2 + \arctan (x - T) / Z_2,$$

$$\phi_4 = \pi/2 + \arctan (x + T) / Z_2,$$

$$r_1^2 = (x + T + \overline{Z_2 - Z_1} \cot \theta)^2 + Z_1^2,$$

$$r_2^2 = (x - T - \overline{Z_2 - Z_1} \cot \theta)^2 + Z_1^2,$$

$$r_3^2 = (x - T)^2 + Z_2^2,$$

and $r_4^2 = (x + T)^2 + Z_2^2.$

The various symbols used in (3) are shown in figure 1. The first part in (3) represents the case of constant density contrast.

3. Method of interpretation

Apart from the constants a_0 , a_1 and a_2 , defining the density function with depth the parameters of the trapezoidal model to be interpreted are Z_1 , Z_2 , T and θ . The constants a_0 , a_1 and a_2 are found by the least squares fitting of the density-depth values to a quadratic function. The density depth values can be obtained from the measured density values in bore-holes, or densities deduced by seismic methods or densities deduced otherwise.

The nonlinear optimization technique using the Marquardt algorithm is adopted to find out the unknown parameters, as the anomaly equation is too lengthy to develop any simple method of interpretation. The basis of the method is that an initial model is assumed and the various parameters are successively improved so as to minimize the

where Δg_{obs} and Δg_{cal} are observed and calculated anomalies respectively and N_{obs} is the total number of observations. The method of the Marquardt algorithm has been discussed by many workers (Radhakrishna Murthy and Bhaskara Rao 1982, Coles 1976). The necessary system of simultaneous equations is given by

$$\sum_{k=1}^4 \sum_{i=1}^{N_{\text{obs}}} \frac{\partial \Delta g(i)}{\partial P_j} \frac{\partial \Delta g(i)}{\partial P_k} (1 + \delta_{jk} \lambda) dP_k$$

$$= \sum_{i=1}^{N_{\text{obs}}} [\Delta g_{\text{obs}}(i) - \Delta g_{\text{cal}}(i)] \frac{\partial \Delta g(i)}{\partial P_j} \quad \text{for } j = 1 \text{ to } 4, \quad (5)$$

where $\delta_{jk} = 1$ for $j = k$,
 $= 0$ for $j \neq k$,

and P_k represents one of the parameters Z_1 , Z_2 , T and θ . Using the notation,

$$F_3 = \sin \theta \ln(r_4/r_1) + \cos \theta (\phi_1 - \phi_4), F_4 = \sin \theta \ln(r_3/r_2) - \cos \theta (\phi_2 - \phi_3)$$

$$F_5 = \sin 2\theta \ln(r_4/r_1) + \cos 2\theta (\phi_1 - \phi_4), F_6 = \sin 2\theta \ln(r_3/r_2) - \cos 2\theta (\phi_2 - \phi_3),$$

$$F_7 = \sin 3\theta \ln(r_4/r_1) + \cos 3\theta (\phi_1 - \phi_4), F_8 = \sin 3\theta \ln(r_3/r_2) - \cos 3\theta (\phi_2 - \phi_3),$$

$$F_9 = x + T + Z_2 \cot \theta, F_{10} = x - T - Z_2 \cot \theta,$$

$$F_{11} = (Z_2 - Z_1) \operatorname{cosec}^2 \theta, F_{12} = x + T + (Z_2 - Z_1) \cot \theta,$$

$$F_{13} = x - T - (Z_2 - Z_1) \cot \theta, F_{14} = (x + T) \cos \theta - Z_2 \sin \theta,$$

$$F_{15} = (x - T) \cos \theta + Z_2 \sin \theta,$$

$$G_1 = F_{10}/r_2^2 - F_9/r_1^2, G_3 = -F_9/r_1^2,$$

$$G_4 = F_{10}/r_2^2, G_5 = (F_9 \cot \theta - Z_1 \operatorname{cosec}^2 \theta)/r_1^2,$$

$$G_6 = -(Z_1 \operatorname{cosec}^2 \theta + F_{10} \cot \theta)/r_2^2, G_7 = Z_1 \cot \theta/r_1^2 + Z_1 \cot \theta/r_2^2,$$

$$G_8 = (x - T)/r_3^2, G_9 = Z_1 \cot \theta/r_1^2 + (x + T)/r_4^2,$$

$$G_{10} = (x - T)/r_3^2 - Z_1 \cot \theta/r_2^2, G_{11} = Z_2/r_4^2 - F_{12} \cot \theta/r_1^2,$$

$$G_{12} = Z_2/r_3^2 + F_{13} \cot \theta/r_2^2, G_{13} = Z_1/r_1^2 + Z_1/r_2^2,$$

$$G_{14} = Z_2/r_4^2 + Z_2/r_3^2, G_{15} = Z_1/r_1^2 - Z_2/r_4^2,$$

$$G_{16} = Z_2/r_3^2 - Z_1/r_2^2, G_{17} = (x + T)/r_4^2 - F_{12}/r_1^2,$$

$$G_{18} = F_{13}/r_2^2 - (x - T)/r_3^2, G_{19} = -(Z_1 F_{11}/r_1^2 + Z_1 F_{11}/r_2^2),$$

$$G_{21} = -Z_1 F_{11}/r_1^2, G_{22} = Z_1 F_{11}/r_2^2,$$

$$G_{23} = F_{11} F_{12}/r_1^2, \text{ and } G_{24} = -F_{11} F_{12}/r_2^2$$

the partial derivatives in (5) are given by

$$\begin{aligned} \frac{\partial \Delta g}{\partial Z_1} = & 2\gamma a_0 [-(\phi_1 - \phi_2) - Z_1 G_1 + F_1 (G_5 \sin \theta + G_3 \cos \theta) \\ & - F_2 (G_6 \sin \theta - G_4 \cos \theta)] + \frac{2\gamma a_1}{2} [-2Z_1 (\phi_1 - \phi_2) \\ & - Z_1^2 G_1 + F_1 \{ -\sin \theta + F_1 (G_5 \sin 2\theta + G_3 \cos 2\theta) \} \\ & - F_2 \{ -\sin \theta - F_2 (G_6 \sin 2\theta - G_4 \cos 2\theta) \}] \\ & + \frac{2\gamma a_2}{3} [-3Z_1^2 (\phi_1 - \phi_2) - Z_1^3 G_1 + F_1 \{ -Z_1 \sin \theta - F_1 \sin 2\theta \\ & + F_1^2 (G_5 \sin 3\theta + G_3 \cos 3\theta) \} - F_2 \{ -Z_1 \sin \theta + F_2 \sin 2\theta \\ & + F_2^2 (G_6 \sin 3\theta - G_4 \cos 3\theta) \}] \\ \frac{\partial \Delta g}{\partial Z_2} = & 2\gamma a_0 [(\phi_4 - \phi_3) + Z_2 G_8 - Z_1 G_7 + F_3 \cos \theta \\ & + F_1 (G_{11} \sin \theta + G_9 \cos \theta) + F_4 \cos \theta - F_2 (G_{12} \sin \theta - G_{10} \cos \theta)] \\ & + \frac{2\gamma a_1}{2} [2Z_2 (\phi_4 - \phi_3) + Z_2^2 G_8 - Z_1^2 G_7 + \cos \theta \{ (Z_2 - Z_1) \sin \theta \\ & + F_1 F_5 \} + F_1 \{ \sin \theta + F_5 \cos \theta + F_1 (G_{11} \sin 2\theta + G_9 \cos 2\theta) \} \\ & + \cos \theta \{ (Z_2 - Z_1) \sin \theta - F_2 F_6 \} - F_2 \{ \sin \theta + F_6 \cos \theta \\ & - F_2 (G_{12} \sin 2\theta - G_{10} \cos 2\theta) \}] + \frac{2\gamma a_2}{3} \left[3Z_2^2 (\phi_4 - \phi_3) \right. \\ & + Z_2^3 G_8 - Z_1^3 G_7 + \cos \theta \left\{ \frac{(Z_2^2 - Z_1^2)}{2} \sin \theta + F_1 (Z_2 - Z_1) \sin 2\theta \right. \\ & + F_1^2 F_7 \left. \right\} + F_1 \{ Z_2 \sin \theta + (Z_2 - Z_1) \cos \theta \sin 2\theta \\ & + 2F_1 F_7 \cos \theta + F_1 \sin 2\theta + F_1^2 (G_{11} \sin 3\theta + G_9 \cos 3\theta) \} \\ & + \cos \theta \left\{ \frac{(Z_2^2 - Z_1^2)}{2} \sin \theta - F_2 (Z_2 - Z_1) \sin 2\theta + F_2^2 F_8 \right\} \\ & - F_2 \{ Z_2 \sin \theta + (Z_2 - Z_1) \cos \theta \sin 2\theta - F_2 \sin 2\theta \\ & \left. - 2F_2 F_8 \cos \theta + F_2^2 (G_{12} \sin 3\theta - G_{10} \cos 3\theta) \} \right] \\ \frac{\partial \Delta g}{\partial T} = & 2\gamma a_0 [Z_2 G_{14} - Z_1 G_{13} + F_3 \sin \theta + F_1 (G_{17} \sin \theta + G_{15} \cos \theta) \\ & + F_4 \sin \theta - F_2 (G_{18} \sin \theta - G_{16} \cos \theta)] \\ & + \frac{2\gamma a_1}{2} [G_{14} Z_2^2 - G_{13} Z_1^2 + \sin \theta \{ (Z_2 - Z_1) \sin \theta + F_1 F_5 \} \end{aligned}$$

$$\begin{aligned}
& + F_1 \{ F_5 \sin \theta + F_1 (G_{17} \sin 2\theta + G_{15} \cos 2\theta) \} \\
& + \sin \theta \{ (Z_2 - Z_1) \sin \theta - F_2 F_6 \} - F_2 \{ F_6 \sin \theta - F_2 (G_{18} \sin 2\theta \\
& - G_{16} \cos 2\theta) \}] + \frac{2\gamma a_2}{3} \left[G_{14} Z_2^3 - G_{13} Z_1^3 + \sin \theta \left\{ \frac{(Z_2^2 - Z_1^2)}{2} \sin \theta \right. \right. \\
& + F_1 (Z_2 - Z_1) \sin 2\theta + F_1^2 F_7 \left. \right\} + F_1 \{ (Z_2 - Z_1) \sin \theta \sin 2\theta \\
& + 2F_1 F_7 \sin \theta + F_1^2 (G_{17} \sin 3\theta + G_{15} \cos 3\theta) \} \\
& + \sin \theta \left\{ \frac{(Z_2^2 - Z_1^2)}{2} \sin \theta - F_2 (Z_2 - Z_1) \sin 2\theta + F_2^2 F_8 \right\} \\
& - F_2 \{ (Z_2 - Z_1) \sin \theta \sin 2\theta - 2F_2 F_8 \sin \theta \\
& + F_2^2 (G_{18} \sin 3\theta - G_{16} \cos 3\theta) \} \left. \right], \text{ and}
\end{aligned}$$

$$\begin{aligned}
\frac{\partial \Delta g}{\partial \theta} = & 2\gamma a_0 [-G_{19} Z_1 + F_3 F_{14} + F_1 \{ \cos \theta \ln (r_4/r_1) + G_{23} \sin \theta \\
& - \sin \theta (\phi_1 - \phi_4) + G_{21} \cos \theta \} - F_4 F_{15} - F_2 \{ \cos \theta \ln (r_3/r_2) \\
& + G_{24} \sin \theta + \sin \theta (\phi_2 - \phi_3) - G_{22} \cos \theta \}] \\
& + \frac{2\gamma a_1}{2} [-G_{19} Z_1^2 + F_{14} \{ (Z_2 - Z_1) \sin \theta + F_1 F_5 \} \\
& + F_1 \{ (Z_2 - Z_1) \cos \theta + F_{14} F_5 + F_1 (2 \cos 2\theta \ln (r_4/r_1) \\
& + G_{23} \sin 2\theta - 2 \sin 2\theta (\phi_1 - \phi_4) + G_{21} \cos 2\theta) \} \\
& - F_{15} \{ (Z_2 - Z_1) \sin \theta - F_2 F_6 \} - F_2 \{ (Z_2 - Z_1) \cos \theta \\
& - F_6 F_{15} - F_2 (2 \cos 2\theta \ln (r_3/r_2) + G_{24} \sin 2\theta \\
& + 2 \sin 2\theta (\phi_2 - \phi_3) - G_{22} \cos 2\theta) \}] \\
& + \frac{2\gamma a_2}{3} \left[-G_{19} Z_1^3 + F_{14} \left\{ \frac{(Z_2^2 - Z_1^2)}{2} \sin \theta + F_1 (Z_2 - Z_1) \sin 2\theta \right. \right. \\
& + F_1^2 F_7 \left. \right\} + F_1 \left\{ \frac{(Z_2^2 - Z_1^2)}{2} \cos \theta + F_{14} (Z_2 - Z_1) \sin 2\theta \right. \\
& + 2F_1 (Z_2 - Z_1) \cos 2\theta + 2F_1 F_7 F_{14} + F_1^2 (3 \cos 3\theta \ln (r_4/r_1) \\
& + G_{23} \sin 3\theta - 3 \sin 3\theta (\phi_1 - \phi_4) + G_{21} \cos 3\theta) \left. \right\} \\
& - F_{15} \left\{ \frac{(Z_2^2 - Z_1^2)}{2} \sin \theta - F_2 (Z_2 - Z_1) \sin 2\theta + F_2^2 F_8 \right\} \\
& - F_2 \left\{ \frac{(Z_2^2 - Z_1^2)}{2} \cos \theta - F_{15} (Z_2 - Z_1) \sin 2\theta - 2F_2 (Z_2 - Z_1) \cos 2\theta \right.
\end{aligned}$$

$$+ 3 \sin 3\theta (\phi_2 - \phi_3) - G_{22} \cos 3\theta \} \Big]. \quad (6)$$

To start with, the program requires the initial values of Z_1 , Z_2 , T and θ to be known. In the interpretation of real anomalies, the initial values of various parameters may be taken from seismic or bore hole data. The objective function $F(1)$ as defined by (4) is found for the initial model. λ is initially given a value of 0.5. The derivatives in (5) are calculated using (6). Equations (5) are solved for the increments dZ_1 , dZ_2 , dT and $d\theta$. If Z'_1 , Z'_2 , T' and θ' are the initial values of Z_1 , Z_2 , T and θ , then the improved values Z_1^* , Z_2^* , T^* and θ^* are given by

$$Z_1^* = Z'_1 + dZ_1, Z_2^* = Z'_2 + dZ_2, T^* = T' + dT, \text{ and } \theta^* = \theta' + d\theta.$$

The objective function $F(2)$ is again calculated using these values. If $F(2) \leq F(1)$, then the step is a success and the value of λ is decreased by a factor of 1/2 and the value of $F(2)$ is assigned to $F(1)$. The method is iterated until some convergence-criterion is met. If $F(2) > F(1)$ at any stage, the value of λ is doubled and the normal equations (4) are again framed and solved for the increments dZ_1 , dZ_2 , dT and $d\theta$. These are added to the values of Z_1 , Z_2 , T and θ at the previous successful step and the objective function $F(2)$ is calculated. This procedure is repeated until $F(2) \leq F(1)$. The usual convergence criterion is that the value of the objective function is below a certain tolerance limit or the increments of various parameters are negligible.

4. Examples

A synthetic anomaly of 20 km length (figure 2a) sampled at an interval of 1 km for $Z_1 = 1$ km, $Z_2 = 4$ km, $T = 2$ km, $\theta = 45^\circ$, $a_0 = -0.4$, $a_1 = 0.08$ and $a_2 = -0.005$ is interpreted by this method. To interpret this anomaly, the initial values of the parameters are taken as follows: $Z_1 = 0.5$ km, $Z_2 = 3.5$ km, $T = 1.5$ km and $\theta = 40^\circ$. The exact values of a_0 , a_1 and a_2 are assumed. The initial model and the corresponding gravity anomaly are also shown by dotted line in figure 2a. The convergence of the method is illustrated in figure 2b by plotting the values of the objective function, λ , and the parameters of the model with respect to iteration number. The values of various parameters at 20th iteration are as follows: $Z_1 = 1.00$ km, $Z_2 = 3.99$ km, $T = 2.07$ km and $\theta = 45.72^\circ$.

To study the correlation between various parameters, the gravity anomalies are interpreted for different initial models. Though the interpreted values of Z_1 and Z_2 are nearly equal to the actual values in many cases, the values of T and θ are quite different from their actual values (table 1). This shows a strong correlation between the parameters T and θ . In other words, many values of T and θ could satisfy the interpretation within a given tolerance of error. Thus there is a trade-off between the errors and the interpreted values of the parameters T and θ . In the interpretation of real anomaly, it is therefore necessary to fix one of these parameters to deduce the other accurately.

The method is also applied to interpret the gravity anomalies over graben-type sedimentary basin. Figure 3 shows the interpretation of gravity anomaly profile over

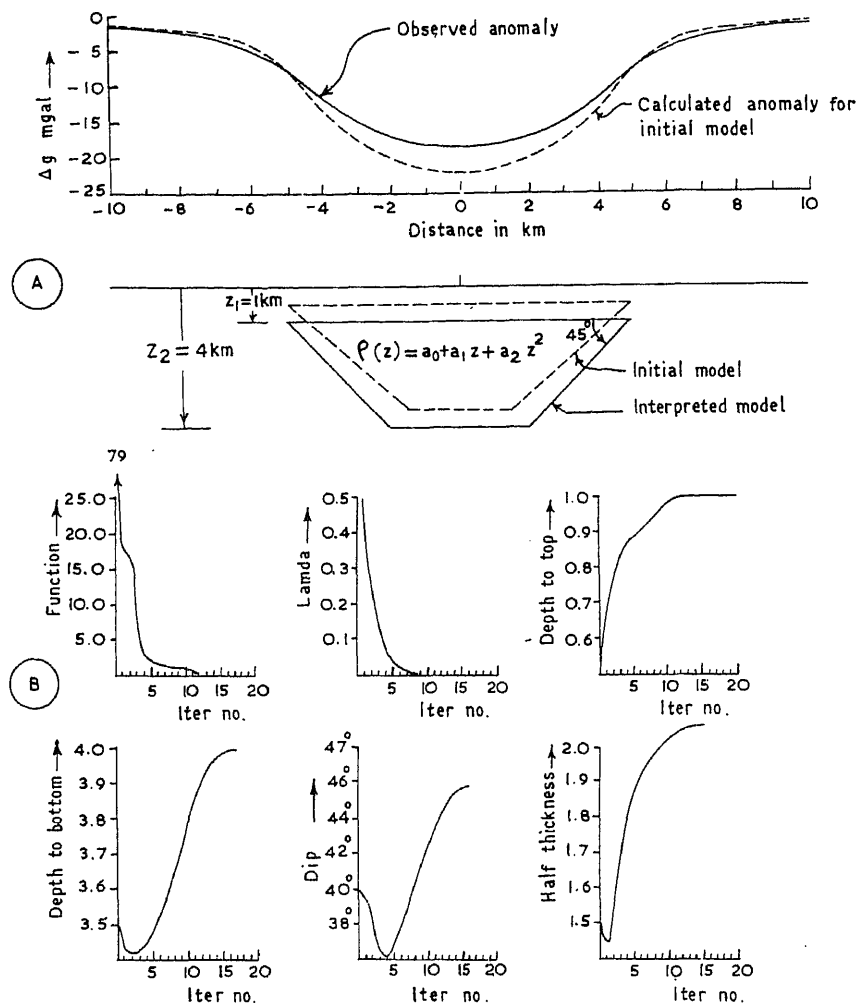


Figure 2. Interpretation of gravity anomalies over a trapezoidal model with quadratic density function.

Table 1. Interpretation of gravity anomalies for different initial models

Model	Function	λ	Z_1	Z_2	θ	T
Initial model	78.94	—	0.5	3.50	40.0	1.5
Interpreted model	0.001	0.0003	1.0	3.99	45.72	2.07
Initial model	431.97	—	0.5	3.50	40.0	2.5
Interpreted model	0.1026	0.01562	0.992	3.784	48.21	2.441
Initial model	47.79	—	0.5	3.0	55.0	2.5
Interpreted model	0.082	0.0009	1.012	3.77	56.35	2.905
Initial model	493.28	—	0.5	2.5	55.0	1.5
Interpreted model	0.0309	0.0002	1.009	3.874	50.63	2.508
Initial model	373.8	—	0.5	3.5	30.0	1.5
Interpreted model	0.1978	0.0312	0.969	3.848	41.75	1.907

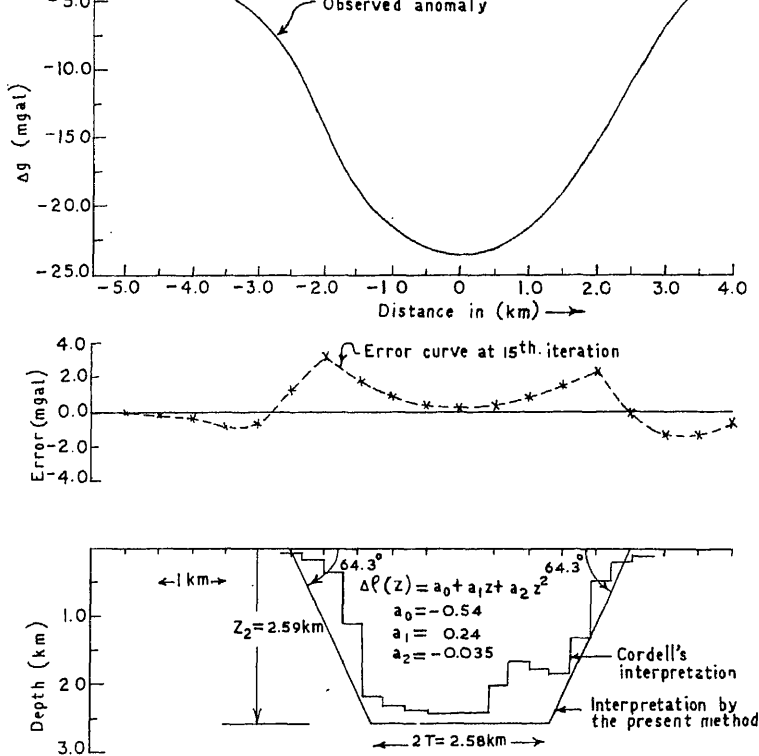


Figure 3. Interpretation of gravity profile over San Jacinto graben structure, California, with quadratic density function (after Cordell 1973).

San Jacinto graben structure, California, with quadratic density function. This profile was originally interpreted by Cordell (1973) with exponential decrease of density contrast with depth. The profile is 9.5 km in length, sampled at 0.5 km interval. The origin of a trapezoidal model is taken at the minimum anomaly point. The constants of the quadratic density function obtained by least squares fitting of density-depth values corresponding to the case of $\lambda = 0.5 \text{ km}^{-1}$ (Cordell 1973) are as follows: $a_0 = -0.54$, $a_1 = 0.24$ and $a_2 = -0.035$. The initial values of an outcropping trapezoidal model are taken to be $Z_2 = 2.5 \text{ km}$, $T = 2.0 \text{ km}$, $\theta = 60^\circ$. In order to use the program directly, Z_1 is fixed and assigned a value of 10^{-5} km to avoid any possible singularities. For these values of the parameters, the value of the objective function is 679.4. The value of λ , which is 0.5 at the beginning, continuously decreased upto 0.0625 at the 4th iteration and again increased to 0.125 at the 5th iteration and remained constant thereafter. The program is terminated at 15th iteration as there are no significant changes in the values of the parameters. The values of various parameters at the end of 15th iteration are as follows: $Z_1 = 10^{-5} \text{ km}$, $Z_2 = 2.59 \text{ km}$, $T = 1.29 \text{ km}$, $\theta = 64.3^\circ$, $\lambda = 0.125$ and $F(1) = 29.45$. The trapezoidal model so obtained and the Cordell's model are drawn in figure 3. These are in close agreement. The residuals at the end of 15th iteration are also shown in figure 3. The residuals remain to be 2 to 3 mgals at $\pm 2 \text{ km}$ from the origin.

These are not quite unexpected in the interpretation of real anomaly, as we are simplifying the interpretation by neglecting local inhomogeneities in density and also the fault plane is approximated by a straight line. Nonetheless, the interpretation is quite stable and acceptable as it is carried out by a model defined by a minimum number of parameters.

5. Conclusions

A synthetic and a real anomaly profile over a trapezoidal model wherein the density contrast decreases with depth is interpreted. The decrease of density contrast is approximated by a quadratic function. The density values known from borewells or by seismic methods might be used to construct a quadratic density function. This is a simple and generalized representation of decrease of density contrast with depth. Thus, the cases of decrease of density contrast linearly and exponentially with depth could easily be accounted by the quadratic function, because the former is a particular case of quadratic function, and the first few terms in the series expansion of the latter constitute a quadratic function. Thus, the choice of quadratic function to represent the decrease of density contrast with depth is justified. It has also a further advantage in that the anomaly equation is derived in closed form for quite different geometrical models.

The anomaly equation could easily be programmed as it contains simple arctan and logarithmic terms. The method converges readily and gives fairly accurate values for Z_1 and Z_2 . However, the parameters T and θ are correlated. Different interpretations could be arrived at for these parameters within a given tolerance of error. It is, therefore, necessary to fix one of these parameters by geological or other considerations to find the other accurately. The method proposed here is useful to interpret gravity anomalies of various structures approximating trapezoidal models, particularly graben-type sedimentary basins, where the densities of the sediments at different depths are reasonably known.

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Origin and depositional environment of the china clay deposits of south Kerala

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Abstract. Results of petrographic, mineralogic and granulometric studies of the sedimentary china clay deposits of Trivandrum district in south Kerala are discussed in terms of their origin and depositional environment. While mineralogic data indicate a khondalitic source for the kaolins as against the hitherto postulated leptynitic source, petrographic evidences suggest that most of the kaolinite were transported from the weathering crust and deposited. Granulometric data and SEM scanning of sand grains from the clays indicate their deposition in a fluvial/low energy littoral environment under conditions of subaqueous agitation.

Keywords. China clay; Warkalli beds; kaolinite; khondalite; leptynite; pelagic suspension; fluvial environment.

1. Introduction

The upper Tertiary sedimentary sequence in Trivandrum district, south Kerala belonging to the Warkalli beds of Mio-Pliocene age contains a column of kaolin clays over the current bedded variegated sandstones, grits, lignite beds etc (Foote 1883; Menon 1966). The kaolin clay beds attain a maximum thickness of about 4.5 m at Thonnakkal in Trivandrum district and the adjoining areas and prove to be of refractory quality raw material (Soman 1984). In some of the clay mines at Thonnakkal (figure 1) more than one bed of kaolin is observed, and the clay bed in most cases is covered by a laterite cover with a maximum thickness of upto 8–10 m (figure 2). Based on the presence of heavy minerals like sillimanite, ilmenite, anatase and graphite in the clays and sandstones at the Karruchal section, south east of Trivandrum and in the sediments of the Warkalli beds, Menon (1966) suggested that the clays were derived from leptynites. However, sillimanite, ilmenite, anatase and graphite are rare in the leptynites compared to their content in the khondalite *sensu stricto*. The purpose of this paper is to present the petrographic, mineralogic and size analysis data on the sedimentary kaolin clays of Trivandrum district and to discuss their origin and depositional environment.

2. Petrography and mineralogy

Investigations of the clay occurrences in south Kerala show that the kaolin incorporates almost entirely the fine grained kaolinite (above 95%) with occasional columnar and sheaf-like aggregates (figure 3a) having $N_p = 1.564$. Aggregates of kaolinite resembling vermicular forms of upto 0.2×0.004 mm size (figure 3b) and oval-shaped kaolinite within fine kaolinite matrix are also seen under the microscope.

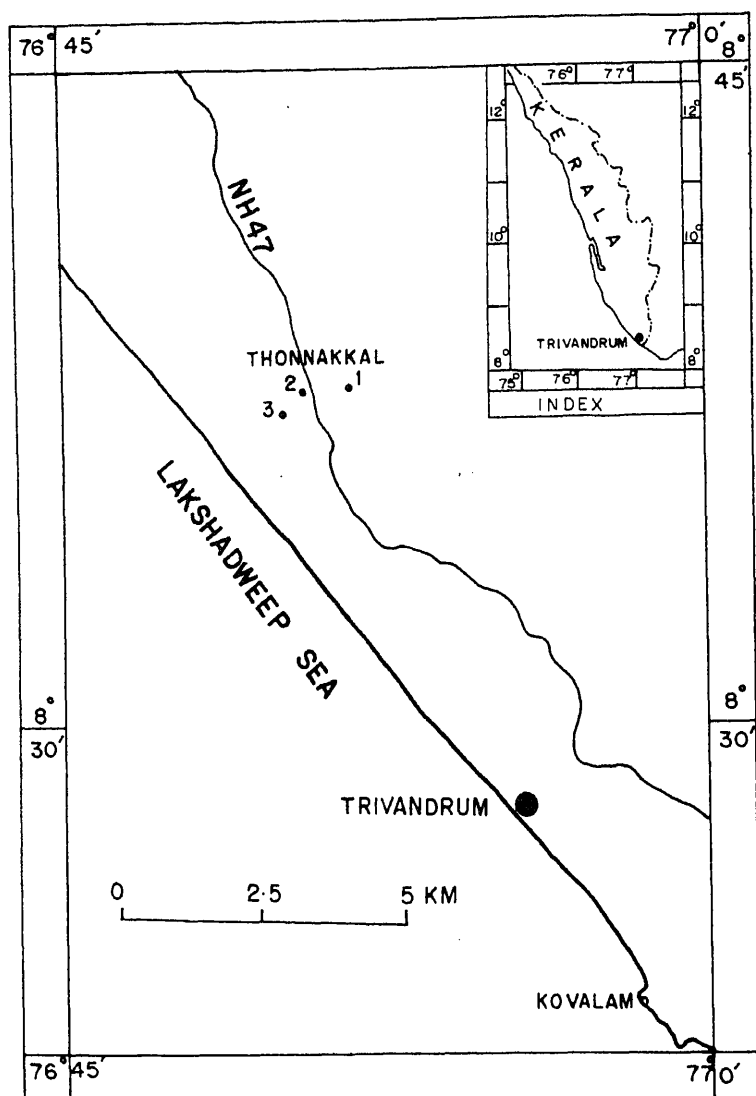


Figure 1. Location map of the china clay mines at Thonnakkal, south Kerala: 1. English-India clay mine, 2. Industrial enterprise clay mine, 3. Standard clay mine.

Preponderance of kaolinite in the clays is substantiated by DTA and x-ray data (figure 4). SEM photomicrographs and x-ray data showing higher first order spacings (eg. $7.225\text{\AA}$) suggest poor crystallinity of the kaolinite (Soman 1984; Grim 1962). Thin and polished section studies show the presence of quartz, gibbsite (pseudomorphous after kaolinite), allophane, goethite, hematite, sillimanite, feldspars and graphite in

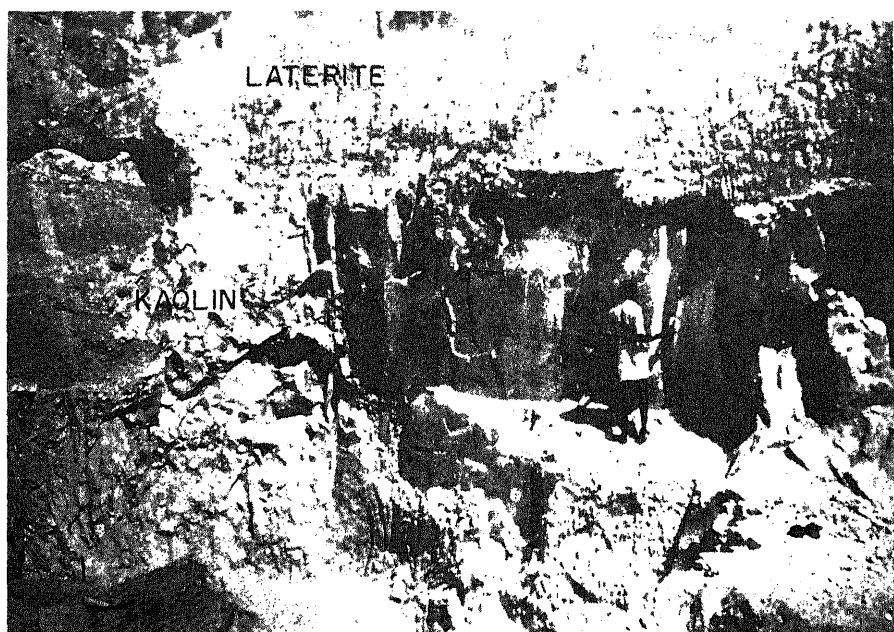


Figure 2. Field photograph showing thick laterite cover over kaolins. English-India Clay mine, Thonnakkal.

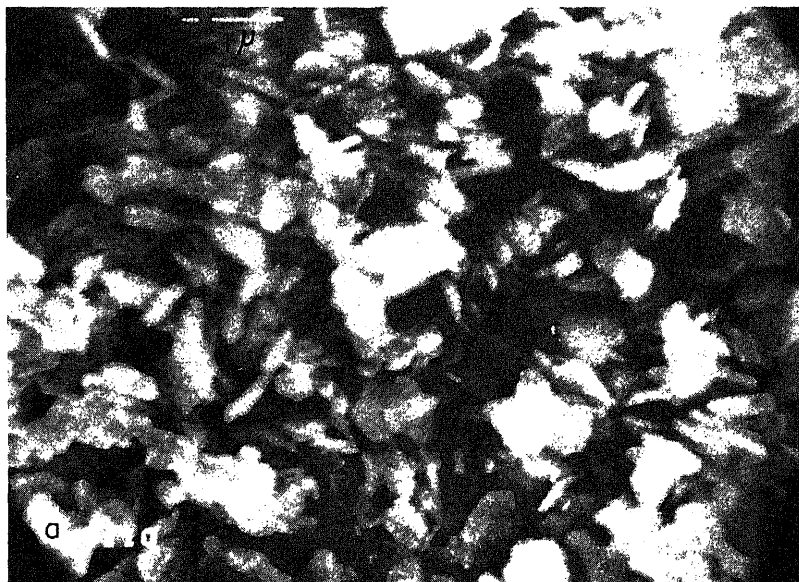


Figure 3. (a).

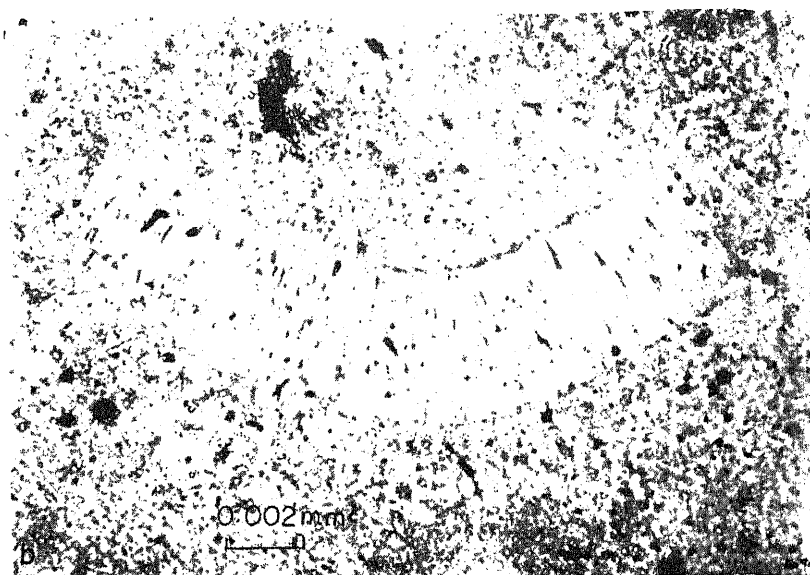
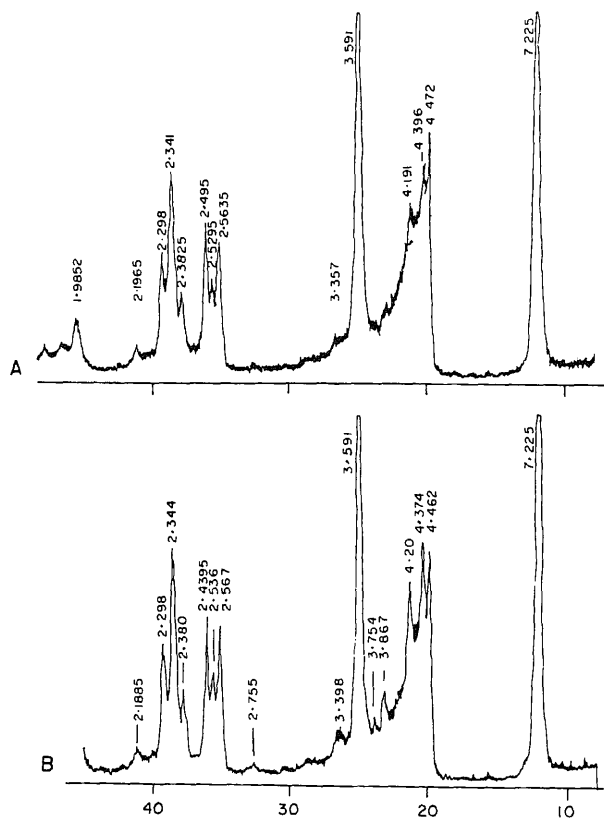


Figure 3. (a) SEM photomicrograph of sheaf-like aggregates of kaolinite. Photograph taken in an ISI-60A SEM with 15 kV accelerating potential and gold-coating of the samples. (b) Photomicrograph of kaolinite showing vermicular forms. Plane polarized light.



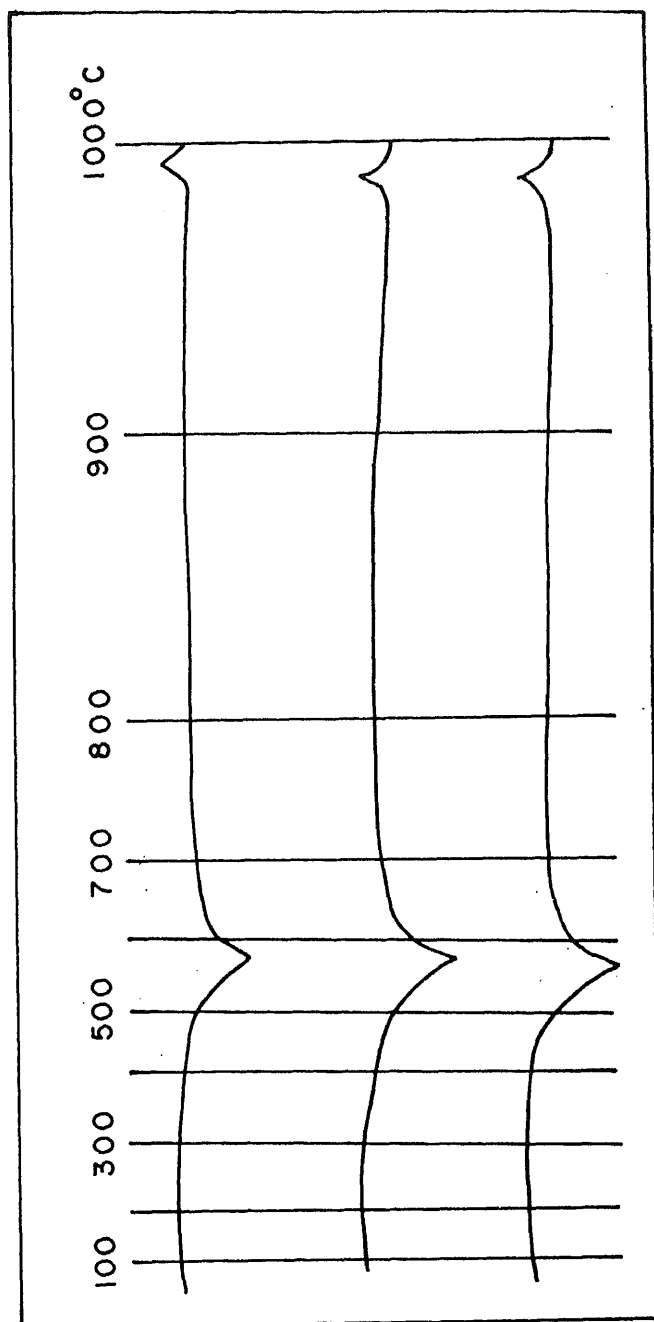
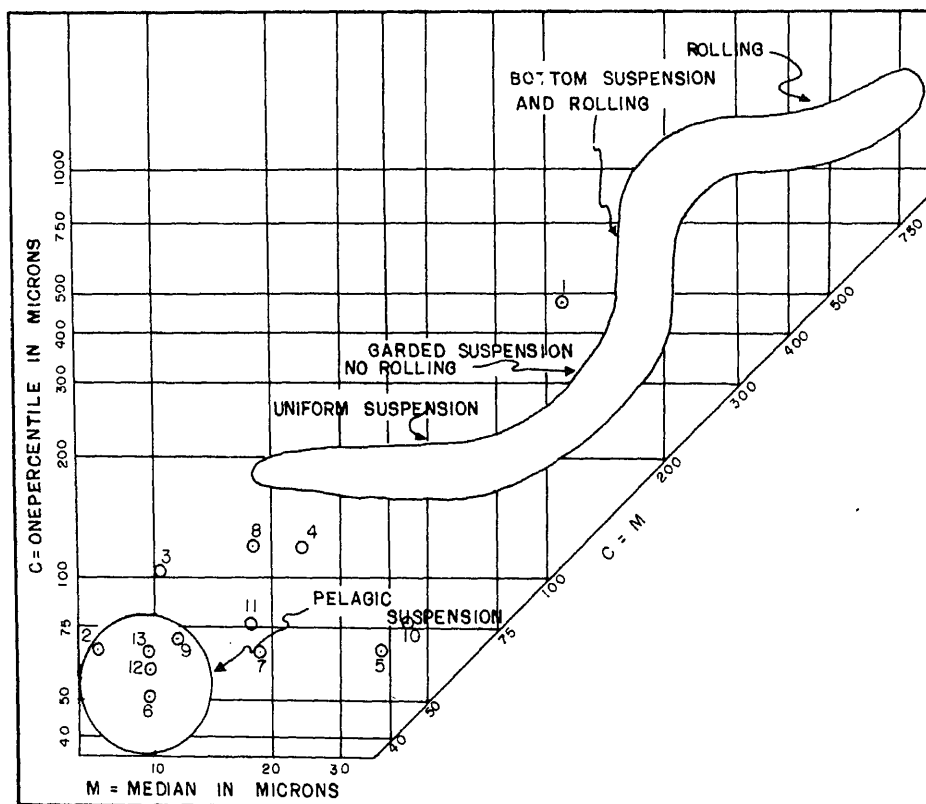


Figure 4b. DTA curves of the kaolin clays.

sedimentary clays compared to its well crystalline nature in the primary clays from the present-day weathering crust (Soman and Slukin 1985) also support the view that kaolinite was transported.

3. Size parameters

Size analysis of 13 clay samples from three vertical profiles belonging to three clay mines, located laterally apart, from Thonnakkal area was carried out according to the method recommended by Carver (1971). Plots of first and fiftieth percentile values of the size analysis in a C-M diagram (Passega 1957, 1977; Vandenberghe 1975) mostly fall around the bottom left corner of pelagic suspension (figure 5). This would indicate deposition in still waters with or without gentle agitation. Passega (1977) showed that such a situation can result in shallow waters also when a turbidity current becomes gradually dissipated and left only with clay particles. SEM examination of sand grains from a clay sample shows the presence of V shaped impact pits in them (figure 6) which implies a subaqueous agitation in the sedimentation environments (Margolis and Kennet 1971). The density and abundance of the presently observed V-shaped impact



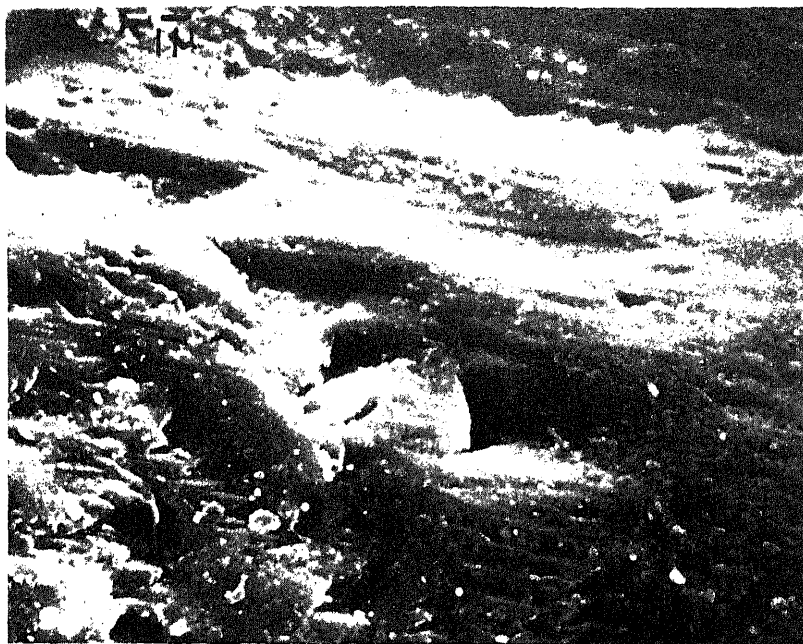


Figure 6. SEM photomicrograph of sand grain from kaolins showing V-shaped impact pits.

pits correspond to typical fluvial/low energy littoral environment (Carter 1984). The presence of current-bedded clastic sediments above and below the clay beds in south Kerala also indicates the existence of shallow water environment with high concentration of suspended sediments during Warkalli sedimentation cycle.

4. Conclusion

The available data suggest that the source material of the kaolin deposits of south Kerala originated from the weathering crust on the khondalite-migmatite complex during Mio-Pliocene times and these were deposited in a fluvial/low energy littoral environment under conditions of subaqueous agitation. This is compatible with the suggestion of Raha *et al* (1983) that the Warkalli group of sediments could possibly have developed as different facies components in a coastal environment. The presence of oval-shaped kaolinite aggregates and poor crystallinity of the kaolinite further suggest that kaolinite itself was transported, while the presence of graphite indicate that the transportation distance of the material could not have been long.

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Some theoretical aspects of electric field and precipitation growth in a finite thundercloud

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Abstract. A time-dependent numerical model allowing a simulation of the electric field and precipitation growth in a thundercloud of finite dimensions is described. It is found that slower growth rate (compared to an infinite thundercloud) of the electric field in a finite thundercloud permits larger size growth and higher terminal velocities of hydrometeors owing to an enhancement in precipitation intensity. Calculations also show that a higher maximum of the electric field is needed to slow down the larger particles produced in a thundercloud of finite dimensions. In particular, these solutions also include contribution of screening charge transport in thundercloud electrification.

Keywords. Electric-field; hydrometeors; precipitation growth.

1. Introduction

Many mechanisms have been proposed to explain the electrification of thunderclouds and the generation of thunderstorm electricity. (Mason 1972; Paluch and Sartor 1973; Ziv and Levin 1974; Chiu and Klett 1976; Chiu 1978; Mathpal *et al* 1982; Singh *et al* 1985). Generally, in earlier theoretical treatments the cloud has been visualized as an infinite parallel plate capacitor. The aspect of the finite dimensions of a thundercloud has not been considered. Only a few investigators (Illingworth and Latham 1975, 77; Beesley *et al* 1977) have studied the electric field growth in a finite thundercloud and have showed that the assumption of a cloud as an infinite parallel plate seriously overestimates the growth rate of the electric field in thunderclouds.

Ziv and Levin (1974) and Levin *et al* (1977) demonstrated a direct coupling between the precipitation and electrical development in thunderclouds. The growth of hydrometeors and the charge separation in thunderclouds have been reported as related processes (Holmes *et al* 1980; Toshikatsu 1980; Singh *et al* 1985). Experimental evidence for the accretion of supercooled water-droplets as hail pellets in the presence of electric field has also been shown by Aufdermaur and Johnson (1972). In other words, micro-physical interactions contributing to the main development of the electric field are associated with the strength of the electric field. This indicates that a greatly exaggerated growth rate of the electric field in a thundercloud will also influence the precipitation intensity and the charge acquired by hydrometeors; this is because the growth of hydrometeors and the charge transferred to them depend upon the strength of the ambient electric field. Such calculations, showing the development of precipitation intensity and electric field in a finite thundercloud, are not available in the literature. It is, therefore, imperative to study the electric field build-up and the precipitation development processes in a thundercloud of finite dimensions. The effects

associated with hydrometeors falling through a vertical electric field have been considered precisely. Results based on the present calculations show a prodigious effect of the finite dimensions of the thundercloud on their precipitation and electrical activity. Such considerations also explain the reality of microphysics and electrical interactions in a thundercloud.

2. Mathematical approach

We consider a cloud of cylindrical shape containing precipitation particles and supercooled water droplets (Illingworth and Latham 1977; Mathpal and Varshneya 1983; Singh *et al* 1984). Figure 1 shows the dimensions of the thundercloud for the present analysis. It has been assumed that a cloud accumulates positive charge at the top and negative charge at the bottom by inductive charging mechanism, while in the intervening zone charge varies linearly with height (Mathpal and Varshneya 1983; Singh *et al* 1984). The growth rate of an electric field in a finite thundercloud (figure 1) can be written as (Singh *et al* 1984)

$$\frac{dE}{dt} = \frac{(1-C)}{\epsilon_0 \pi D^2} \cdot \frac{dQ}{dt}, \quad (1)$$

where

$$C = \left[\frac{[(L+H)^2 + D^2]^{1/2} - (L^2 + D^2)^{1/2}}{(L^2 + D^2)^{1/2} - L} - \frac{D^2}{4L} \ln \left[\frac{(L^2 + D^2)^{1/2}}{(L^2 + D^2)^{1/2} - L} \right] \right] / [H + L/2]. \quad (2)$$

In the present study the particles are divided into five ($N = 5$) size intervals i.e. R_1, R_2, R_3, R_4 and R_5 (Ziv and Levin 1974; Singh *et al* 1985). The first interval R_1 denotes cloud particles while the remaining intervals represent precipitation particles. The size of the cloud particles remains constant and precipitation particles grow by collection of cloud particles. For the detailed description of (i) distribution of

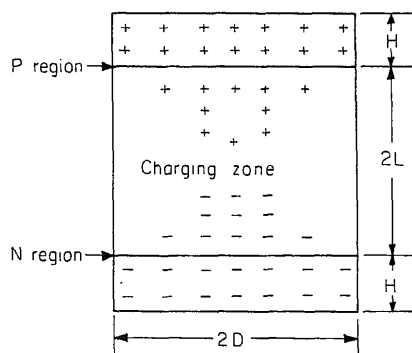


Figure 1. Cloud size and charge distribution within it [D, L and H represent the radius, length of charging zone on each side from the centre of the cloud and the height of lower

precipitation liquid water content, W_p , and that of cloud water content, W_c , (ii) estimation of number density of precipitation and cloud particles, (iii) terminal velocities of precipitation and cloud particles, the reader should refer to Ziv and Levin (1974) and Singh *et al* (1985).

Winn *et al* (1978) reported the presence of screening charge layers at cloud boundaries, negatively charged downdraft and strong association between field structures and air motions. Vonnegut (1955), Moore (1977), Mathpal *et al* (1982) and Singh *et al* (1984) have also suggested that convective transport of negative screening charge into the lower region of a thundercloud through its outer periphery contributes very much to the electrification of thunderclouds. Recently Singh *et al* (1985) have shown the contribution of screening charge transport in the microphysical processes of thunderclouds. But in this study also the finite dimensions of thunderclouds have not been taken into account. Adopting the contribution of negative charge transport and allowing interactions among all N categories of particles, the rate of total negative charge accumulation in a finite thundercloud of radius D may be given by

$$\frac{dQ}{dt} = \pi D^2 \left[- \left[\sum_{i=2}^{N \geq 2} n_i Q_i (V_i - V_1) + J_d(t) \right] + J_s(t) \right], \quad (3)$$

where n_i —number density, Q_i —charge, V_i —fall velocity, of particles in the interval i and V_1 —fall velocity of particles in interval 1 i.e. cloud particles. $J_d(t)$ and $J_s(t)$ are discharge and screening charge current densities at instant t , respectively. Discharge current density can be expressed as (Levin and Ziv 1974; Singh *et al* 1985)

$$J_d(t) = (10^{-8}/3) [\exp(6.7 \times 10^{-6} E(t)) - \exp(6.7 \times 10^{-6} E_0)], \quad (4)$$

where E_0 and E denote the initial value of the electric field (fair weather electric field) and the ambient electric field in a thundercloud respectively. The screening charge current density which depends upon downdraft velocity, U , may be determined from (Mathpal *et al* 1982; Mathpal and Varshneya 1983; Singh *et al* 1984, 85).

$$J_s(t) = [(1.218 f_1 f_2 \varepsilon_0 E(t) W_c) / r_c \rho_c] [U + V_E] \quad (5)$$

Here, f_1, f_2, r_c, ρ_c are fractional constant of electric field, fractional constant of saturation space charge on cloud particles, radius of the cloud particles at boundary and density of those cloud particles, respectively.

Finally, substituting (3) into (1), the growth rate of the electric field in a finite thundercloud may be represented as

$$\frac{dE}{dt} = \frac{(1-C)}{\varepsilon_0} \left[- \left[\sum_{i=2}^{N \geq 2} n_i Q_i (V_i - V_1) + J_d(t) \right] + J_s(t) \right]. \quad (6)$$

The equations describing the variations of the particles sizes and charges are given by (Ziv and Levin 1974; Singh *et al* 1985):

$$\frac{dR_j}{dt} = \frac{\pi}{3\rho_p} \left(\frac{R_j + R_1}{R_j} \right)^2 (V_j - V_1) n_1 \rho_c R_1^3 C_1 C_2, \quad (7)$$

$$\frac{dQ_j}{dt} = \pi \left[\frac{1}{2} \sum_{k=j+1}^{N \geq j+1} (R_j + R_k)^2 (V_k - V_j) n_k C_1 C_3 (1 - C_2) \Delta Q_{jk} - \sum_{i=1}^{j-1 \geq 1} (R_i + R_j)^2 (V_j - V_i) n_i C_1 C_3 (1 - C_2) \Delta Q_{ij} \right], \quad (8)$$

and

$$\Delta Q_{ij} = [\gamma E(t) R_i^2 \cos \theta + (\omega - 1) Q_j + \omega Q_i] [1 - e^{-t} c/\tau], \quad (9)$$

where

$$\omega = A_2/(A_2 - A_1); \gamma = A_3/(A_2 - A_1). \quad (10)$$

The subscripts, i, j, k denote the size of different particles where $i < j < k$. Other parameters used in these equations are described in the list of symbols at the end.

However, the effective precipitation rate is represented as

$$p = \sum_{i=2}^N n_i M_i V_i$$

or

$$p = \sum_{i=2}^N W_i V_i \quad (11)$$

3. Results and discussion

Equations (6), (7), (8) and (9) were solved simultaneously by the Runge-Kutta method. In all computations that follow, values of involved parameters, i.e. $N = 5$, $C_1 = .50$, $\theta = 45^\circ$, $C_3 = .50$, $t_c/\tau = \infty$, $\rho_p = .6 \times 10^3 \text{ kg m}^{-3}$, $\rho_c = 1 \times 10^3 \text{ kg m}^{-3}$, $E_0 = 500 \text{ V m}^{-1}$, $R_1 = r_c = 10 \text{ } \mu\text{m}$, $R_4 = 200 \text{ } \mu\text{m}$, $L = 1.5 \text{ km}$, $H = .50 \text{ km}$, have been considered (Ziv and Levin 1974; Mathpal and Varshneya 1983; Singh *et al* 1984, 85). The coalescence efficiencies C_2 have been determined from Whelpdale and List (1971). The terminal velocities of hydrometeors are computed from Ziv and Levin (1974).

The effect of cloud geometry on size growth, terminal velocity and charge of hydrometeors of the fourth category has been illustrated in figures 2a–c respectively. The behaviour of the electric field build up in an infinite and finite thundercloud has been shown in figure 2d. This figure illustrates that variation in cloud width from infinity to a realistic value, reduces field growth appreciably. For example, in case of an infinite thundercloud, we obtain an electric field, $E = 355 \text{ kV m}^{-1}$ in 1000 sec which reduces to only 156 kV m^{-1} within the same time interval for a thundercloud having radius 1.25 km . It has been noted that reduced field growth rate in a finite thundercloud allows higher growth in size and terminal velocity of hydrometeors (figures 2a, b). Moreover, because the charge transferred to the particles is proportional to the strength of the electric field, hydrometeors charging rate also reduces in a finite thundercloud in comparison with an infinite thundercloud. In brief one can say that consideration of the finite dimensions of a thundercloud affects all microphysical interactions.

Figure 2d also exhibits the effect of thundercloud width on maximum field strength reached in a thundercloud. We obtain maximum values of electric fields, $E_{\max} \approx 385 \text{ kV m}^{-1}$ in 800 sec, $E_{\max} \approx 417 \text{ kV m}^{-1}$ in 1080 sec and $E_{\max} \approx 437 \text{ kV m}^{-1}$ in 1220 sec for cloud with $D = \infty$, 2.0 and 1.25 km respectively. This shows that reduction in cloud width considerably decreases electric field growth rate but finally a

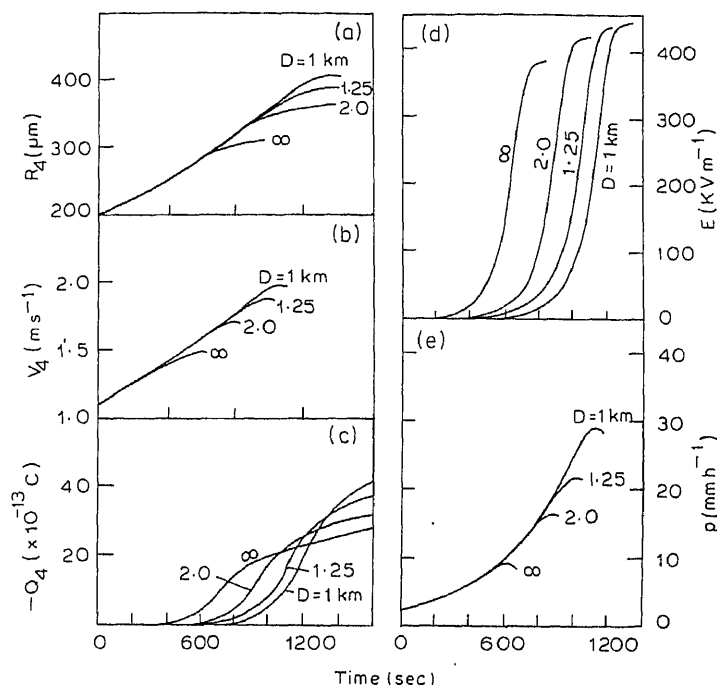


Figure 2. Effect of cloud width D , on (a) size growth, (b) terminal velocity, (c) charging rate, (d) electric field, and (e) precipitation rate ($W_p = W_c = 0.5 \text{ gmm}^{-3}$).

slowed down. An increase in size of hydrometeors leads to the possibility of relatively stronger electrical forces for slowing them down. In other words, rapid charge separation constitutes rapid field development which soon suspends the particles. This early levitation of charged particles causes a smaller maximum value of the electric field which occurs in an infinite thundercloud. In fact, in a narrow thundercloud the breakdown conditions are reached late in the cloud life time. The present results also explain the possibility of the occurrence of lightning from small thunderclouds as observed by Rawlins (1982).

Figure 2e shows the precipitation development in infinite and finite thunderclouds. From the present calculations, the maximum precipitation intensities $p_{\text{max}} = 9.2, 16.30$, and 22 mm h^{-1} are obtained for $D = \infty, 2.0$, and 1.25 km respectively. The larger particles with higher terminal velocities associated with slower field growth rate in finite thunderclouds enhance precipitation intensity significantly. Comparison of precipitation intensity in infinite and finite thunderclouds indicates that the assumption of a thundercloud of infinite dimensions very much underestimates precipitation intensity.

Finally, compiling our results we conclude that introduction of the finite geometry of a thundercloud decreases the electric field growth rate very severely but increases the precipitation development and the maximum value of the electric field. These calculations can also be used to assess the effect of electrical forces on the electric field and precipitation growth in thunderclouds of finite dimensions. The calculated values of maximum electric field and precipitation intensity in finite thunderclouds are consistent with available observations (Winn and Moore 1971; Winn *et al* 1974; Illingworth and Latham (1977); Christian *et al* 1980).

List of symbols

A_1, A_2, A_3	charge transfer coefficients given by Davis (1964),
C_1, C_2, C_3	collision, coalescence and separation efficiencies, respectively,
Q_{ij}	charge transferred by interaction of particles i and j .
M_i	mass of the particle in interval i
R_j	radius of particles in interval j ,
t_c	contact time of two interacting particles,
τ	relaxation time of charge carriers on the interacting particles,
θ	angle between the line of centres of colliding particles and the ambient electric field,

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Global features of sea-level pressure distribution in April and July associated with contrasting situations of Indian summer monsoon

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Abstract. Global sea-level pressure distribution has been analysed for the months of April and July for 5 years of contrasting situations of Indian summer monsoon, comprising of two drought years (1972 and 1974), a flood year (1975) and two normal monsoon years (1970 and 1973). Mean monthly sea-level pressure data at about 400 stations have been used in the study. Prominent features of pressure departures from long-term normals have also been noted. It is observed that the month of April shows more prominent contrasting features than July. In April, the high pressure centres over USSR and the North Pacific move considerably eastward during poor monsoon years, while a breakaway cell of Icelandic Low goes deep south. Both the high pressure areas over south Indian Ocean and Australia are stronger in good monsoon years. In July, the subtropical high pressure zone over the southern Indian Ocean is stronger and the Australian high is more eastward, in good monsoon years.

Keywords. Southwest monsoon; global sea-level pressure; drought; pressure departures.

1. Introduction

Indian economy is very closely linked to the performance of the southwest monsoon, which contributes about 78 % of the total rainfall over the country, during the four months from June to September. Though there are large spatial variations in the monsoon rainfall, the total rainfall received by the country appears to be fairly consistent, at least in the statistical sense, with a mean of 85.3 cm and coefficient of variation of 9.5 % (Mooley and Parthasarathy 1984). However, it needs no emphasis that even these apparently small year-to-year variations have a phenomenal effect on agricultural and human activities in the country, so much so that a departure beyond $\pm 12\%$ of the normal is associated with widespread flood/drought conditions. The date of onset and the duration of the active monsoon also play an important role in rainfall variations.

It is well known that the Asian summer monsoon is produced primarily by the seasonal shifting of thermally-produced planetary belts of pressure and winds under continental influences. The migration of the equatorial trough leads to a pressure gradient from the equator toward the Indian subcontinent, which has considerable influence on the transport of moisture and clouding toward the country. Thus, it is obvious that any mechanism which affects the surface pressure pattern in the region has a direct bearing upon the Indian monsoon rainfall. In this context, it would be useful to examine the nature of global surface pressure distribution associated with anomalous rainfall conditions over India, to identify certain large-scale patterns that may be significantly contributing to the rainfall variations.

Some studies have been made to understand the relation between the mean monthly

global and hemispheric surface pressure distribution and the Indian monsoon rainfall. These studies essentially reduce the spatial distribution of the surface pressure into several principal components and then attempt to establish a relation with the rainfall. Paolino and Shukla (1981), who used Northern Hemispheric (NH) sea-level pressure data for 53 years (1925–77) did not find any evidence of a statistically significant relation between the sea level pressure anywhere over the NH during any month or season of the year and the monsoon rainfall over India. However, Chang and Yasunari (1982), who used global surface pressure data of July for 106 years (1871–1976), reported that the 2nd component explained 19% of the total variance, reflecting a pressure seesaw between the subtropical oceans and the Afro-Eurasian continents, which seemed to be largely responsible for the strength of the Asian monsoon. They also reported that the 3rd component accounted for 14% of the total variance, representing a north-south (or east-west) shift of the main centres of action, and was closely connected with the monsoon circulation. Parthasarathy (1984), using the data during the 106-year period 1871–1976, found a significant relationship between All-India monsoon rainfall and the 2nd and 3rd principal components of NH sea-level pressure field in July, but not in January or April. He also analysed the relationship between 4 sea-level pressure centres of action and the Indian rainfall during the 80 years 1899–1978, and concluded that the Pacific high pressure during the preceding March–April–May was positively correlated with the monsoon rainfall, and that the Aleutian low pressure during the previous December–January–February was negatively correlated. He further reported that the mean pressure departures in these pressure centres of action during drought or flood years were generally less than 1 mb.

In the present study, the mean monthly global sea-level pressure distribution has been analysed for April and July, and an attempt is made to present a synoptic view of the contrasting features associated with poor and heavy monsoon situations over India.

2. Data and analysis

The years 1972 and 1974 are known to have witnessed widespread drought conditions all over the country, ranking as the 4th and 13th most severe drought years in the last hundred years, while the year 1975 was a heavy monsoon year, ranking 9th among large-scale excess years in the same period (Mooley and Parthasarathy 1984). The years 1970 and 1973 were normal monsoon years, with slightly excess rainfall. Seasonal

Table 1. All-India summer monsoon rainfall (June to September) and its departure from normal (after Mooley and Parthasarathy 1984).

Year	All-India summer monsoon rainfall (mm)	Departure from normal (%)
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rainfall values and the percentage departures from normal for these five years are given in table 1 (after Mooley and Parthasarathy 1984). To examine the global pressure distribution features in these five typical years, the authors have collected mean monthly sea-level pressure data at about 400 stations well-spread over the world (figure 1), for April and July from the Monthly Climatic Data for the World (NOAA/WMO). The features during April hold some predictive value for the following monsoon season while July coincides with the active period of the monsoon, giving some insight into the circulation mechanism. Detailed global charts of pressure distribution have been prepared for these two months of all the 5 years mentioned above. South of 60°S, isobars could not be drawn as the data availability is meagre in that area. The authors have also computed the pressure departures from the long-term normals given by WMO (1971) and demarcated areas of positive and negative pressure departures on all the charts, by shading the areas under negative pressure anomalies. These charts are then critically examined to identify consistent contrasting features associated with heavy/poor monsoon situations over India.

3. Results

The following are some of the salient features noticed on the global sea-level pressure charts.

3.1 April

In this month, significant contrast could not be seen in respect of the Arctic High and the Aleutian Low, except that the former was fairly strong (1028 mb) in the two years 1975 (flood) and 1973 (normal). The seasonal low pressure area over the Indo-African

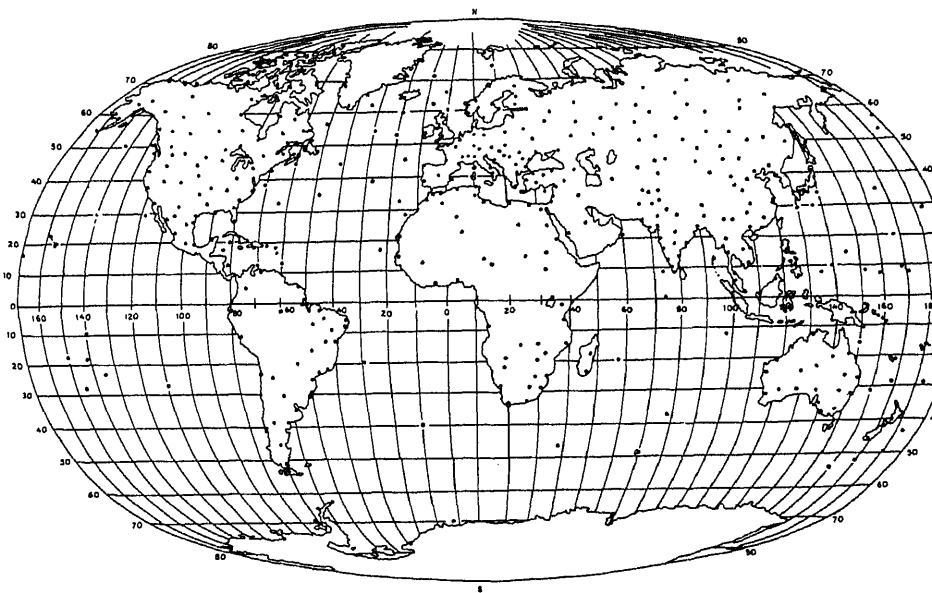


Figure 1. Locations of the stations for which data have been used.

region also does not show marked differences between poor and good monsoon years. Other features that have shown some contrast are detailed below for each year.

(a) 1972: The Icelandic Low is well-established over North Atlantic (1007 mb), extending from the north-east Canadian Coast to Sweden (figure 2). Subtropical high pressure centres are over the eastern sides of north central Pacific (1021 mb) and north central Atlantic (1022 mb) oceans, and over North Korea and Japan (1023 mb) with east-west extensions into the Pacific High and central USSR. The Southern Hemisphere exhibits its characteristic smooth pressure pattern with high pressure areas off the south American Coast (1023 mb) and over South Australia (1021 mb), the latter having a slight zonal extension into the adjoining Indian Ocean.

It has been observed that pressure departures, negative as well as positive, are relatively weak all over the globe (within $\pm 2-3$ mb). However, it can be seen that there are broad areas of negative departures over the western parts of North America and the adjoining Pacific, Europe and central USSR, north of Japan, Africa and South Indian Ocean and adjoining areas, and the eastern portions of South America.

(b) 1974: The Icelandic Low (1007 mb) extends southwestward along the northern portions of the North American Coast (figure 3). Another zonally extended subpolar low (1006 mb) is seen over the northern parts of USSR. Besides the subtropical high pressure areas over the North Pacific (1026 mb) and the North Atlantic (1020 mb), smaller high pressure cells are present over the British Isles (1024 mb) and central USSR (1017 mb). The subtropical high pressure zones in the Southern Hemisphere are zonally extended over the South Pacific (1020 mb) and the South Indian Ocean (1020 mb).

Very strong positive pressure departures are seen over the British Isles, south of New Zealand, Peru coast, far north-east of USSR and in the Indian Ocean south of Masquarine Island. Strong negative anomalies are seen over north central USSR, west

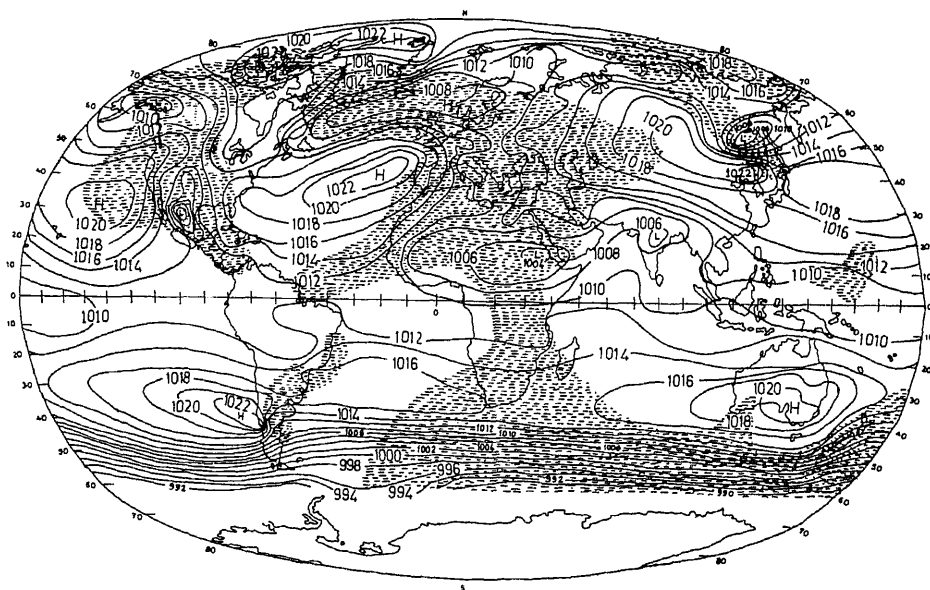


Figure 2. Mean sea-level pressure (mb) for April 1972.

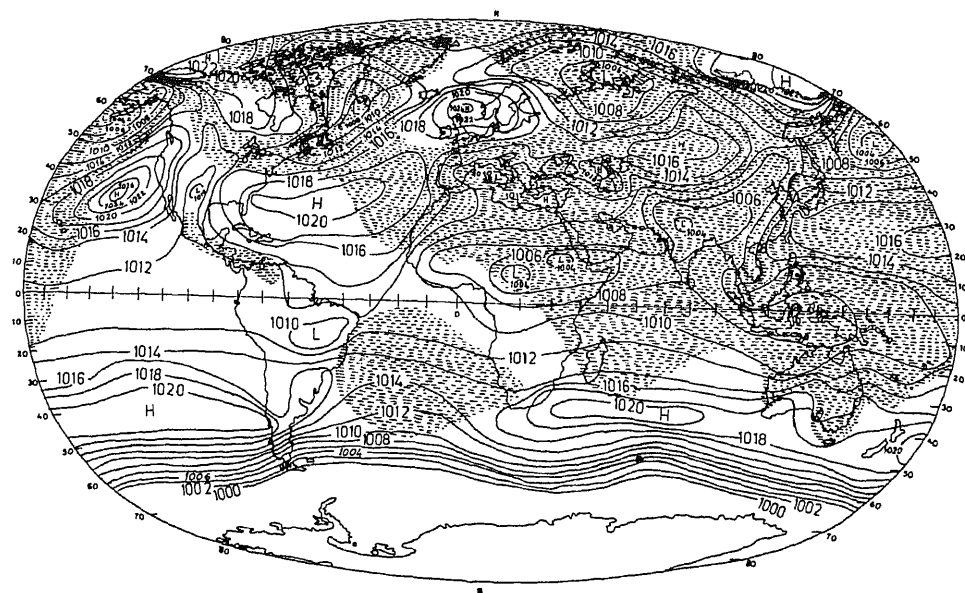


Figure 3. Mean sea-level pressure (mb) for April 1974.

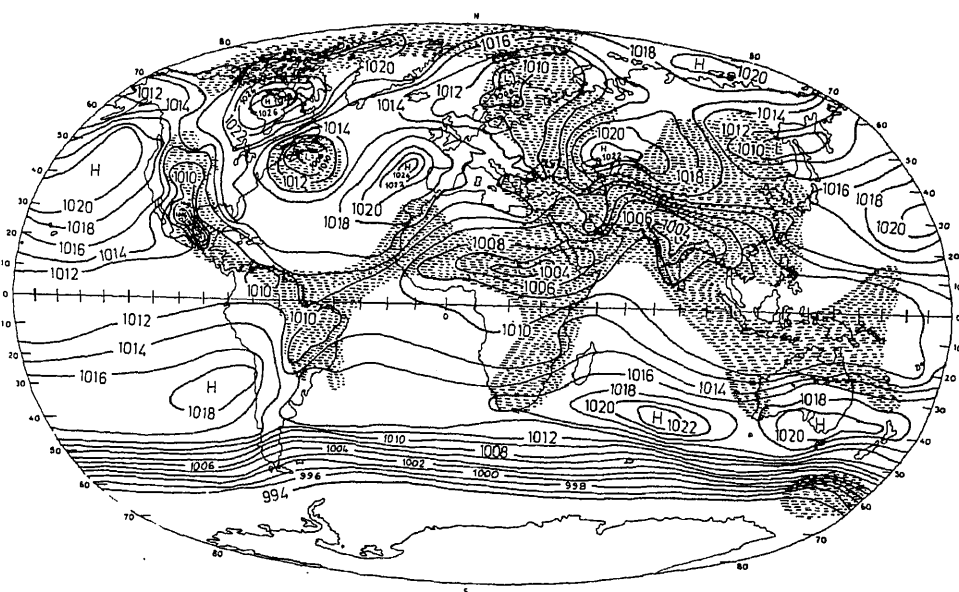


Figure 4. Mean sea-level pressure (mb) for April 1975.

of Greenland, Australia, west Pacific, north Africa and adjoining parts of Eurasia and equatorial Indian Ocean.

(c) 1975: The Icelandic Low is broken into two cells, one north of Sweden (1008 mb) over Finland and the other over the North Atlantic.

westward over USSR. The subtropical high over the South Pacific is slightly towards north (25°S). The high pressure area over south Indian Ocean extends zonally over almost the whole of the Eastern Hemisphere between 30° and 40°N , encompassing South Australia and New Zealand and is fairly strong (1020 and 1022 mb).

A very interesting feature shown by the pressure departures in this year is the positive anomalies over the ocean regions except in the Icelandic Low and a few other localized areas. Very strong positive anomalies occur over Northeast Canada (+8 mb) and the southern tip of South America (+13 mb). South Asia and its neighbouring continental areas are conspicuous by their negative pressure departures.

(d) 1970: The Icelandic Low is broken into two cells (figure 5), one over north central Atlantic (1008 mb) and the other over Sweden (1006 mb). Well-established subtropical high pressure areas are present over the North Pacific (1022 mb) and Central USSR (1024 mb). The high over the North Atlantic is pushed eastward (1024 mb). The subtropical high in South Pacific is fairly strong (1020 mb). The South Indian Ocean and South Australia are covered by zonally extended high pressure cells (1018–1020 mb).

A relatively larger area over the globe is covered by positive pressure departures. This is particularly significant over the Southern Hemisphere where negative anomalies are confined to South Australia and the adjoining region. In the Northern Hemisphere, Canada, eastern USA and the adjoining North Atlantic, Central Africa and India, Europe and northern USSR show negative pressure departures.

(e) 1973: Icelandic Low comprises of two cells (figure 6), over North Central Atlantic (1002 mb) and Sweden (1006 mb). The subtropical high in the North Pacific is well-defined (1022 mb) while that over the North Atlantic occurs as two separate cells, one considerably to the north near the UK (1026 mb) and the other near the southeast coast of USA (1017 mb). The subtropical continental high over central USSR is also fairly

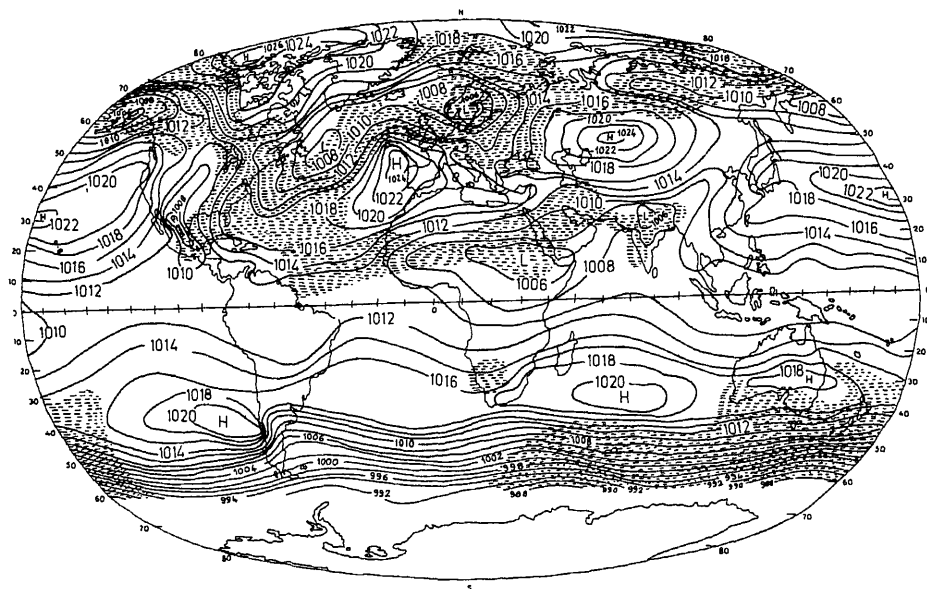


Figure 5. Mean sea-level pressure (mb) for April 1970.

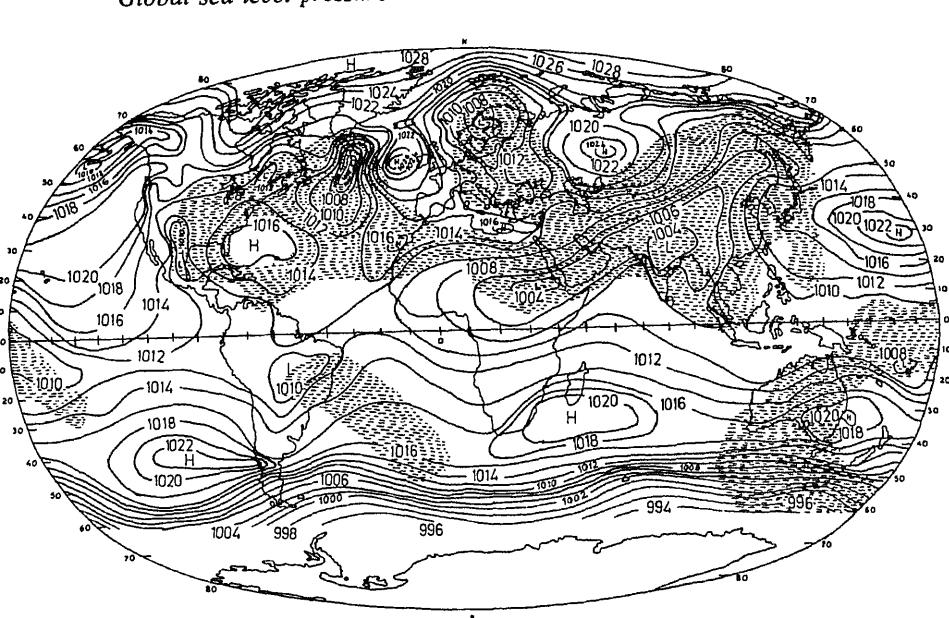


Figure 6. Mean sea-level pressure (mb) for April 1973.

strong (1024 mb). In the Southern Hemisphere, the sub-tropical high pressure zones over South Pacific, South Indian Ocean and Southwest Australia are prominent (1020–1022 mb).

A larger area in the Southern Hemisphere is covered by positive pressure departures. In the Northern Hemisphere, the polar region and the North Pacific are the only major areas covered by positive anomalies. Prominent negative anomalies are seen over the areas under the Icelandic Low while strong positive anomalies are mainly over the polar region.

3.2 July

In July, none of the pressure systems in the Northern Hemisphere, either seasonal or semi-permanent, show significant differences between anomalous monsoon years. The contrasting features observed in the Southern Hemisphere are described below.

(a) 1972: In the Southern Hemisphere, the subtropical high pressure zone is composed of three distinct cells (figure 7) over the South Pacific (1019 mb), the east coast of South Africa (1022 mb) and Australia (1022 mb).

The whole of the Southern Hemisphere is covered by negative pressure departures, except the eastern parts of the South Atlantic and the South Indian Ocean, and their adjoining continental areas of Africa and Australia. In the Northern Hemisphere, the North Pacific and the adjoining east Asia, the eastern portions of North Atlantic and the adjoining West Africa and the Icelandic Low show negative anomalies. Strong positive anomalies are seen over northern Europe and the Arctic High.

(b) 1974: The pressure features over the Southern Hemisphere in this year were similar to those in 1972, except that the South Pacific High was much stronger (1030 mb) in 1974. Two other high pressure cells, one zonally extended over the South Atlantic protruding into the South Indian Ocean (1023 mb) and the other relatively weak over Australia (1019 mb), are conspicuous.

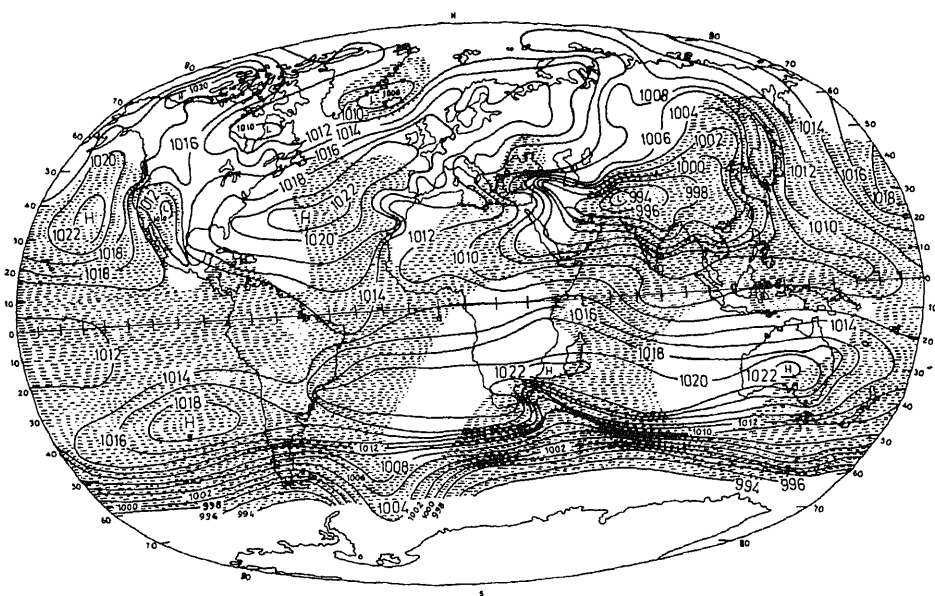


Figure 7. Mean sea-level pressure (mb) for July 1972.

A major part of the Eastern Hemisphere is covered by negative pressure departures. They are particularly high over the northern subpolar lows. The pressure anomalies suggest that the high pressure zones in the North and South Pacific as well as the Atlantic are stronger than normal.

(c) 1975: In the Southern Hemisphere, the subtropical high pressure zone is marked by four well-marked cells (figure 8), in the South Pacific, the South Atlantic, the South Indian Ocean and Australia (1020–1022 mb).

The whole of Africa, Indian Ocean, equatorial Atlantic, and Australia are covered by negative pressure departures. A major part of North America and Greenland are also covered by high negative anomalies. Most of the Pacific Ocean, South America, northern Europe and USSR show positive departures. It may also be noted that the Indian region shows positive departures at all stations.

(d) 1970: The subtropical high in the Southern Hemisphere is composed of four distinct cells, as in 1975. Those over Australia and the South Pacific are relatively strong (1024 mb).

The subpolar regions in both the hemispheres have shown negative pressure departures. Major parts of the Pacific and Atlantic oceans are marked by positive anomalies. The eastern parts of India and the adjoining areas, as well as Australia show positive anomalies. The western and central parts of the Indian Ocean and the adjoining Africa show negative anomalies.

(e) 1973: In the Southern Hemisphere, the South Pacific High considerably moved southwestward and merged with the Australian High, forming a strong high pressure cell (1026 mb) southeast of Australia (figure 9). The high pressure cell over the South Indian ocean is well defined (1022 mb).

The major part of the Northern Hemisphere is covered by negative pressure departures. Positive anomalies are predominantly seen over parts of Canada, central

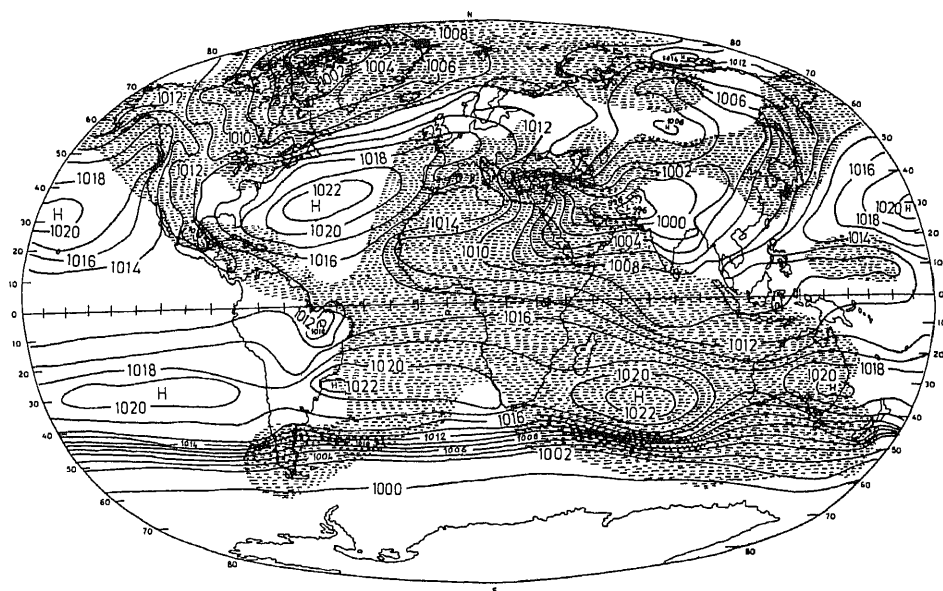


Figure 8. Mean sea-level pressure (mb) for July 1975.

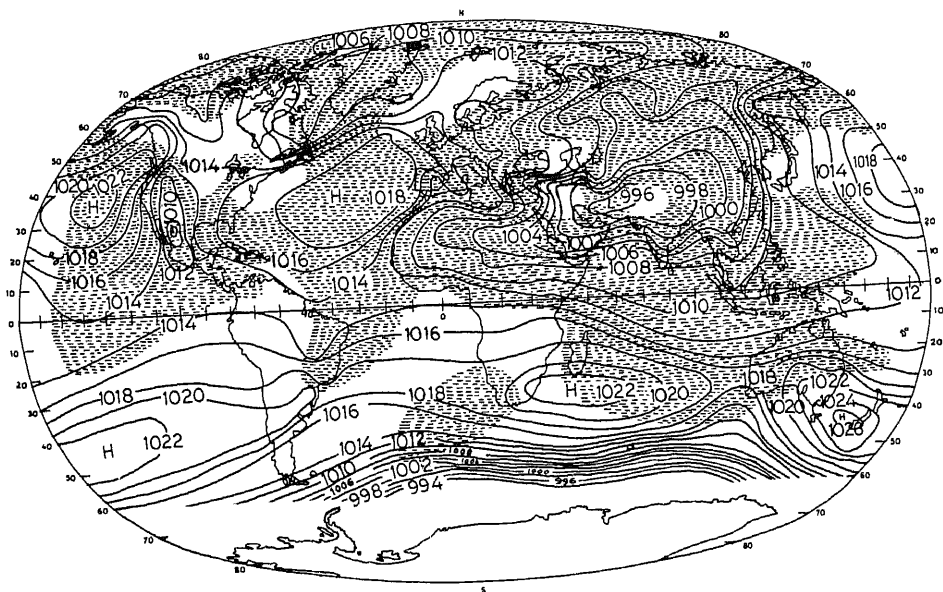


Figure 9. Mean sea-level pressure (mb) for July 1973.

parts of the North Pacific and the north of the British Isles. In the Southern Hemisphere, the Indian Ocean, Australia and the northeastern parts of Brazil show negative anomalies. Significantly high positive anomalies are seen over New Zealand

4. Discussion

From the nature of the pressure distribution described above, some specific features of contrast can be pointed out as follows.

During April, the continental high over USSR as well as the North Pacific High are found to have shifted considerably eastward during poor monsoon years. Particularly in the drought year 1972, the USSR High reaches as far east as Korea (figure 2). During heavy and good monsoon years, these highs are considerably west of their usual positions. The Icelandic Low is poleward during April and covers contiguous areas in poor monsoon years whereas in good monsoon years it prominently breaks into two cells, one going deep south and the other remaining near the subpolar regions. The one coming south pushes part of the North Atlantic high pressure centre considerably northeastward over to the British Isles. In this month, the subtropical highs in the Southern Hemisphere over the South Indian Ocean and South Australia are both strong and well-established in good monsoon years while in drought years they are localized to only one region.

Regarding the prominent features of the pressure departures in April, the area covered by positive anomalies over a major part of the oceans is markedly larger in heavy monsoon years. Over the South Indian Ocean, the pressure departures are consistently positive in heavy and good monsoon years and negative in poor monsoon years. In the North Atlantic near Greenland, the departures are significantly negative in poor monsoon years and positive in good monsoon years. It is worth noting here that the overall pressure anomalies during the well-known drought year 1972 are weak all over the globe in April.

In July, the subtropical high pressure cell over the South Indian Ocean is not well-defined in drought years whereas it is quite marked and covers a larger area east of South Africa and Malagasy in good monsoon years. The Australian High is also further eastward during good monsoon years. Consistent differences in the pressure anomaly pattern for poor and good monsoon situations could not be noticed in July. However, it may be mentioned that the anomalies are positive over the Indian region particularly on the eastern part and over the Bay of Bengal during good monsoon years.

It is also to be noted here that no significant differences could be noticed regarding the seasonal lows over the Indo-African region in April and over Pakistan in July.

5. Conclusions

A comparative study of the global sea-level pressure distribution and pressure anomalies in April and July for poor and good monsoon situations over India, leads to the following conclusions. (i) The month of April shows more prominent contrasting features between poor and good monsoon years than July. (ii) In April, the high pressure centres over USSR and the North Pacific move considerably eastward during poor monsoon years. (iii) In April, a breakaway cell of the Icelandic Low comes deep south during good monsoon years. (iv) In April, the subtropical high pressure centres over the South Indian ocean and Australia are more well-established and stronger in good monsoon years. (v) Pressure departures over oceanic areas are predominantly positive in April for good monsoon years. (vi) In July, the subtropical high pressure cell

over the South Indian ocean is more organised and stronger during good monsoon years. (vii) The Australian High is more eastward in July during good monsoon years. (viii) Significant differences in the pressure departures for July are not noticeable. (ix) Pressure departures are weak all over the globe in April of the well-known drought year 1972. (x) There are no significant differences in the characteristics of the seasonal lows over the Indo-African region and Pakistan for good and poor monsoon situations.

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***In situ* stress measurements using hydraulic fracturing method**

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Abstract. The first *in situ* stress measurements in India were carried out in 176 m deep borehole by employing deep-hole wireline hydraulic fracturing equipment. The results reveal that the direction of maximum compression is oriented at N 35° E, and the existence of high-near-surface horizontal stresses compared with the overburden pressure. This can be attributed to the continuing convergence of Indian and Eurasian plates. Earthquakes experienced near Hyderabad could be due to refracturing of some NNE or NE trending mineralized fractures which persistently occur in the area, as these fractures are critically oriented with respect to the present-day stress field.

Keywords. *In situ* stresses; hydraulic fracturing; breakdown pressure; refrac pressure; shut-in pressure; natural planes of weakness; maximum compression.

1. Introduction

The Indian continent is characterized by high seismicity in the Himalayas and the adjoining north-eastern region. The Himalayan region is one of the most seismically active belts of the world and has experienced several major earthquakes during historical times (Khattari *et al* 1984). During the past hundred years two earthquakes of magnitude 8·7 and seventeen of magnitude greater than 7·0 have occurred in north-eastern India besides innumerable minor earthquakes (Gaur 1983; Gupta *et al* 1984). Until recently, the Indian shield was considered as generally aseismic. However, with the improvement of the seismic network it is now recognized that widespread, although minor, seismicity exists throughout peninsular India (Indra Mohan *et al* 1981). During 1964-70 six earthquakes of magnitude about 6·0 occurred in widely separated locations on peninsular India (Chandra 1977). In the Hyderabad region, four minor earthquakes of magnitude not exceeding 4·5 occurred during the past 15 years (Kanungo *et al* 1978; Rastogi and Chadha 1984). The high seismicity in the Himalayas and north-eastern India, and the widespread minor seismicity in peninsular India can be attributed to the existence of high tectonic stresses at the collision zone which progressively diminish southwards. These stresses are caused by the continuing convergence of the Indian and Eurasian plates which over the past 40 million years have resulted in the folding and uplift of the collision zone into one of the loftiest mountain ranges of the world. A precise knowledge of the stress regime (magnitude as well as direction of the *in situ* stresses) prevailing in different tectonic segments of the

structures, planning of large-scale hydraulic fracturing operations to stimulate oil and gas wells, extracting geothermal energy, disposing industrial liquid wastes into the subsurface rocks etc.

Although the importance of the knowledge of the *in situ* stresses has long been recognized in the country, no direct stress measurements were made so far due to lack of expertise. In order to fill this gap, capabilities have been developed at the National Geophysical Research Institute for carrying out *in situ* stress measurements by hydraulic fracturing in deep boreholes (upto 500 m deep) of 3–5 inches diameter. The first *in situ* stress measurements in India were carried out recently at Hyderabad in a 176 m deep borehole using the wireline hydraulic fracturing equipment. The methodology and the results of these first measurements are presented and discussed in this paper.

2. Experimental technique

2.1 Wireline hydrofracturing system

A schematic of the Bochum Wireline Hydrofrac System (Rummel and Baumgartner 1985) used in the tests is shown in figure 1. The equipment comprises of double straddle packer assembly, winch and pumping unit with a heavy duty tripod, 500 m long 7-conductor electromechanical cable and a high pressure rubber hose mounted on a 4-wheel trailer. The equipment also includes the pressurising system and electronic data recording system.

The straddle packer assembly consists of two nylon reinforced rubber elements, an injection unit of 36 cm long, a push-pull valve assembly and a pressure release valve assembled on a steel mandrel. The active sealing length of the packer elements is about 1 m, and a test interval (70–120 cm long) can be had by using injection units of suitable lengths. Packers used in the present investigation can effectively seal the test interval for injection pressures as high as 300 bars. However, special rubber packers are available for injection pressures of upto 400 bars. The wireline hydrofrac equipment includes a novel feature i.e., a push-pull valve assembly by which fluid pressure can be applied either into the packers or into the test interval using only one pressure line. The push-pull valve mounted on the straddle packer assembly is activated to connect the pressure line either to the packers or to the test interval by applying or releasing the tension in the cable respectively.

A pressure release valve, mounted on top of the push-pull valve, is connected to the packers and guarantees fast and complete deflation of the packers. This is especially important in dry boreholes where the packers may remain partially inflated even after venting the system at the surface. The linkage between the logging cable and the packer assembly is provided by the cable head. A pressure transducer with an integrated amplifier is housed within the cable head, and measures downhole either by the packer pressure or fluid pressure in the test interval depending upon the position of the push-pull valve.

The winch and the pumping system, mounted on a 4-wheel trailer are driven by a 22 kW diesel engine and a hydraulic pump (figure 2). The winch drum is integrated

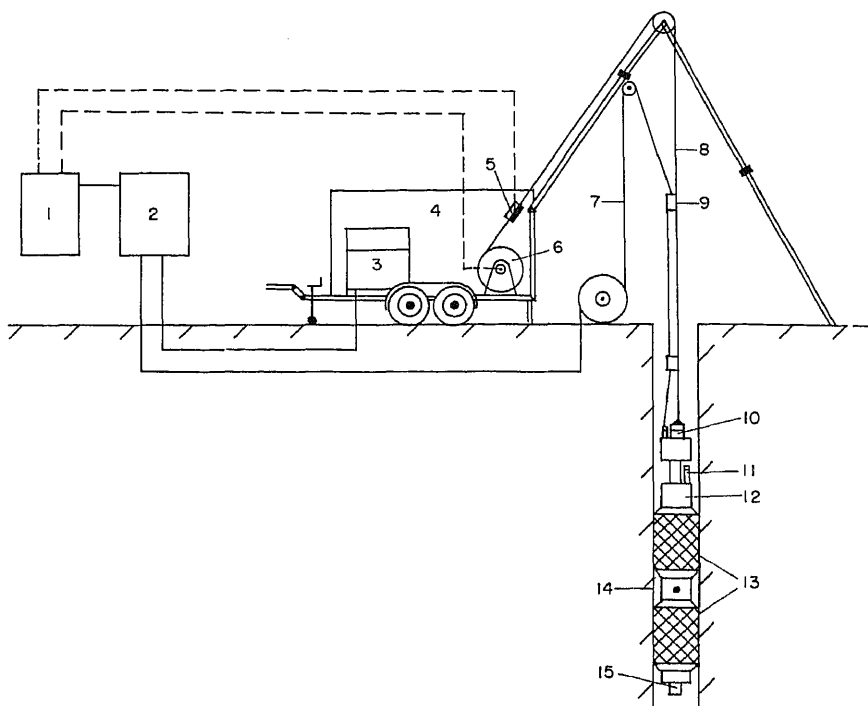


Figure 1. Schematic diagram of the Bochum wireline hydrofrac system. 1. Electronic recording system 2. Pressure and flow control panel 3. Pumping unit 4. Trailer mounted winch and pumping unit 5. Rotational transducer to indicate depth 6. Cable drum 7. High pressure hose 8. Seven conductor logging cable 9. Clamp to connect hose to cable 10. Pressure transducer 11. Release valve 12. Push-pull valve 13. Packer elements 14. Fracture interval 15. Steel mandrel over which the hydrofrac tool (10–13) is assembled.

with hydraulic motors and a brake. The pulling capacity of the winch is set to 20 kilo Newtons (KN), but can be tuned to 40 KN. The system incorporates automatic cable wind-up, and an electronic depth monitoring device. Cable tripping speed can be varied upto about 0.8 m/sec. The pumping system consists of a hydraulic motor and a duplex plunger pump and provides a maximum pumping rate of 10 l/min at pressures of upto 500 bars. Winch and pump operations are remote controlled. The packer assembly is tripped to depth by a heavy-duty aluminium tripod. Strike and dip of the hydrofractures induced in the test intervals are obtained by using an impression packer assembly in conjunction with a single shot borehole orientation tool.

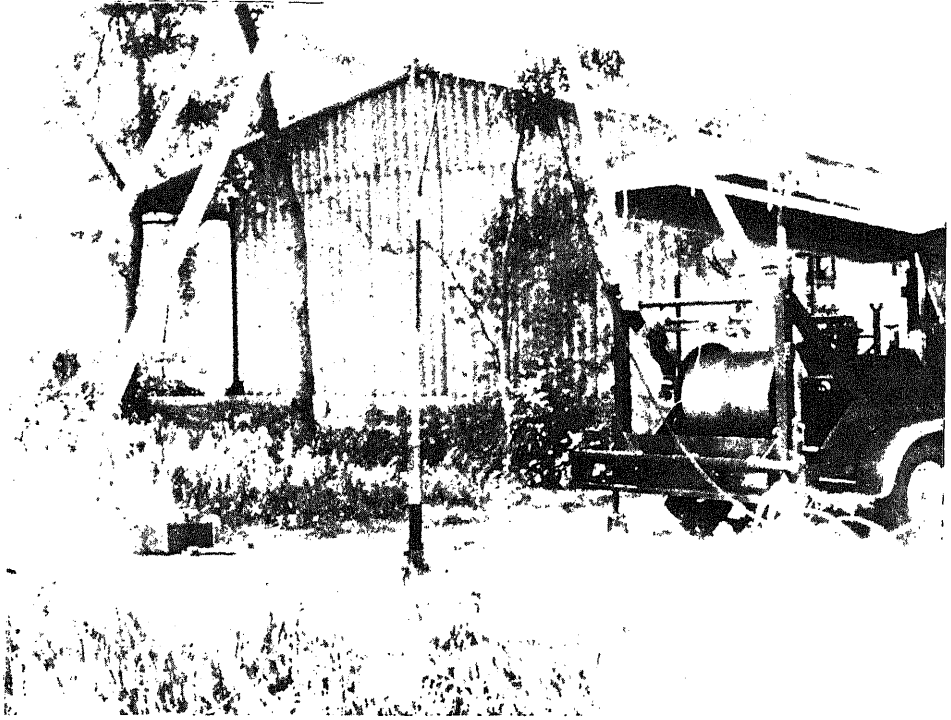


Figure 2. A view of the trailer mounted wireline hydrofrac equipment.

2.2 Testing procedure

Prior to conducting hydrofrac tests, the cores recovered from the borehole are critically examined to determine the nature and location of natural planes of weakness (NPW) and the fracture logs are prepared. Generally intact sections of the borehole which are free from NPW are selected for the frac tests. After the straddle packer assembly is tripped to the maximum predetermined depth and the packers are set, the test interval is instantaneously pressurized to 25 bars for determining rock permeability. Then, the fracture is induced and propagated at the full pumping rate in several phases with 10 to 20 litres of water being injected during each phase. The tests yield the hydrofrac data i.e. breakdown pressure P_C (frac initiation), the re-opening pressure P_R and the shut-in pressure P_{SI} . Shut-in pressure is also obtained from the slow-rate pumping tests. In all cases water recovered after venting is measured as a function of time for a period upto 30 minutes in some cases. This test procedure is repeated after the packer assembly is moved upwards to the next pre-determined test interval. After the completion of all the frac tests, imprint/impression of each induced fracture is obtained by the wireline impression packer unit.

3. Geology

The area consists essentially of one major rock formation viz pink and grey granites and forms a part of the huge granitic batholith that covers an area of about

5000 km² in the Telengana Region of Andhra Pradesh. These granites are presumed to be a part of the peninsular gneissic complex of Precambrian age (2 billion years). The granites have, in general, a NW–SE trend of foliation which is also the regional trend of Dharwars. Two important major structural features i.e. pink alaskites striking N 30° W–S 30° E and basic dykes trending E–W occur in the area. Another set of structural features comprising of mineralized fractures such as aplite veins, pegmatite veins, epidote veins and quartz reefs (mentioned here in the decreasing order of their age) are found in this area. These features, particularly epidote veins and quartz reefs, exhibit consistent NE–SW trends. Two prominent sets of joints are abundantly present in the area. One set strikes in N 60° W–S 60° E direction while the other set trends N 10° E–S 10° W. Detailed lithology and fracture logs were prepared by carefully examining the cores recovered from the borehole (Gowd *et al* 1985). Considering the fracture logs, twelve hydrofrac test intervals were selected in the intact zones.

4. Results and discussion

A typical pressure-time record of the hydrofrac test is shown in figure 3 which indicates the initiation of a fracture (frac test), propagation of the fracture (refrac test) and slow-rate pumping test at 117 m depth during several pressurisation tests. The values of P_C , P_R and P_{SI} were determined from the pressure time record. Similarly, tests were conducted at other 11 test intervals and the respective P_C , P_R and P_{SI} were determined. The results reveal that P_C required to initiate fractures in intact granite (upto a depth range of 150 m) range between 63 and 167 bars, re-opening pressures between 34 and 110 bars, and P_{SI} just sufficient to keep the fractures open against the normal stress, range between 24 and 78 bars (table 1).

Frac impression data reveal that vertical fractures were induced at eight test intervals while pre-existing inclined fractures re-opened at the other four test locations. The strike and dip of these fractures are given in table 1. The azimuth of the induced vertical fractures is presented in Fig. 4 and demonstrates that the

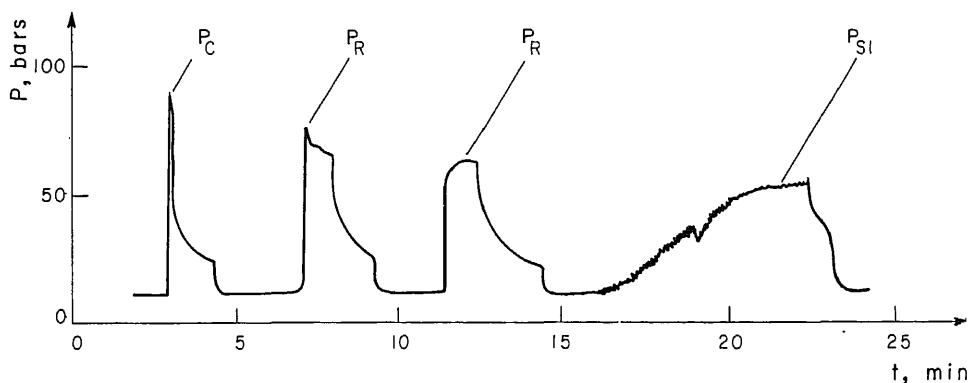


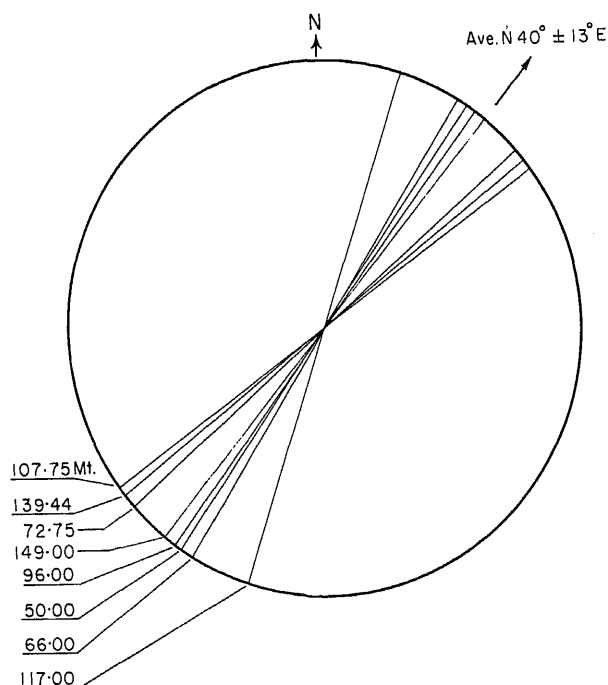
Figure 3. Pressure-time record of the hydrofrac tests conducted at 117.00 m depth in the

Table 1. Frac data obtained in the N.G.R.I. campus borehole, Hyderabad.

Test No.	Depth (m)	P_C (bars)	P_R (bars)	P_{SI} (bars)	T (bars)	Frac α°	Orientation d°
H. 1	43.80	90	34	24	56	56	62
H. 2	46.75	79	48	36	31	345	14
H. 3	50.00	94	76	45	18	35	90 (V)
H. 4	66.00	66	53	40	13	33	90 (V)
H. 5	72.75	81	63	45	18	50	90 (V)
H. 6	76.00	63	52	30	11	351	52
H. 7	96.00	126	87	57	39	37	90 (V)
H. 8	107.75	119	81	57	38	55	90 (V)
H. 9	111.40	85	76	57	9	39	36
H. 10	117.00	88	62	53	26	19	90 (V)
H. 11	139.44	123	71	56	52	52	90 (V)
H. 12	149.00	167	110	78	57	40	90 (V)

P_C : Breakdown pressure; P_R : Refrac pressure; P_{SI} : Shut-in pressure; T : *in situ* tensile strength ($P_C - P_R$); α : Orientation of frac plane (measured eastward from the north); d : dip of frac plane with respect to the horizontal.

induced vertical fractures are consistently oriented in NE-SW quadrant throughout the 50–150 m depth section investigated, average of all these eight orientations being $N 40^\circ \pm 13^\circ E$.

**Figure 4.** Orientation of the vertical fractures induced during the hydrofrac tests at various depths in the N.G.R.I. campus borehole, Hyderabad.

The procedure followed to derive stress data from hydraulic fracturing pressure records is the same as that used by other hydrofrac groups (Rummel *et al* 1983; Haimson and Lee 1984). First, the magnitude of the principal horizontal stresses was computed from the hydrofrac data using the classical interpretation method (Hubbert and Willis 1957) which is based on the assumption that the strike of the induced vertical fractures coincides with the direction of the maximum horizontal principal stress. The following reactions were used in this classical approach.

$$S_h = P_{SI}$$

$$S_H = 3P_{SI} - P_R - P_O,$$

where S_H and S_h are the maximum and minimum principal horizontal stresses respectively and P_O is the pore fluid pressure. The values of S_H and S_h are evaluated and the same are plotted as a function of depth in figure 5 (circles). The best straight line fits (broken lines) to the data points are also shown in the figure.

The principal stresses were also computed from the shut-in pressure data only using the computer method developed by Cornet and Valette (1984), and Baumgartner *et al* (1986). This non-classical i.e. P_{SI} method allows one to consider the pressure data of the inclined fracture also. The results reveal that the principal horizontal stresses vary with depth as per the following equations

$$S_H \text{ (bars)} = 27 + 0.45 Z \text{ (meters)},$$

$$S_h \text{ (bars)} = 26 + 0.25 Z \text{ (meters)}.$$

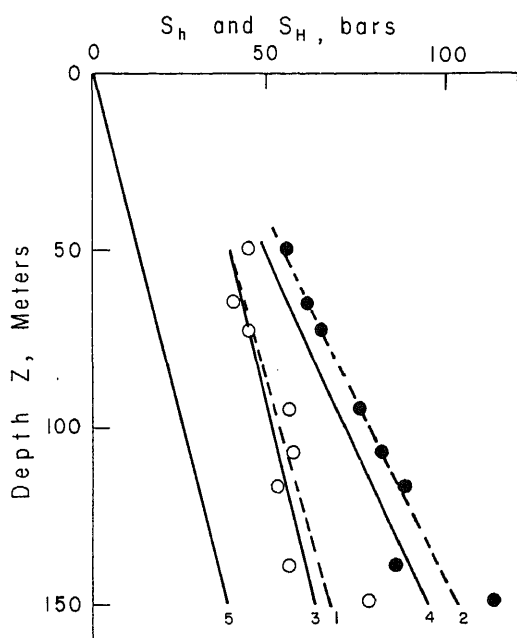


Figure 5. Maximum and minimum principal horizontal stresses (S_H , S_h) as a function of

The plots of S_H and S_h are also shown in figure 5 (continuous lines). These equations reveal that the maximum and minimum principal horizontal stresses are about 72 bars and 51 bars at 100 m depth, and about 117 bars and 76 bars at 200 m depth respectively, whereas the vertical stress (overburden pressure) is about 26 bars and 52 bars only at these depths. Thus the results demonstrate the existence of high near-surface horizontal stresses compared with the overburden pressure, indicating that they are of tectonic origin.

The computations based on the P_{SI} method have also yielded the direction of the maximum principal horizontal stress as N 35° E, which closely agrees with the average azimuth of the induced vertical fractures (N 40° ± 13° E). First motion data of the earthquakes that occurred recently near Hyderabad also indicates the existence of horizontal compression in the northeast direction (Kanungo *et al* 1978). The observed direction of the maximum horizontal compression is in agreement with the general understanding of the north-eastward motion of the Indian continent. This suggests that the collision of the Indian continent with the Eurasian plate 40 million years ago not only resulted in the folding and the uplift of the collision zone into one of the loftiest mountain ranges of the world i.e. the Himalayas but was also responsible for inducing high intraplate stresses in the Indian shield as far south as Hyderabad. However, this statement needs to be confirmed by further measurements in different parts of the Indian shield.

Mineralized fractures such as epidote veins, quartz reefs etc predominantly strike in the northeast quadrant in the Hyderabad region (Sitaramayya 1969; Kanungo

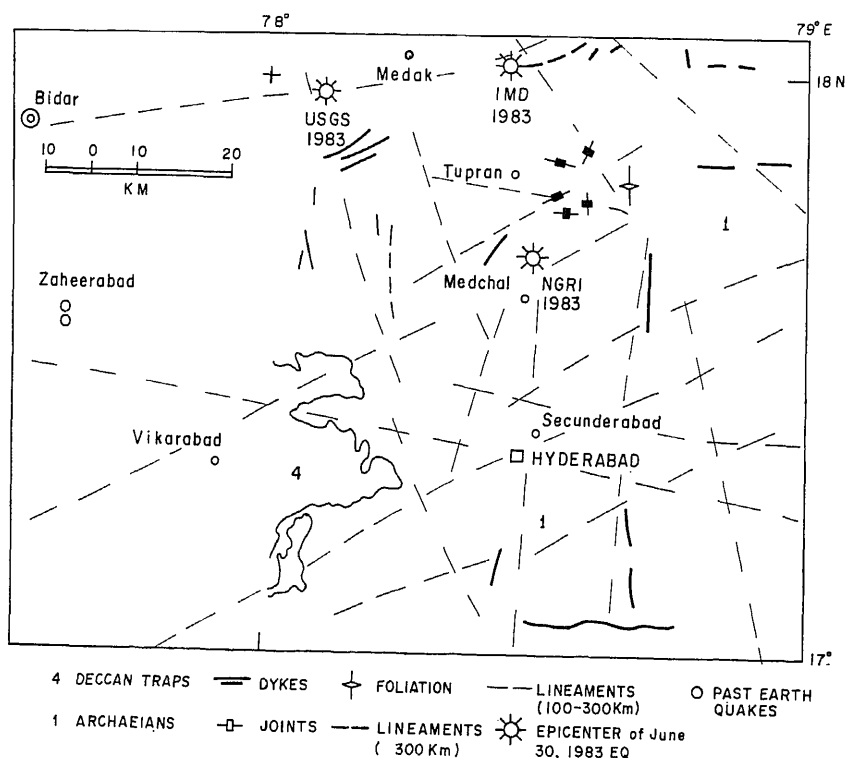


Figure 6. Mega lineament map of Hyderabad region.

et al 1975). Mega-lineament map of the Hyderabad area (figure 6) also shows strong trends in the NNE and NE directions. It appears that the intraplate stresses in the area whose magnitude at depth can be gauged from the observed high near surface stresses, are sufficient to cause time-dependent deformations, which, over a period of 50–100 years will result in the accumulation of strain energy that is capable of refracturing or giving rise to seismic slip along some of these pre-existing weak planes since they are most favourably oriented with respect to the present day stress field. This would also explain the recent occurrence of four minor earthquakes (magnitude ≥ 4.5) in the area as well as the earthquake that occurred near Hyderabad in 1876.

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Computation of synthetic seismograms for plane layered earth models including absorption and dispersion phenomena

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Abstract. A computer program package has been written in FORTRAN-IV language and tested successfully on an ICL 1904S computer. This program enables one to compute synthetic seismograms for layered earth models. The provision for studying the effect of absorption and dispersion of seismic waves has been made with Subroutines. The present program utilizes eight Subroutines and requires about 35 K core memory. A set of examples are illustrated for absorption and dispersion models. An exponential decay of amplitude has been used for the absorption model. This method is based on the plane wave propagation in a flat-layered earth system. Normal incident P-waves are used to eliminate the effect of other phases. Change in shape of reflected waves is observed in absorption model due to damping of energy of higher frequencies. Lack of resolution is found between closely spaced reflections at higher frequencies. The effect of dispersion on seismic waves decreases the time of primary reflections as well as amplitudes of the seismic waves.

Keywords. Synthetic seismograms; computer programs; absorption; dispersion.

1. Introduction

The problem of wave propagation from a seismic source in a layered medium can be addressed either through ray theory methods or by using wave theory approach. In the latter approach a direct numerical evaluation of integral representing the frequency domain solution is required. In this domain, approximate computational methods have been developed by Pinnery (1965), Fuchs (1968), Fuchs and Muller (1971), Bouchon and Aki (1977) and Bouchon (1979) and have introduced the concept of a discretized wave field to obtain the solution in the near field of the source. In the past, a number of techniques for computation of synthetic seismograms have been developed (Wuenschel 1960; Trorey 1962; Claerbout 1968; Trietal and Robinson 1966; Robinson 1968; Robinson and Trietal 1977; Khattri *et al* 1978; Kennet and Kerry 1979; Kennet 1980; Kennet and Illingworth 1981). They have considered different phenomena associated with the earth's crust. In most of these cases, effects of seismic energy loss have been ignored, but Trorey (1962) introduced an approach which allowed the inclusion of attenuation. Khattri *et al* (1978) considered the frequency-dependent absorption and velocity free from dispersion. Acoustic wave propagation in porous media has been studied by several workers (Biot 1962 a, b; Deresiewicz and Rice 1962, 1964; Deresiewicz and Levv 1967; Deresiewicz 1974 and Burridge and Vargas 1979). Deresiewicz (1974) showed that the presence of liquid in the pores causes a decrease in the phase velocity and motion is attenuated. More recently, there has been considerable interest for problems of absorption and dispersion. Ganley (1981) considered such

problems and gave a method to generate synthetic seismograms which incorporate these effects. His technique can be used to produce normal incidence, plane wave seismograms for flat-layered earth models. It is essentially a modification of communication theory approach to synthetic seismograms.

In this article, we have developed a set of computer programs for the computation of synthetic seismograms based on the theory proposed by Ganley (1981) as it is straightforward and includes the effect of absorption and dispersion. These seismograms are useful in the study of effect of changes in lithology. This program contains eight Subroutines which are listed with their functions in table 1. Input and output of the Subroutines are given in table 2. This program requires 35 K core memory for the type of cases taken up in the present study. The program has been tested on an ICL 1904S computer. We have presented the results of an assumed model which contains sandy oil, gas and water overlain and underlain by shale to form a trap. Bed rock is assumed to be sandstone as a lower half space. In computation, an unit spike is taken as a signal and source-receivers configuration is assumed at the surface.

2. Computational procedure

Our programming formulation has been taken from the method given by Ganley (1981). The computation of synthetic seismograms is based on the well-known absorption model of exponential decay of amplitude with distance which is given by;

$$A = A_0 \exp(-\alpha z) \quad (1)$$

where A_0 is the initial amplitude and A , the amplitude at depth z , α the absorption coefficient which is frequency-dependent, usually written as

$$\alpha = |w|/2cQ, \quad (2)$$

where $|w|$ is the absolute angular frequency, c the phase velocity and Q the quality factor.

Table 1. List of Subroutines required for the computation of synthetic seismograms.

Subroutines	Function
SYNTH 1	Calculates attenuation, velocity and quality factor as a function of frequency.
SYNTH 2	Makes velocity complex function of frequency and computes reflection and transmission coefficients, using output of SYNTH 1.
WIDSYNTH	Computes reflection and transmission coefficients without considering the frequency effect.
SYNTH 3	Computes spectrum of upgoing and downgoing waves using output from SYNTH 2/WIDSYNTH.
SYNTH 4	Computes synthetic seismograms on the basis of spectrum obtained from SYNTH 3.
REFL	Provides complex conjugate to use in NLOGN Subroutine.
NLOGN	Performs the Fourier transformation.
MAXSN	Finds the maximum value to normalize the desired output.

Table 2. Variables used in different Subroutines.

Subroutines	Variables	
	Input	Output
SYNTH 1	W=angular frequency NOM=number of points C=phase velocity Q=quality factor NL=number of layers in a model	ALP=attenuation coefficient CW=frequency-dependent velocity QD=frequency-dependent Q
SYNTH 2	C,CW,Q,QD,NOM,NL, W,DEN=density of layer	Complex VC=velocity RC=reflection coefficient TC=transmission coefficient
WIDSYNTH	C,Q,DEN,NOM,NL	Complex VC,RC,TC
SYNTH 3	RC,TC,ALP,W,NOM,NL, TH=thickness of layer TR=travel time in layer	U=spectrum of the upgoing wave
SYNTH 4	U,NOM,NL,F=linear frequency.	AMP=amplitude PHA=phase
REFL	U,NOM	RF(=U with its complex conjugate)
NLOGN	NT=integer number and RF	AMP
MAXSN	NN=2 xNOM, AMP, PHA	XMAX=maximum value of amplitude/phase

We have used Futterman's (1962) theory as it provides an excellent model of attenuation and dispersion that is in good agreement with experimental data. It is based on the principle of superposition and linearity of absorption coefficient as a function of frequency. However, some other absorption-dispersion pair could also be used.

Using the Futterman's (1962) absorption-dispersion pair, the absorption coefficient α can be written as,

$$\alpha(w) = w/2cQ' = w/2c_0Q_0. \quad (3)$$

The frequency-dependent phase velocity is given by

$$c(w) = c_0 \left[1 - \frac{1}{\pi Q_0} \ln \frac{\nu \omega}{\omega_0} \right]^{-1} \quad (4)$$

and

$$Q'(w) = Q_0 \left[1 - \frac{1}{\pi Q_0} \ln \frac{\nu \omega}{\omega_0} \right] \quad (5)$$

At ω_0 frequency (approximately zero), there is no dispersion and the phase velocity approaches c_0 , quality factor Q_0 , as $\omega \rightarrow \omega_0$. The reflection and transmission coefficient will be real at that frequency. However, the principle of causality is violated if Q depends on ω linearly as it tends to infinity. For non-zero frequency, velocity is made a complex function. The real part of velocity is given by

$$V_r = c/2[1 + (1 + 1/Q^2)^{-1/2}], \quad (6)$$

and the complex velocity is

$$V(\omega) = V_r \left[1 + \frac{i \operatorname{sgn}(\omega)}{2Q} \right], \quad (7)$$

where $\operatorname{sgn}(\omega) = 1$ for $\omega > 0$ and -1 for $\omega < 0$. V_r is the real velocity which is constant for a particular layer and $V(\omega)$ is the complex velocity which is not constant for the layer but it varies with frequency and quality factor of that layer. For dispersive case complex velocity is used to calculate the reflection and transmission coefficients as follows:

$$R = (\rho_2 V_2 - \rho_1 V_1) / (\rho_1 V_1 + \rho_2 V_2) \quad (8)$$

and

$$T = (2\rho_2 V_2) / (\rho_1 V_1 + \rho_2 V_2), \quad (9)$$

where ρ is the density of the layer. If U_i and D_i are the spectrums of the upgoing wave and downgoing waves respectively in the i th layer, these are related as

$$D_i = \frac{\exp(\alpha d_i) \exp(i\omega d_i / c_i)}{T_i} (D_{i+1} + R_i U_{i+1}) \quad (10)$$

and

$$U_i = \frac{\exp(-\alpha d_i) \exp(-i\omega d_i / c_i)}{T_i} (R_i D_{i+1} + U_{i+1}) \quad (11)$$

where d_i is the thickness of the i th layer. The ratio of the spectrums of upgoing and downgoing waves are given by

$$Y_i = U_i / D_i = \exp(-2\alpha d_i) \exp(-2i\omega d_i / c_i) \left(\frac{R_i + Y_{i+1}}{1 + R_i Y_{i+1}} \right). \quad (12)$$

For an n -layer model $U_{n+1} = 0$ and therefore $Y_{n+1} = 0$. Equation (12) can be successively applied from the bottom layer to the surface until Y_1 is determined.

If R_0 is the reflection coefficient of the surface as seen from above, then $-R_0$ will be the reflection coefficient of the surface for upgoing wave in the first layer, and

$$D_1(\omega) = 1 - R_0 U_1(\omega). \quad (13)$$

With knowledge of $U_1(\omega)$ and $D_1(\omega)$, the Fourier transform of synthetic seismogram, $X(\omega)$ at the surface is given by

$$\begin{aligned} X(\omega) &= U_1(\omega) + D_1(\omega), \\ &= (1 - R_0) U_1(\omega) + 1 \end{aligned} \quad (14)$$

Therefore, given an n -layer model, it is possible to calculate the Fourier transform (FT) of the surface synthetic seismogram for an initial downgoing spike at the surface. This FT can then be multiplied by the FT of the wavelet if we wish to use a wavelet different from a spike as the input wavelet.

We have developed computer programs based on the above equations. Part-wise calculations are performed using different Subroutines (table 1). Subroutine SYNTH 1 calculates the attenuation coefficient, frequency-dependent velocity and modified quality factor depending on equations (3), (4) and (5). Subroutine SYNTH 2 first makes the velocity as a complex function, then reflection and transmission coefficients are calculated for dispersive case using (6) to (9).

We have also computed synthetic seismograms without the effect of dispersion. For this purpose, a separate Subroutine WIDSYNTH is developed. After computations of reflection and transmission coefficients of different layers, we have calculated the ratio of the spectrum of the upgoing and downgoing waves with the help of Subroutine SYNTH 3, which is based on (12). Surface layer synthetic seismograms in frequency domain, are computed using (13) and (14). This is performed in Subroutine SYNTH 4. Time domain synthetic seismograms are computed using Subroutine NLOGN. Subroutine REFL is used to find out the complex conjugate of the frequency function, calculated only at positive frequencies. Maximum values of the amplitude and phases are obtained with Subroutine MAXSN, which are used to normalize the output.

3. Examples of synthetic seismograms

In this exercise a number of theoretical data were assumed for different layers to check the validity of the programs in the close proximity of the real field situation. We used a model whose relevant parameters are given in table 3. Velocity, density, thickness and quality factors have been used as an input data. Q values and phase velocities given in table 3 are assumed at zero frequency. Lower and upper cut-off frequencies are set to be 0.001 Hz and 125.0 Hz respectively. Since the amplitude for negative values of frequency (ω) is the complex conjugate of the corresponding positive values, computations are made for positive frequencies at 65 points including $\omega = 0$. The interval between consecutive samples in the frequency domain is 1.953 Hz ($= 125/64$). Further the number of samples obtained at Nyquist interval of 4 msec is required to be an integral power of 2 in order to apply Fourier

Table 3. Various parameters of the model used in computation of synthetic seismograms.

Lithology	Velocity (m/sec)	Density (gram/cc)	Thickness (meter)	Quality factor
Shale	2700	2.23	200	30.9
gas-sand	1700	2.05	25	21.4
oil-sand	3040	2.29	30	56.0
water-sand	3536	2.32	58	64.8
Shale	3049	2.32	105	39.2
sandstone	4090	2.45	—	75.4

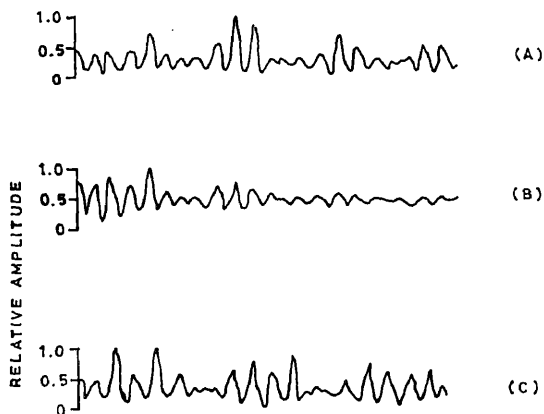
Transform in our case for time domain calculation. The number of samples in both frequency and time domains should be equal. Thus it yields $128 (= 2^7)$ samples in both the domains.

3.1 Non-dispersive model

In this model two cases have been considered, viz (i) without absorption and (ii) with absorption. Figure 1a shows the amplitude spectrum of a non-dispersive model without absorption. The maximum amplitude of oscillations has been found in the frequency range greater than 50 Hz. In this spectrum, constructive and destructive interferences are recognized by maxima and minima respectively. Figure 1b illustrates the amplitude spectrum of the non-dispersive dissipative model. It is clear from this case that amplitudes are highly attenuated at frequencies greater than 60 Hz. At lower frequencies, the attenuation is not very much pronounced. Figures 2a and 2b show the corresponding phase spectra of the model. It is observed that phases vary between ± 1.0 radian. The oscillations in phase spectra in the dissipative case show high attenuation of phase in the higher frequency range greater than 60 Hz, which may be attributed to attenuation of the wave-form within this domain.

The corresponding synthetic seismograms have been computed from the frequency spectra using inverse Fourier transform and are shown in figures 3b and 3c. Time section is also illustrated against two-way travel time in msec (figure 3a).

AMPLITUDE SPECTRUM



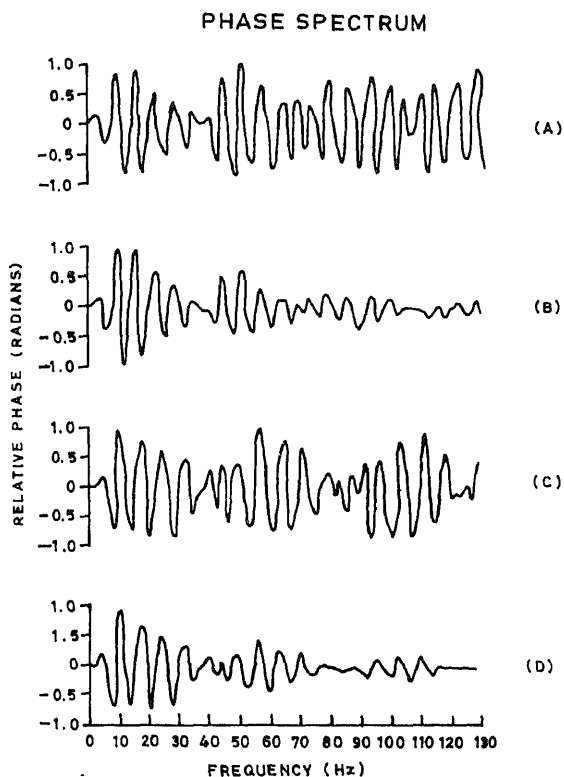


Figure 2. Phase spectra of various models. (a) and (b). Non-dispersive models without and with attenuation. (c) and (d). Dispersive models without and with attenuation.

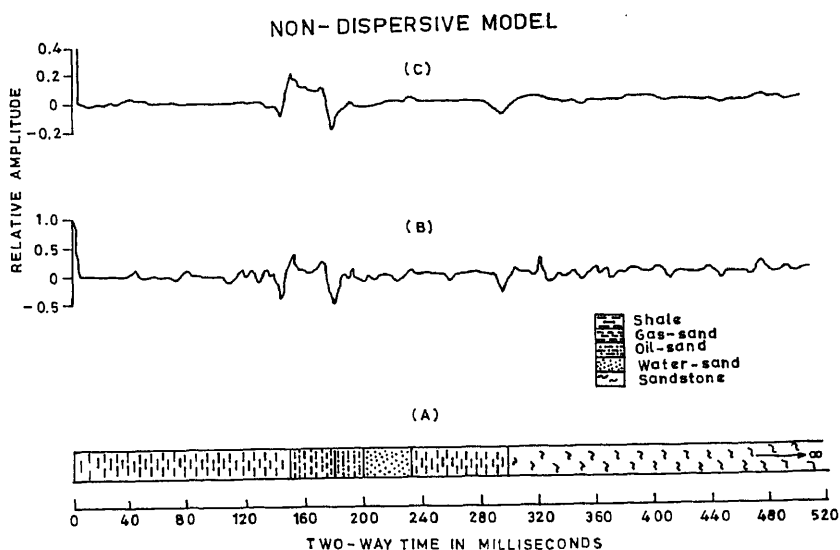


Figure 3. Synthetic seismograms along with geological section for non-dispersive model. (a) Geological section against two-way time in msec. (b) and (c). Synthetic

Figures 3b and 3c represent loss-less and dissipative seismograms for a nondispersive case. It is clear from these diagrams that boundaries are well marked by reflection amplitudes. High amplitudes are observed at shale-gas and gas-oil boundaries. However, amplitudes are attenuated much for the dissipative case as shown in figure 3c.

3.2 *Dispersive model*

In this model, effects of dispersion (i) with absorption and (ii) without absorption are considered separately as in the previous model. Figures 1c and 1d illustrate the nonabsorptive and absorptive cases of amplitude spectra. In this case, maximum amplitude of oscillations is observed in the frequency range less than 30 Hz. The period of oscillations is approximately similar as in the case of nondispersive model, but the maxima and minima positions are displaced which illustrates interference patterns at another frequencies in comparison to the previous model. In the absorption model (figure 1d) attenuation is more pronounced at higher frequencies greater than 70 Hz than that of the dissipative case of nondispersive model. Figures 2c and 2d show the corresponding phase spectra of loss-less and dissipative models. Phase variations in these cases are found between $+1.0$ and -0.8 radian. Phases are highly attenuated for dissipative model at higher frequencies range greater than 60 Hz. Phase variation in the dispersive loss-less model (figure 2c) is low in comparison to nondispersive loss-less model (figure 2a) in two frequency ranges around 80 Hz and 120 Hz.

Figure 4 shows the synthetic seismograms of the dispersive model. Time section in this case is identical as was considered for nondispersive model to compare arrivals of primary reflections (figure 4a). In this case, reflection positions are found to be displaced showing the effect of dispersion. The pattern of seismograms is similar to nondispersive case with decreased amplitudes for both the nondissipative and dissipative cases (figures 4b and 4c). It is observed that dispersion effect produces 15 msec time difference in primary reflections in comparison to nondispersive case.

3.3 *Discussion and conclusion*

The concept of utilizing the sedimentation models to study the possibilities of determining stratigraphic variations and locating stratigraphic traps through the study of synthetic seismograms has been presented. Stratigraphic traps are important for the accumulation of petroleum, resulting from the change in lithology and environment of deposition.

Amplitude spectra of nondispersive and dispersive models show identical period of oscillations but maximum amplitude positions are observed at different frequencies, viz 52 Hz in nondispersive case and 26 Hz in dispersive case. Maxima and minima positions are not coincident over the entire frequency range except 50–60 Hz. The exception of this frequency range may be explained by the fact that the maximum contribution to reflected signals, which arise from shale-gas and oil-gas boundaries, does not contain low frequency component and the

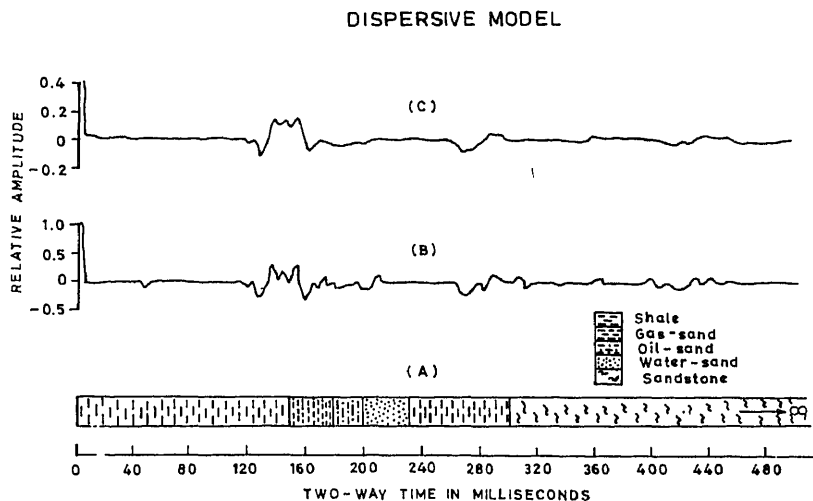


Figure 4. Synthetic seismograms alongwith geological section for the dispersive model (a). Lithological section against two-way time in msec. (b) and (c). Synthetic seismograms without and with absorption.

through virtually unaffected. The closest similarity between the two spectra observed at frequencies less than 8 Hz. It is clear from figure 1 that absorption is more pronounced in the dispersive case as compared to the nondispersive model. Figure 2 shows the variation of phases for different models. In the case of loss-less dispersive/nondispersive model, phase variations are approximately between ± 1 radian. Change in phases for both the dissipative models has been observed at higher frequency greater than 60 Hz, which may be attributed to the absorptive effect. However, the cause for the change in phase in dispersive model (figure 2) around 80 Hz and 120 Hz is not precisely known.

Synthetic seismograms (figures 3 and 4) show similarity in the oscillations. It is found that the presence of gas produces high amplitude reflections in both the models. The amplitude in dispersive case is less in comparison to nondispersive case. The effect of dispersion is to decrease the arrival time of primary reflection and wave amplitudes. Absorption effect in both cases creates change in shape of reflections by damping the energy of higher frequencies. It also broadens the pulse and lack of resolution is observed between closely-spaced reflections at higher frequencies. It is clear from the present discussion that the frequency domain calculation is useful to evaluate the spectral response of elastic and dissipative earth models.

The present computer program package enables one to study the effect of changes in lithological characters by generating the synthetic seismograms for plane-layered earth system. The required core memory is suitable for moderately sized computers. These programs are stored in the disk file for ready use whenever necessary. We are now trying to study the effects of oblique incident wave on the interfaces using modified Subroutines. Attempts are being made to study the effects of scattering and diffraction on the seismic waves for different geological models. The results of these investigations would be published elsewhere.

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Magnetic anomalies over thin plates and their analysis

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Abstract. The magnetic anomaly over a two-dimensional thin horizontal plate is similar to the first horizontal derivative of the magnetic anomaly over a thick dipping dike of infinite depth extent but with a different direction of magnetization. Hence, the magnetic anomalies of thin plates may be integrated along the profile and the pseudomagnetic potential anomaly thus obtained may be interpreted using any standard method for interpreting dike anomalies. Expressions for the Fourier amplitude and phase spectra of the magnetic anomaly over a thin plate are also derived and procedures to evaluate the parameters of the plate from the spectra are formulated.

Keywords. Thin plate; magnetic interpretation; Fourier transform; pseudo-magnetic potential.

1. Introduction

Quantitative interpretation of isolated magnetic anomalies is usually carried out by assuming the sources to be of simple geometric shapes such as dikes, sheets, steps etc. The anomalies caused by undulations in the basement surface are often attributed to a prism model, but the thin plate model appears to be a better approximation due to the small thickness-to-depth ratio of the suprabasement feature. The authors are not aware of any published work relating to the interpretation of magnetic anomalies over thin plates except the work of Telford *et al* (1976) and Nettleton (1976), which is only elementary in nature.

In the present work, the mathematical expression for induced magnetic anomalies over thin plates is derived. Two methods for calculating the depth, width and susceptibility of the plate are suggested. In the first method, the anomaly is integrated with distance to obtain a profile of pseudo-magnetic potential denoted by $P(x)$. This resembles the magnetic anomaly of a dike with a different direction of magnetization and hence, any standard method of dike interpretation may be used to obtain the body parameters. The curve matching technique of Gay (1963), the method of characteristic curves of Koulomzine *et al* (1970) or Atchuta Rao and Ram Babu (1981), the computer method of Rao *et al* (1973), the spectral method of Bhimasankaram *et al* (1977) or Rao *et al* (1978) etc., are a few methods for interpreting magnetic anomalies over thick dikes. In the second method, the Fourier spectra of the anomaly are analyzed to evaluate the parameters of the plate.

2. Theory

2.1 General expression for the magnetic anomaly over a thin plate

Table 1. Equivalents of the amplitude coefficient (C) and index parameter (Q) for the plate and the dike.

Anomaly (F)	Amplitude coefficient	Index parameter
(a) for thin plate	C_p	Q_p
T	$2kTtB^2$	$2I_p - 90$
Z	$2kTtB$	I_p
H	$2kTtB \cos a$	$I_p - 90$
(b) for thick dike	C_d	Q_d
T	$2kTB^2 \sin(\delta)$	$2I_d - \delta - 90$
Z	$2kTB \sin(\delta)$	$I_d - \delta$
H	$2kTB \sin(\delta) \cos a$	$I_d - \delta - 90$

The general expression for the magnetic anomaly $F_p(x)$ over the plate ef may be written as follows:

$$F_p(x) = C_p \left[\frac{(x+b) \sin Q_p + h \cos Q_p}{(x+b)^2 + h^2} - \frac{(x-b) \sin Q_p + h \cos Q_p}{(x-b)^2 + h^2} \right] \quad (3)$$

2.2 Type curves of plate anomalies

Equation (3) gives the magnetic anomaly in either the total (ΔT), vertical (ΔZ), or horizontal (ΔH), field at any point $P(x)$ on a line perpendicular to the strike of a two-dimensional thin horizontal plate of width $2b$ situated at a depth h below the line of observation. It may be noted from (3) that the amplitude of $F_p(x)$ depends on C_p whereas its shape depends on Q_p, b and h . A few sets of type curves over the plate are presented in figure 2.

The ΔT anomaly over a thin plate ($b/h=2$), striking east-west is shown in figure 2a for different inclinations of the earth's field. It may be noted from figure 2a that at equator ($i=0^\circ$), the magnetic anomaly is a symmetric low flanked on either side by a small high of equal amplitude. At the pole ($i=90^\circ$), the curve is a mirror image along x -axis of the ΔT curve for $i=0^\circ$. In intermediate latitudes ($i=30^\circ$ and 60°), the curve consists of a low towards the northern end.

The ΔT anomaly over a plate ($b/h=2$; $i=30^\circ$) for various strike directions (figure 2b) shows that when the strike of the plate is along north-south ($=0^\circ$), the anomaly is a symmetric high flanked on either side by a small low. As the strike changes from north-south to east-west, the northern low enhances while the southern low diminishes.

Variations in the ΔT anomaly over a plate when the width of the plate changes may be noted from figure 2c when $a=0^\circ$ and $i=30^\circ$. It is seen that the anomaly stretches horizontally as the width increases and for half-widths equal to or greater than 3 depth units, the low and the high occur approximately over the edges of the plate.

2.3 Relation between magnetic anomalies over thin plates and thick dikes

The magnetic anomaly over a thick dipping dike of the same width as the thin plate

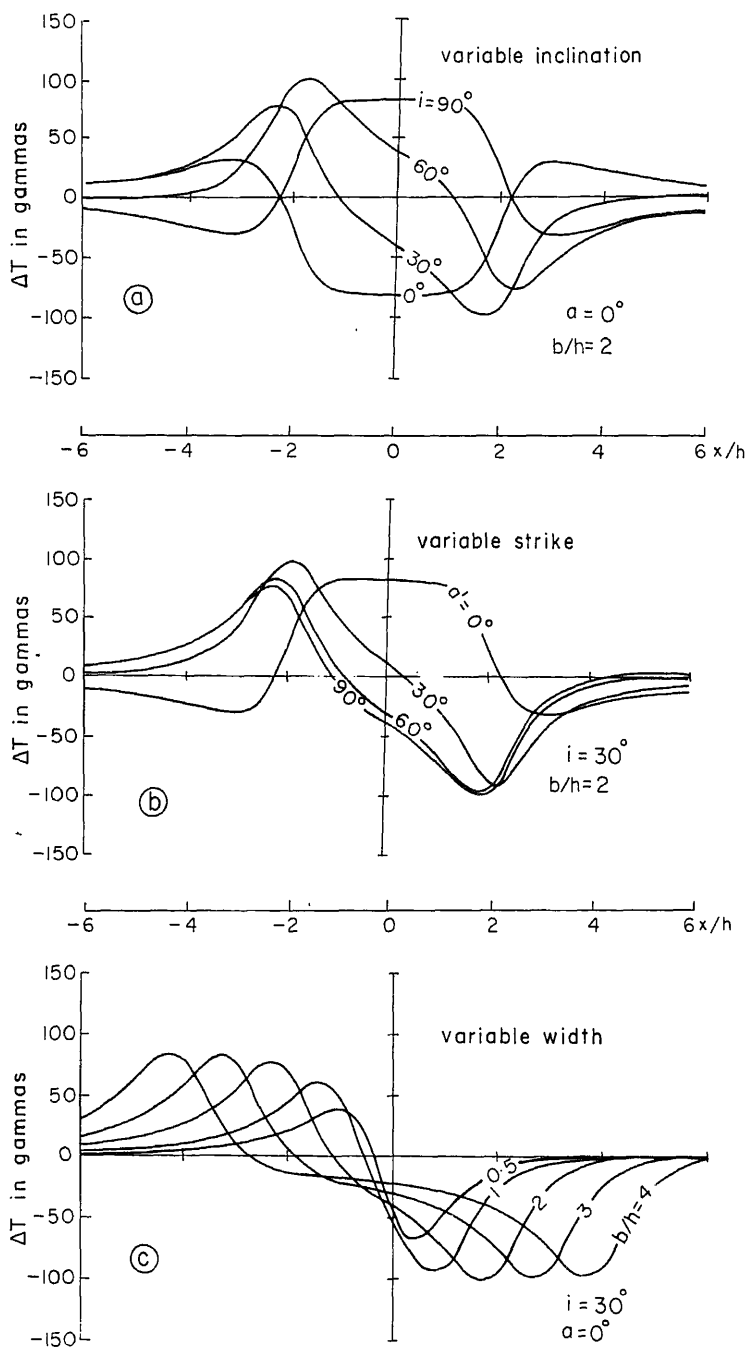


Figure 2. Model magnetic anomalies over a two-dimensional thin plate; (a) variable inclination, (b) variable strike, and (c) variable width.

$$F_d(x) = C_d \left[\cos Q_d \left(\arctan \frac{x+b}{h} - \arctan \frac{x-b}{h} \right) + 0.5 \sin Q_d \ln \frac{(x+b)^2 + h^2}{(x-b)^2 + h^2} \right], \quad (4)$$

where the coefficients C_d and Q_d are defined in table 1 in terms of the inclination (i), azimuth (a) of the profile and dip (δ) of the dike etc. The horizontal derivative $F_{dx}(x)$ may now be written as follows:

$$F_{dx}(x) = C_d \left[\cos Q_d \left(\frac{h}{(x+b)^2 + h^2} - \frac{h}{(x-b)^2 + h^2} \right) + \sin Q_d \left(\frac{(x+b)}{(x+b)^2 + h^2} - \frac{(x-b)}{(x-b)^2 + h^2} \right) \right], \quad (5)$$

when Q_p and Q_d are identical, the following identity may be written from (3) and (5).

$$\frac{F_p(x)}{C_p} = \frac{F_{dx}(x)}{C_d}. \quad (6)$$

If the effective dips of magnetization of the dike and the plate are denoted as I_d and I_p respectively, then under the condition $Q_p = Q_d$, $I_p = I_d - \delta/2$ for ΔT anomaly, and $I_p = I_d - \delta$ for ΔZ and ΔH anomalies.

2.4 Analysis of plate anomalies

The magnetic anomaly [$F_p(x)$] over a plate may now be computed using (3) when the parameters of the plate are known. But, in practice, it is necessary to find the parameters of the plate when $F_p(x)$ is known. We suggest the following two methods for finding the parameters of the plate from $F_p(x)$.

2.5 The pseudo-magnetic potential method

Integrating $F_p(x)$ with respect to x , we obtain the expression for $P(x)$, the pseudo-magnetic potential anomaly as follows:

$$P(x) = \int_{-\infty}^x F_p(x) dx \\ = C_p \left[\cos Q_p \arctan \left(\frac{x+b}{h} - \arctan \frac{x-b}{h} \right) \right]$$

Again under the condition $Q_p = Q_d$ the following identity may be written from equations (4) and (7):

$$\frac{P(x)}{C_p} \equiv \frac{F_d(x)}{C_d}.$$

It may be inferred from (8) that the function $P(x)$ is identical to $F_d(x)$ (except the amplitude factor). Hence, any technique available for interpreting d anomalies can be used to interpret the plate anomalies by making use of function $P(x)$. In the example presented here, the curve matching technique of (1963) is adopted.

2.6 The spectral method

The Fourier transform $\bar{F}(w)$ of $F_p(x)$ may be written as follows:

$$\bar{F}(w) = \int_{-\infty}^{\infty} F_p(x) \exp(-jwx) dx.$$

Using the following transform pairs (Erdelyi 1954)

$$h/(x^2 + h^2) \rightarrow \pi \exp(-wh),$$

and

$$x/(x^2 + h^2) \rightarrow \pi j \exp(-wh),$$

and the Fourier shift theorem, the final expression for $\bar{F}(w)$ the Fourier transform of $F_p(x)$ may be written as follows:

$$\bar{F}(w) = 2j\pi C_p \exp(-wh) \exp(jQ_p) \sin wb. \quad (10)$$

Since, for field data, the origin is unknown, the distances are measured from a reference point which is at a distance D from the origin. In this case, (10) may be rewritten as follows:

$$\bar{F}(w) = 2\pi j C_p \exp(-wh) \exp[j(Q_p - wD)] \sin wb. \quad (11)$$

The amplitude $A(w)$ and phase $\phi(w)$ of $F(w)$ may be written from (11) as follows:

$$A(w) = 2\pi C_p \exp(-wh) \sin wb, \quad (12)$$

$$\text{and} \quad \Phi(w) = \pi/2 + w/D - Q_p. \quad (13)$$

It may be noted from (12) that the amplitude $A(w)$ is an exponentially decaying sinusoid (figure 3). $A(w) = 0$ whenever wb is equal to $n\pi$ ($n = 0, 1, 2, \dots$). $A(w) = 0$ successively at two frequencies w_1 and w_2 , then

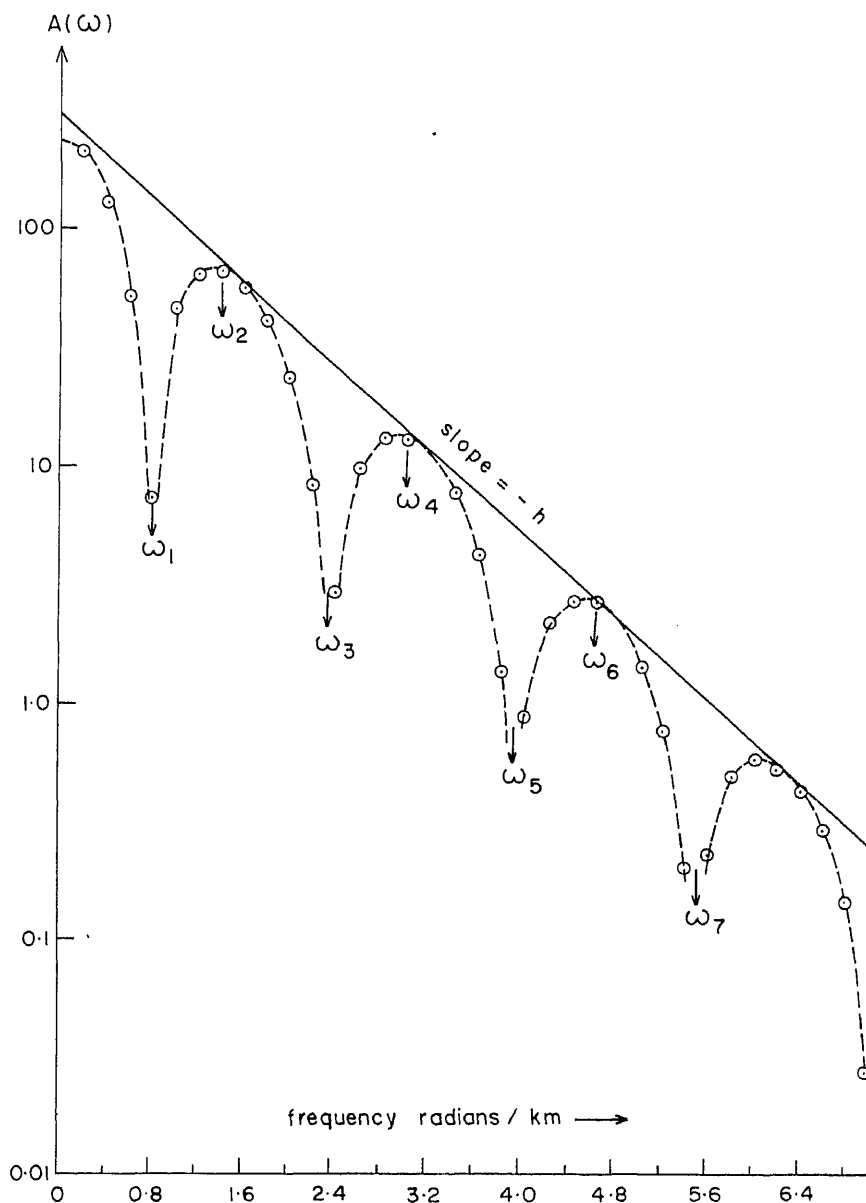


Figure 3. Theoretical Fourier amplitude $[A(\omega)]$ spectrum of the magnetic anomaly over a thin plate.

$$\omega_1 b = n\pi, \text{ and } \omega_2 b = (n+1)\pi;$$

so that

$$b = \pi/(\omega_2 - \omega_1).$$

(14)

$$A_I(w) = 2\pi C_p \exp(-wh). \quad (15)$$

A plot of $\ln A_I(w)$ versus w will be a straight line whose slope and intercept are equal to $-h$ and $\ln(2\pi C_p)$ respectively from which h and C_p may be calculated. After finding C_p , the thickness (t) of the plate may be evaluated using table 1 by assuming the susceptibility contrast (k) or vice versa.

The phase $[\phi(w)]$ spectrum will be a straight line whose slope is equal to D , the distance of the origin from the starting point of the profile. The intercept of this line is equal to $(\frac{1}{2}\pi - Q_p)$.

Thus, using the properties of the Fourier spectra, it is possible to evaluate the parameters of the plate.

3. Example

The aeromagnetic anomaly, 10 km south of Tezpur, (Assam state) in north eastern India is shown in figure 4a. The observations were made along a line making an angle of 45° from the magnetic north towards east at an elevation of 1.36 km above sea level. The inclination, declination and intensity of the earth's magnetic field are 39°N , $1/3^\circ\text{W}$ and 47500 nT respectively. The area is covered by Brahmaputra alluvium overlying the tertiary sediments occurring above the archaean basement. The sediments being non-magnetic cannot produce the observed magnetic anomaly. Hence, this anomaly might be due to the basement topography which may be approximated to a thin plate model. Using the two methods presented in this paper, this anomaly has been interpreted in the following way.

3.1 The pseudo-magnetic potential method

The magnetic anomaly was integrated by successive addition to obtain $P(x)$. The function $P(x)$ is normalized to the same scale (2.5 inches) as the theoretical curves for thick dikes (Gay 1963) and compared by changing the distance scale suitably to obtain a good fit with one of the theoretical curves. Since in this case, $i = 39^\circ$, $a = 45^\circ$ and $I = 50^\circ$ hence, $Q_p = 10^\circ$. The $P(x)$ curve should match with one of the dike curves whose $Q_d = 10^\circ$. It may be seen from figure 4b that the $P(x)$ curve fits well with the dike curve whose Q_d and R (the width-to-depth ratio) values are 10° and 5 (extrapolated curve) respectively. The parameters of the plate obtained in this way are presented in table 2.

3.2 The spectral method

The ΔT anomaly of length 31.5 km is digitized at 0.5 km interval and its discrete Fourier transform is computed. From the amplitude $A(w)$ spectrum (figure 5) of the Fourier transform, the depth, width and susceptibility (for $t = 0.5$ km) of the plate are evaluated and are also shown in table 2. The low susceptibility value of 0.006 c.g.s. is characteristic of granitic basement. It may be inferred from our

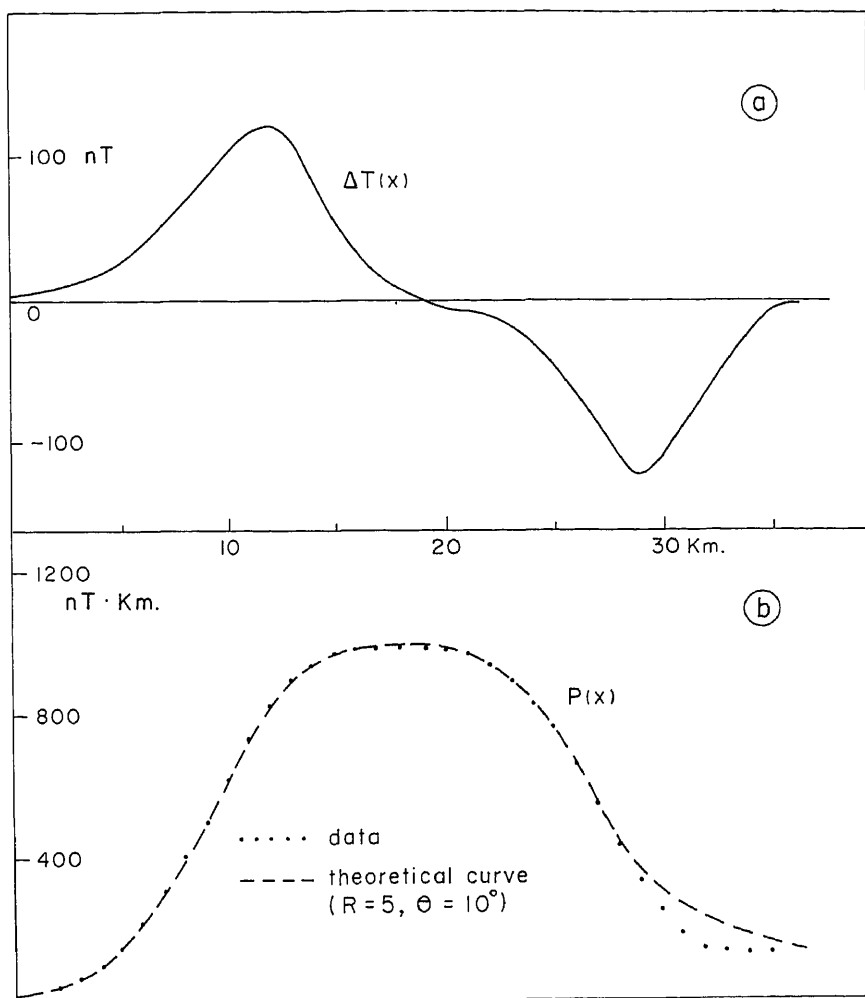


Figure 4. (a) An aeromagnetic anomaly, near Tezpur (Assam state, India), (b) interpretation of $P(x)$ curve using the standard curves of Gay (1963).

Table 2. Results of interpretation of the aeromagnetic anomaly near Tezpur, using the thin plate model.

Parameter	The pseudo magnetic potential method	Spectral method
Depth (h in km) to top (below flight level)	3.36	3.90
Depth (h in km.) to top	2.00	2.54
Width ($2b$ in km)	16.65	15.80
Susceptibility (k) (c.g.s.)	0.006	0.006

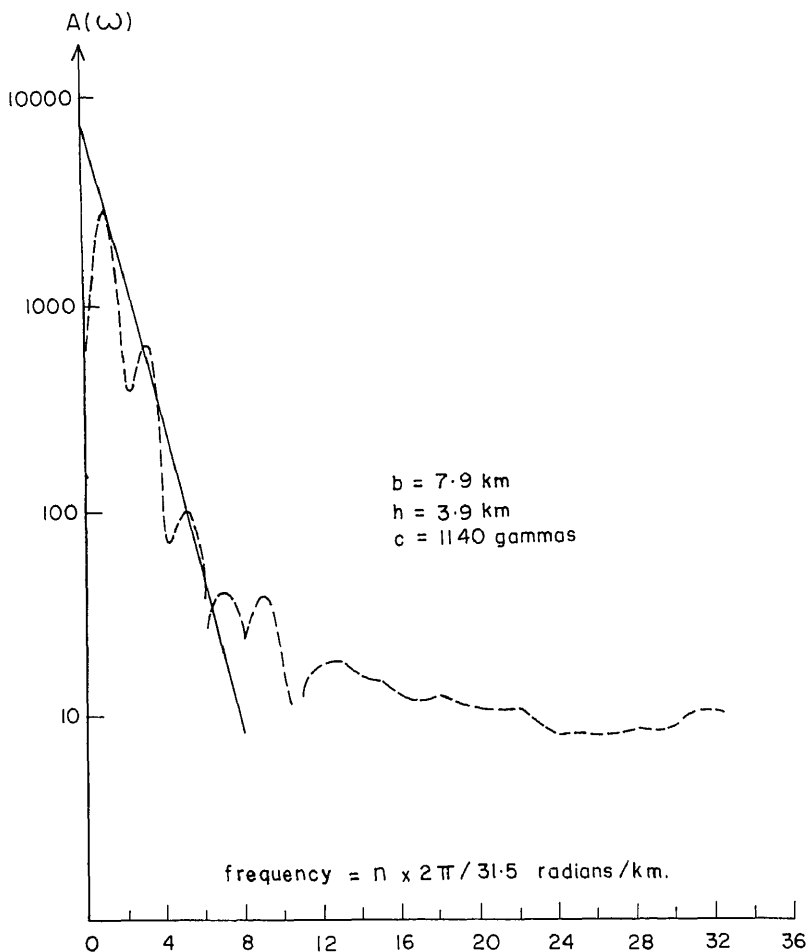


Figure 5. Fourier amplitude $[A(\omega)]$ spectrum of the magnetic anomaly shown in figure 4a.

interpretation that the depth to the basement in this region is around 2.0 km below ground surface.

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Analysis of gravimagnetic anomalies by horizontal gradients

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Abstract. A simple technique for the interpretation of gravimagnetic anomalies by a numerical differentiation is presented. The ratio of horizontal gradient to the observed field was used for the location of the origin and depth to the causative body. This technique was tested by using fundamental expressions due to sphere, horizontal cylinder and vertical fault models and field data over a spherical body. It has been found that the depth to the centre of the body can be determined accurately by this technique.

Keywords. Horizontal gradients; gravimagnetic interpretation; horizontal cylinder; vertical fault.

1. Introduction

Interpretation of gravity anomalies making use of gradients are being increasingly employed. Notable work in this field have been carried out by several workers (Hood 1965; Hood and McClure 1965; Stanley and Green 1976; Radhakrishna Murthy 1980; Achutha Rao *et al* 1981; Gupta 1983).

In the present paper a method is proposed for interpreting some of the gravity and magnetic anomalies by considering only the horizontal gradients. The method is illustrated with a few theoretical and field examples pertaining to sphere, horizontal cylinder and vertical fault. The accuracy of the results obtained by this technique depends upon the accuracy of the initial gravimagnetic data.

2. Basis of method

The vertical component of gravity field (Δg) and that of vertical magnetic field polarized in vertical direction (ΔZ) due to sphere (figure 1a), horizontal cylinder (figure 1b) and vertical fault (figure 1c) are given by Dobrin (1976). The gravity expression in a X-Z plane in general can be defined as

$$\Delta g(x, z) = K' g(x, z) \quad (1)$$

where

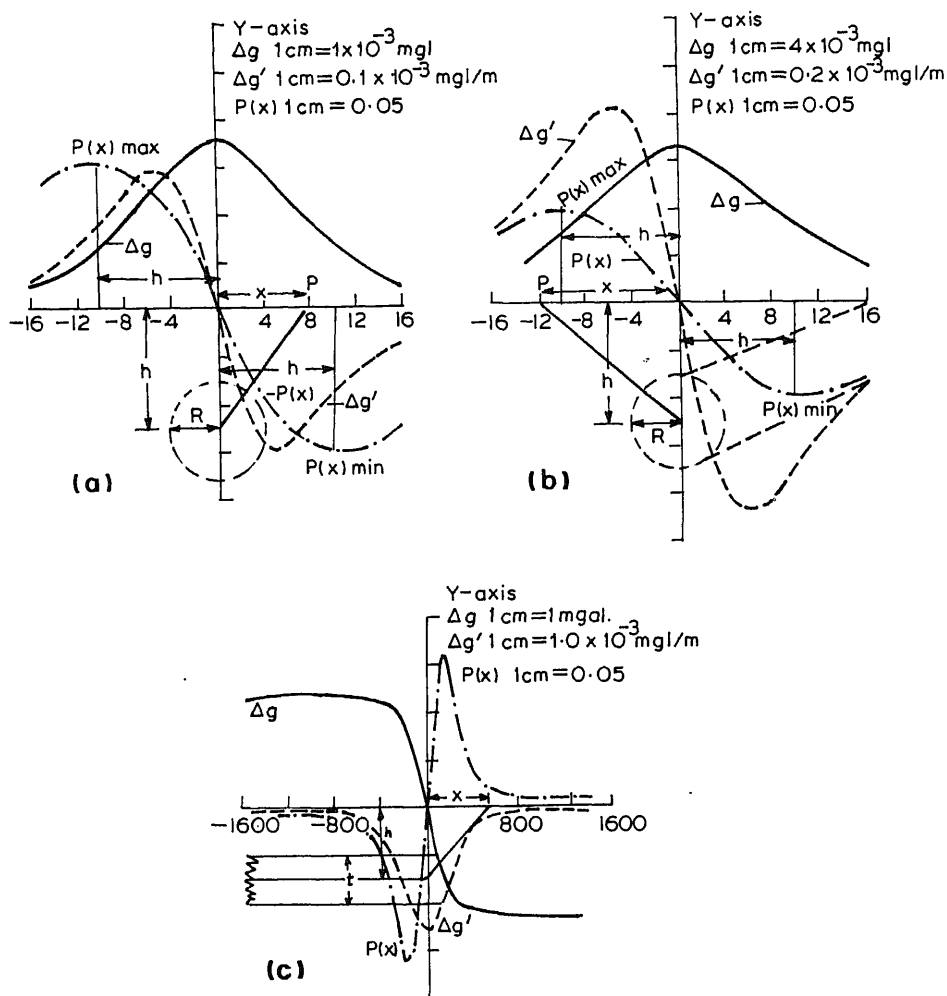


Figure 1. Theoretical curves of Δg , $\Delta g'$ and $P(x)$ for (a) sphere (b) horizontal cylinder and (c) vertical fault gravity models.

Similarly the magnetic expression in a X-Z plane can be written as

$$\Delta Z(x, z) = K_1 Z(x, z) \quad (3)$$

where

$$Z(x, z) = \begin{cases} \frac{(2z^2 - x^2)}{(x^2 + z^2)^{5/2}} & K_1 = 4.19 \times 10^5 R^3 I \quad \text{for a sphere,} \\ \frac{(z^2 - x^2)}{(x^2 + z^2)^2} & K_1 = 6.28 \times 10^5 R^2 I \quad \text{for a horizontal cylinder,} \\ \frac{x}{(x^2 + z^2)^{3/2}} & K_1 = 2 \times 10^5 I \quad \text{for a vertical} \end{cases} \quad (4)$$

where G denotes the universal gravity constant, σ the density contrast, R the radius, I the magnetization, Z the depth to the body, t the thickness of the fault, $\Delta g(x)$ and $\Delta Z(x)$ are the vertical component of gravity and magnetic intensity in milligals and gammas respectively.

By differentiating (2) and (4) with respect to x , the respective horizontal gradients for gravity are

$$\Delta g'(x, z) = K' g'(x, z), \quad (5)$$

where

$$g'(x, z) = \begin{cases} zx/(x^2 + z^2)^{5/2} & K' = -4\pi GR^3\sigma & \text{for a sphere} \\ zx/(x^2 + z^2)^2 & K' = -4\pi GR^2\sigma & \text{for a horizontal cylinder} \\ z/(x^2 + z^2) & K' = 2Gt\sigma\pi & \text{for a vertical fault.} \end{cases} \quad (6)$$

Similarly for a magnetic case

$$\Delta Z'(x, z) = K'_1 Z'(x, z), \quad (7)$$

where

$$Z'(x, z) = \begin{cases} \frac{3x(x^2 - 4z^2)}{(x^2 + z^2)^{7/2}} & K'_1 = 4.19 \times 10^5 R^3 I & \text{for a sphere,} \\ \frac{x(x^2 - 3z^2)}{(x^2 + z^2)^2} & K'_1 = 12.56 \times 10^5 R^2 I & \text{for a horizontal cylinder,} \\ \frac{(z^2 - x^2)}{(x^2 + z^2)^2} & K'_1 = 2 \times 10^5 I t & \text{for a vertical fault,} \end{cases} \quad (8)$$

In each case, dividing the first horizontal derivative of the field by its corresponding gravity and magnetic field, we obtain in the case of gravity

$$P(x) = \Delta g'(x, z) / \Delta g(x, z), \quad (9)$$

where

$$P(x) = \begin{cases} -3x/(x^2 + z^2) & \text{for a sphere} \\ -2x/(x^2 + z^2) & \text{for a horizontal cylinder} \\ z/(\pi/2 - \tan^{-1}(x/z))(x^2 + z^2) & \text{for a vertical fault} \end{cases} \quad (10)$$

In the case of magnetic,

$$Q(x) = \Delta Z'(x, z) / \Delta Z(x, z) \quad (11)$$

where

$$Q(x) = \begin{cases} \frac{3x(x^2 - 4z^2)}{(x^2 + z^2)(2z^2 - x^2)} & \text{for a sphere,} \\ \frac{2x(x^2 - 3z^2)}{(x^2 + z^2)(z^2 - x^2)} & \text{for a horizontal cylinder,} \\ \frac{(z^2 - x^2)}{x(x^2 + z^2)} & \text{for a vertical fault,} \end{cases} \quad (12)$$

where $\Delta g'(x, z)$ and $\Delta Z'(x, z)$ are their respective derivatives with respect to x . $P(x)$ is the ratio of $(\Delta g'/\Delta g)$ and $Q(x)$ is the ratio of $(\Delta Z'/\Delta Z)$.

Using (11) and (12) z can be obtained for gravity as:

$$Z = \begin{cases} \left(\frac{-3x}{P(x)} - x^2 \right)^{1/2} & \text{for a sphere,} \\ \left(\frac{-2x}{P(x)} - x^2 \right)^{1/2} & \text{for a horizontal cylinder,} \\ \frac{2}{\pi P(x)} & \text{for vertical fault} \\ & \text{when } x = 0. \end{cases} \quad (13)$$

In the case of magnetic

$$Z = \begin{cases} \pm x/2 & \text{for sphere at } Q(x) = 0, \\ \pm x/(3)^{1/2} & \text{for a horizontal cylinder } Q(x) = 0, \\ \pm x & \text{for vertical fault } Q(x) = 0. \end{cases} \quad (14)$$

3. Theoretical examples

The technique presented is illustrated with the construction of theoretical curves based on (2), (4), (6) and (8) for sphere, horizontal cylinder and vertical fault. The first horizontal derivatives of the gravimagnetic effects are shown in figures 1a, 1b and 1c and figures 2a, 2b and 2c respectively, which also include the plots of $P(x)$ and $Q(x)$ values.

With the knowledge of the $P(x)$ and $Q(x)$ and using equations (13) and (14) z can be evaluated for any value of x .

In addition to the above analysis the depth of sphere and horizontal cylinder can also be determined from $P(x)$ and $Q(x)$ values. The values of abscissa at which the function acquires an extrema may be found from the condition $dP(x)/dx$ and $dQ(x)/dx = 0$.

The functions $P(x)$ and $Q(x)$ for sphere and horizontal cylinder attain the maximum when x is negative and minimum when x has positive values. The distance from the origin to the maxima or minima of the $P(x)$ and $Q(x)$ curves

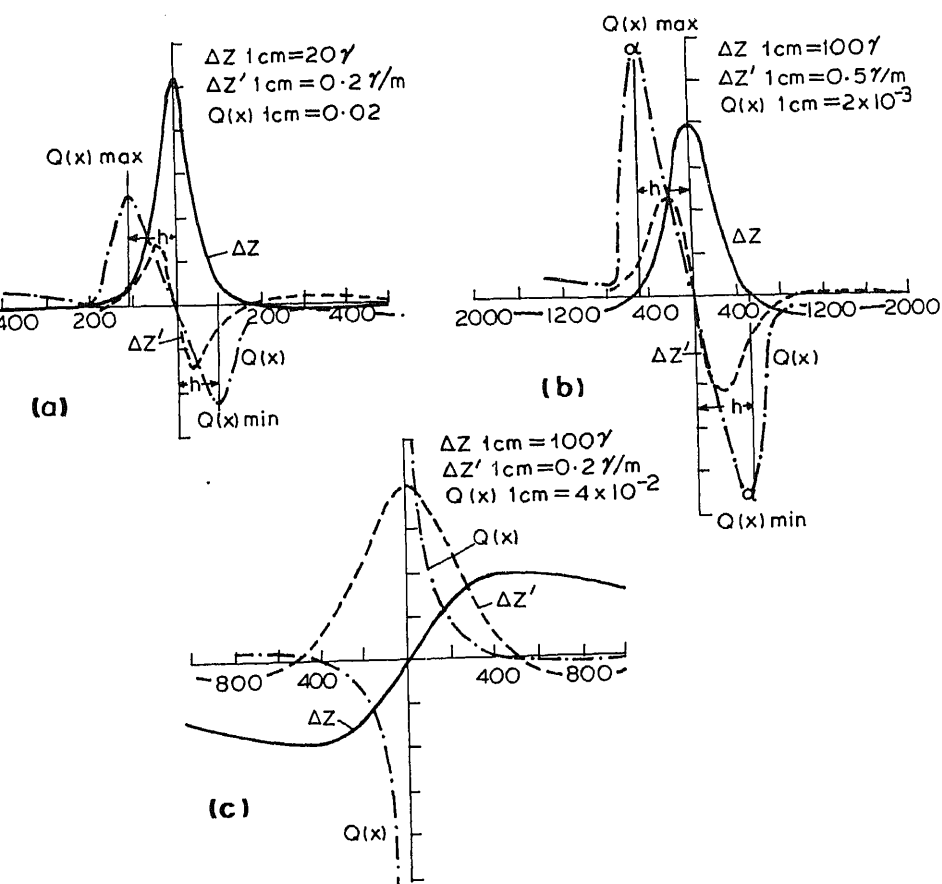


Figure 2. Theoretical curves of ΔZ , $\Delta Z'$ and $Q(x)$ for (a) sphere (b) horizontal cylinder and (c) vertical fault magnetic models.

horizontal cylinder (figure 1b) in the case of gravity and magnetic sphere (figure 2a) whereas in the case of magnetic horizontal cylinder (figure 2b), the distance from origin to maximum ($Q(x) \rightarrow \infty$) or minimum ($Q(x) \rightarrow -\infty$) gives the depth to the centre of the cylinder.

4. Field examples

The applicability of the technique is tested with field data pertaining to the gravity anomaly over salt dome shown in figure 3a (Nettleton 1976) and the magnetic anomaly over profile CC' on the vertical intensity anomaly map of the Bankhura (Verma and Bandyopadhyay 1975) and are shown in figure 3b. In both cases,

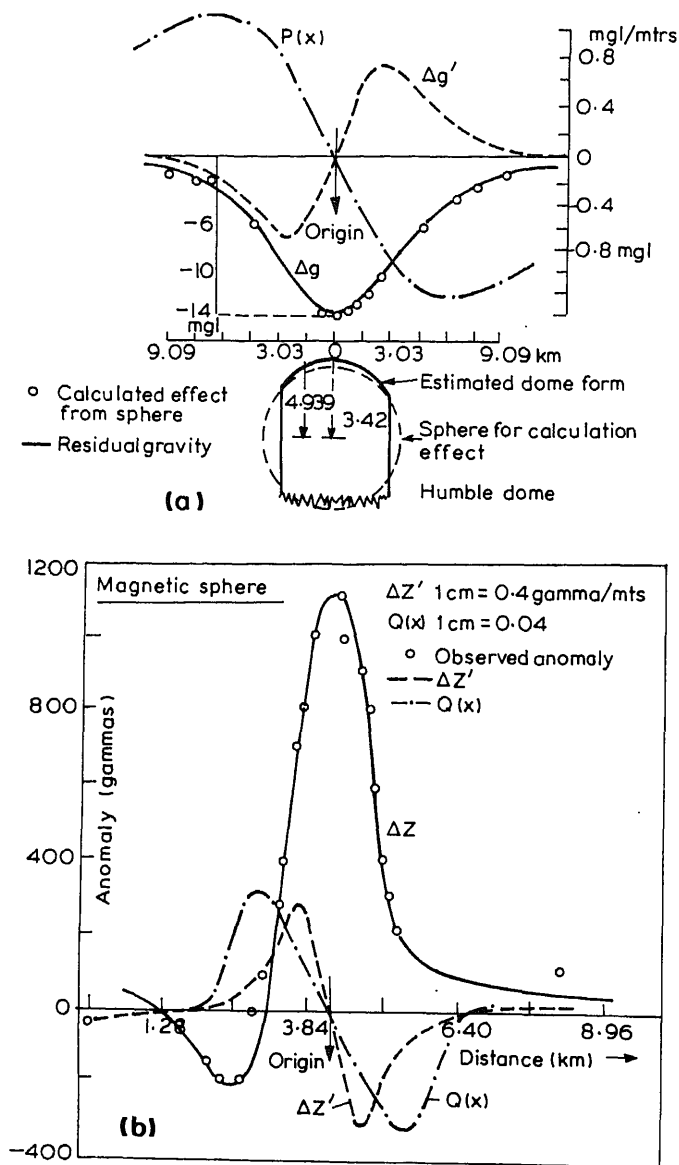


Figure 3. Illustration of horizontal gradients techniques over (a) gravity and (b) magnetic field examples.

5. Discussion of the results

It was verified numerically that (13) and (14) give the exact value of z when using synthetic data. After adding $\pm 5\%$ random error in the synthetic data (table 2), the depth obtained by the present method is within $\pm 5\%$ error of the exact value.

Table 1. Comparative study of depths

Parameters	Gravity (km)		Magnetics (km)		Other values (Mohan <i>et al</i> 1982)
	Calculated values (Present method)	Reported values (Nettleton 1976)	Calculated values (Present method)	Reported values (Rao 75)	
h	4.94	4.94	1.31	1.31	1.30
R	3.42	3.42	1.09	1.00	1.00

Table 2. Theoretical gravity (sphere) results

Model depth (in m)	Using synthetic data (in m)	Using data with random errors $\pm 5\%$ (in m)	Percentage of error
100	100	102	1.96
200	200	198	-1.01
300	300	310	3.23
400	400	397	-0.76
500	500	503	0.63
600	600	609	1.64
800	800	801	0.12
1000	1000	1050	4.76

From the above analysis it is evident that the characteristics of gradients $P(x)$ and $Q(x)$ curves in gravimagnetic analysis can provide fairly accurate estimates of the depths and other parameters of the buried objects.

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Palaeo-tectonic environment of the Precambrian basaltic mafic dykes on the application of chemical discriminants

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Abstract. Dykes of Chitaldurg and Newer Dolerite I are OFB and represent early attempts to rift and to generate oceanic crust. Shimoga dykes are CAB and belong to orogenic phase. Guntur dykes and Ananthapur–Chittoor dykes are OIB and these are related to ‘hot spot’. Newer Dolerite II is CONB and belongs to healing and filling of fractured crust. The Cuddapah dykes are transitional to all magma types and these represent a compound event of rifting and filling of fractured continental crust.

Keywords. Palaeo-tectonic environment; mafic dykes; chemical discriminant.

1. Introduction

The advent of plate tectonics paradigm has stimulated the earth scientists to classify the basaltic magma types in terms of tectonic setting of the Phanerozoic volcanics. The principal tectonic settings and the related basaltic magma types are as follows: (1) Converging plate margin as in the subduction zone of island arc basalts (IAB) characterized by early low potassium tholeiite (LKT) followed mainly by calc-alkaline basalt (CAB) and minor shoshonite (SHO), (2) diverging plate margin as in the mid-oceanic ridge basalt (MORB) characterized by low potassium tholeiitic basalt enriched in Cr, Ni and impoverished in lithophile elements, (3) within plate ocean island basalts (OIB), characterized by alkali basalt and tholeiitic basalt, (4) within plate continental basalt (CONB), characterised by tholeiitic basalt and alkali basalt. The OIB and CONB are together described as within plate basalts (WPB). The marginal basin basalts (MBB), though relatively impoverished in Ni, Cr and relatively enriched in lithophile elements are considered similar to mid-oceanic ridge basalts (MORB), on the basis of major element composition and these together are described as the ocean floor basalts (OFB). The orogenic basalts (ORNB) are calc-alkaline basalts and these are considered similar to and equated to island arc basalts (IAB).

The observed chemical variations of the basaltic magma of different tectonic environments, whether these are due to dynamical partial melting of a homogeneous mantle source or due to heterogeneity of the mantle, is one of the most actively and lively debated topics in the earth sciences (Hanson 1977; Langmuir *et al* 1977, 1980; Hawkesworth *et al* 1977; O’Nions *et al* 1978, 1979, 1981; Sun 1980; Tarney *et al* 1980; Allégre *et al* 1980; Dupre *et al* 1981; Allégre 1982;

The current ideas on the basaltic magma are that the source of MORB is the Low Velocity Zone (LVZ) of the mantle which is severely depleted in highly incompatible elements Cs, Ba, Rb, U, Th, K, La and Nb and the source of OIB is the Lower Mantle (possibly generated through plume-hot spot) which is relatively undepleted in the incompatible elements compared to the whole Mantle while the source of IAT is in the Uppermost Mantle separated from the source of MORB by the subducted lithosphere of oceanic basalt and sediment and is not as depleted as MORB in the incompatible elements. The ratios and the relative variations of $\text{Sr}_{87}/\text{Sr}_{86}$, $^{207}\text{Pb}/\text{Pb}_{204}$, $^{206}\text{Pb}/\text{Pb}_{204}$, $^{143}\text{Nd}/^{144}\text{Nd}$ have been extensively used; the depleted mantle is generally richer in the ratio of $^{143}\text{Nd}/^{144}\text{Nd}$ and poorer in $\text{Sr}_{87}/\text{Sr}_{86}$ while the converse is the case with the undepleted mantle.

Certain groups of elements which are immobile or less mobile in the secondary processes and whose relative concentrations reflect the compositional differences among the magma types erupted at different tectonic environments have been proposed as chemical discriminants by way of diagrams with demarcated fields of environments (Pearce and Cann 1973; Pearce *et al* 1975, 1977; Pearce and Norry 1979; Floyd and Winchester 1975; Pearce 1976, 1982; Wood *et al* 1979; Wood 1980; Thompson *et al* 1980; Gill 1984). The ratios of some elemental pairs, Th/Hf, Zr/Nb, La/Nb, Ba/La, Sr/Nd have also been found helpful as discriminants.

The application of chemical discriminants has been extended to Precambrian metavolcanics. The Precambrian mafic dykes (and other continental basalts) show certain features which merit a closer examination of their palaeo-tectonic environments from the application of chemical discriminants. An attempt is therefore made here to assess the palaeo-tectonic environments of the Precambrian mafic dykes wide-spread in parts of peninsular India.

2. Mafic dykes selected and their geological setting

Seven sets of Precambrian mafic dykes from parts of peninsular India are selected for which the geologic setting is known reasonably well. Though the mafic dykes in some of these areas contain rock types of alkali basalt composition in addition to tholeiitic basalt composition, the former are omitted as they are not important in assessing the tectonic setting (Pearce and Cann 1973; Floyd and Winchester 1975). The selected seven sets satisfy the sub-alkaline nature of the magma as per the discriminant function of Le Maitre (1968). According to this function, $Z = -0.177 \text{SiO}_2 + 0.698 \text{TiO}_2 + 0.418 \text{Na}_2\text{O} + 0.569 \text{K}_2\text{O}$ should have an arithmetic value of less than -2.38 for sub-alkaline rocks. This condition also satisfies the requirement for selecting the analyses for plotting on various diagrams based on chemical discriminants to infer the palaeo-tectonic environment.

The following are the seven sets of mafic dykes selected:

- (a) Mafic dykes of Chitaldurg belt, Karnataka (quartz-normative) (Naqvi *et al* 1972, 1974),
- (b) Mafic dykes of Shimoga belt, Karnataka (quartz-normative) (Naqvi *et al* 1972, 1974),
- (c) Newer Dolerite I of Singhbhum belt (Saha *et al* 1973),
- (d) Newer Dolerite II of Singhbhum belt (Saha *et al* 1973),

- (e) Mafic dykes of Guntur, northeast of Cuddapah basin, A.P. (Ramamohana Rao *et al* 1984),
- (f) Mafic dykes of Ananthapur (Lakshmi Reddy and Murty 1976) and mafic dykes of Chittoor (Chakrapani Naidu and Jayakumar 1980) put together as one set and these are to the west and south of Cuddapah basin, A.P.
- (g) Dyke rocks of Cuddapah basin, A.P. (Vemban 1946).

The trace element data for the sets f and g are taken from Sankaran (1964), and correspond to the ones given for Tiruthani dykes and Cuddapah dykes respectively. The major elements of these sets have been analyzed by classical gravimetric methods of Groves (1937) (sets c, d, e and g) and by rapid methods of Shapiro and Brannock (1962) (sets a, b and f). The trace element determinations on all the sets have been made on emission spectrograph (Ahrens and Taylor 1961). Details of precision and accuracy of these analyses given by the above authors suggest a good degree of reliability of the analytical data.

Figure 1 is a sketch map of peninsular India showing the locations of the areas of the mafic dykes studied. Table 1 gives the chemical composition of the mafic dykes. The precise age data on all the mafic dykes are not yet available. The dykes of Chitaldurg and of Shimoga cut across the metavolcanics, metasediments and granites of these belts. The granites of these belts gave Rb/Sr age of 2500 Ma

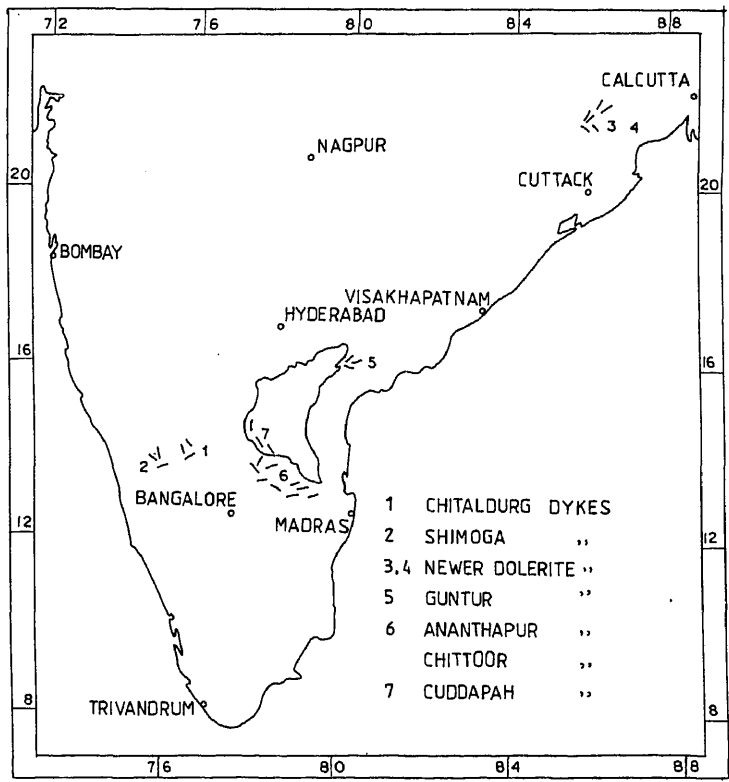


Figure 1. Sketch map showing the locations of the 7 sets of mafic dykes.

Table 1. Major (%) and trace (ppm) element composition and calculated chemical parameters of the seven sets of mafic dykes selected.

	Chitaldurg (AV.5) (1)	Shimoga (AV.7) (2)	Newer Dolerite I (AV.6) (3)	Newer Dolerite II (AV.9) (4)	Guntur (AV.8) (5)	Anantapur- Chittoor (AV.6) (6)	Cuddapah (AV.7) (7)
SiO ₂	52.42	53.52	51.16	51.21	51.03	48.79	48.79
TiO ₂	1.38	1.03	0.79	0.96	1.05	1.84	1.42
Al ₂ O ₃	14.88	14.49	12.73	14.49	14.89	14.51	14.50
Fe ₂ O ₃	2.85	1.72	2.15	2.43	3.07	1.69	3.68
FeO	8.07	9.26	8.42	9.22	8.73	10.36	9.75
MnO	0.02	0.06	0.12	0.19	0.23	0.18	0.41
MgO	7.06	5.74	9.26	6.22	7.91	7.31	6.38
CaO	7.36	8.45	8.77	9.47	9.21	11.56	9.21
Na ₂ O	3.07	2.56	2.74	2.51	1.97	2.15	2.67
K ₂ O	0.66	0.84	0.73	1.00	0.63	0.61	0.63
P ₂ O ₅	0.47	0.17	0.30	0.40	0.46	0.20	0.17
H ₂ O ⁺	—	—	3.18	2.17	0.68	0.70	2.21
Total:	98.24	97.84	100.35	100.27	99.86	100.12	99.82
Co	67	46	78	67	42	54	77
Cr	177	121	858	43	265	57	372
Cu	59	39	96	164	—	105	341
Ga	nd	nd	20	23	11	4	4
Ni	163	93	333	192	122	39	77
Sr	nd	nd	77	122	nd	287	770
Ti	8273	6175	4736	5755	6294	11031	8513
V	226	215	213	193	350	90	203
Y	18	27	20	20	26	3	3
Zr	98	109	67	68	64	100*	100*
Z(-)	6.66	7.21	6.94	7.02	7.12	6.52	6.16
F ₁	0.411	0.438	0.403	0.414	0.419	0.302	0.3471
F ₂ (-)	1.504	1.452	1.519	1.452	1.504	1.530	1.4405
F ₃ (-)	2.387	2.337	2.333	2.324	2.342	2.369	2.2848
Ni/Co	2.43	2.02	4.26	2.88	2.90	0.72	1.00
Zr/Y	5.4	4.03	3.35	3.40	2.46	33.3	33.3

*Value given for one sample is taken for the group as a whole.

(Crawford 1969). Drury (1984) quotes a model Sm-Nd age of 1700 Ma for the dykes of central Karnataka. The Newer Dolerite I and II are intrusive into Singhbhum granite which gave Rb/Sr age of 2950 Ma (Sarkar *et al* 1979). The Newer Dolerite I and II represent two generations of dyke activity probably spread over from ca 1700 Ma to ca 950 Ma (Saha *et al* 1973). The dykes of Guntur, Anantapur-Chittoor cut across the pink granites which are considered older than the Cuddapah sediments. The sills and dykes of the Cuddapah occur as concordant and discordant bodies predominantly within the Vempalle Dolomite Formation. These mafic rocks gave Rb/Sr age of 1583 ± 143 Ma (Crawford and Compston 1973). Some dykes of the Cuddapah cutting across the Vempalle Dolomite Formation and also the Tadepatri Shale Formation gave Rb/Sr age of 980 ± 110 Ma (Crawford and Compston 1973). The above age data on the mafic dykes broadly fall within the Middle Proterozoic.

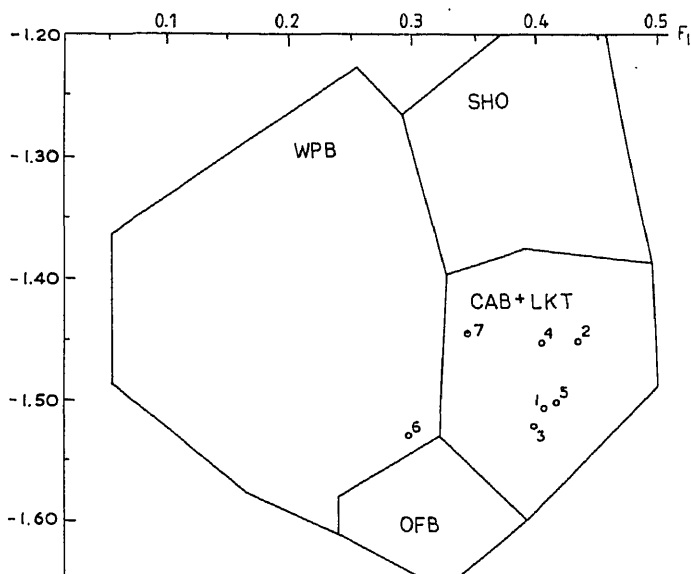
The dyke rocks have prominent trends in different regions. The mafic dykes of Chitaldurg and Shimoga have dominant NNW to NW trend with subsidiary NNE and ENE trend. The Newer Dolerite dykes I and II have dominant NNE to NE trend with a subsidiary NW trend. The Guntur, Ananthapur-Chittoor mafic dykes have dominant ENE trend with subsidiary NNE and NW trend. The Cuddapah dykes have NW or basal N-S trend.

3. Chemical discriminants applied and the palaeo-tectonic environment of the mafic dykes

The chemical data of the seven sets of Precambrian mafic dykes presented in table 1 are utilized in the calculation of chemical discriminants. These are plotted in figures 2 to 6 to arrive at the magma type and the related tectonic environment. In all the diagrams, the points plotted and numbered 1 to 7 correspond to the mafic dykes as follows: 1. Chitaldurg, 2. Shimoga, 3. Newer Dolerite I, 4. Newer Dolerite II, 5. Guntur, 6. Ananthapur-Chittoor, 7. Cuddapah.

The $\text{TiO}_2\text{-K}_2\text{O-P}_2\text{O}_5$ diagram (Pearce *et al* 1975) discriminates the 'primitive' basalts (Jamieson and Clarke 1970, p.183) into ocean floor basalts (OFB) and non-ocean floor basalt (NOFB). All the seven sets plot in the non-ocean floor basalts which correspond to continental basalts.

The functions $F_1\text{-}F_2$ diagram (Pearce 1976) discriminates (figure 2) the basalts of OFB from WPB (which includes OIB) and also those of island arc (CAB + LKT, SHO). Only Ananthapur-Chittoor plots in WPB and the other six plot in

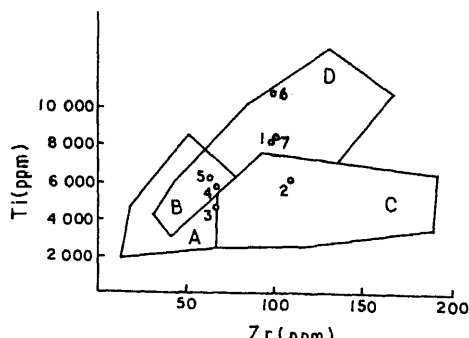


(CAB + LKT) field. The functions F_2 - F_3 diagram (Pearce 1976) discriminates CAB from LKT and also SHO of island arc basalts and also those of OFB. The above six sets now plot in LKT. According to Pearce (1976), the basalts which have $F_1 < 0.33$ are WPB. He further proposes, on the basis of study of basalts of SW England and those of Vesuvius, that the basalts which have $F_1 < 0.33$ may be of 'hot-spot' origin and those > 0.33 may be of island arc origin. The analogy of 'hot-spot' origin to Ananthapur-Chittoor dykes which have $F_1 < 0.33$ is referred to later.

The Ti-Zr-Y, Ti-Zr and Ti-Zr-Sr diagrams (Pearce and Cann 1973) are applicable to fresh and/or altered basaltic rocks, discriminate OFB, CAB, LKT, WPB (which includes OIB) with some overlap in certain region of Ti-Zr diagram. In Ti-Zr-Y diagram, Chitaldurg plots in WPB but close to OFB field and Shimoga plots in CAB. Newer Dolerite I and II and Guntur plot in OFB field. Ananthapur-Chittoor and Cuddapah plot outside the fields. In Ti-Zr (figure 3), Chitaldurg plots in OFB. Shimoga plots in CAB. Newer Dolerite I plots in LKT. Newer Dolerite II and Guntur plot in the overlapping region OFB, LKT, CAB. But Ananthapur-Chittoor and Cuddapah plot in OFB. The Ti-Zr-Sr diagram applicable to fresh rocks, four sets for which Sr data are available are plotted. Newer dolerite I and II plot in OFB and Ananthapur-Chittoor and Cuddapah plot in LKT. The plots of Ti-Zr-Y, Ti-Zr and Ti-Zr-Sr when read together suggest that Shimoga is CAB, while Chitaldurg is not. Others are LKT or OFB or WPB.

The $\text{MgO}/\text{Al}_2\text{O}_3$ -($\text{Na}_2\text{O} + \text{K}_2\text{O}$)/Total $\text{FeO} + \text{TiO}_2$ diagram (Green 1973) gives the trend lines of oceanic tholeiite, ocean island basalt (Hawaiian), besides a demarcated field of calc-alkaline rocks (figure 4). Only Chitaldurg plots right on oceanic tholeiitic trend. Newer Dolerite I plots towards the oceanic tholeiite side. Shimoga, Newer Dolerite II, Ananthapur-Chittoor, Guntur and Cuddapah plot on a line in between the oceanic tholeiite trend and the ocean island (Hawaiian) trend. Among these Shimoga plots closer to the demarcated field of calc-alkaline rocks.

The FeO^* (Total)- $\text{MgO}-\text{Al}_2\text{O}_3$ diagram (Pearce *et al* 1977) discriminates (figure 5) effectively subalkaline basalts into OFB, OIB, CONB and orogenic (equated to Island Arc: ORNB = IAB) basalts. In this diagram OIB field represents Hawaiian type of basalts occurring on the oceanic crust of WPB and CONB (continental) basalts represent rift type basalts of WPB (Pearce *et al* 1977). In figure 5,



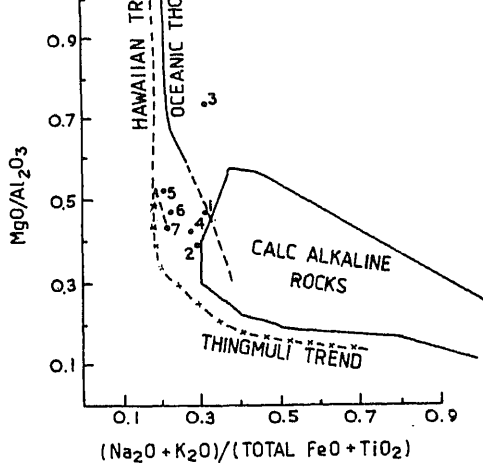


Figure 4. $\text{MgO}/\text{Al}_2\text{O}_3$ – $(\text{Na}_2\text{O} + \text{K}_2\text{O})/(\text{Total FeO} + \text{TiO}_2)$ diagram (after Green 1973).

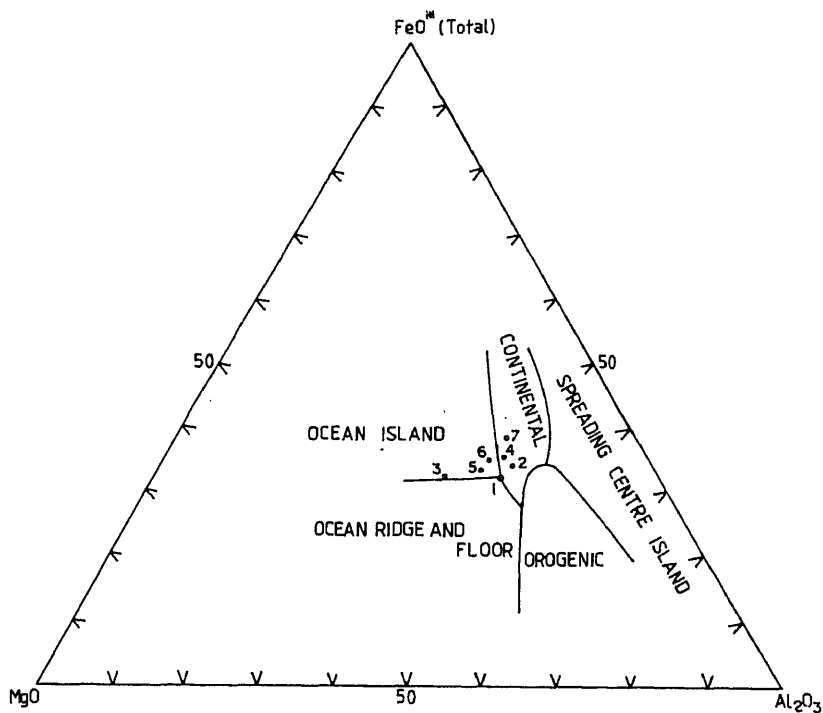


Figure 5. $\text{FeO}^* (\text{Total})$ – MgO – Al_2O_3 diagram (after Pearce *et al* 1977).

Chitaldurg plots at the tri-junction of OFB, OIB and CONB, but Shimoga plots in OIB. Newer Dolerite I plots on the border line of OFB and OIB and Newer Dolerite II plots in CONB. Guntur and Anantapur–Chittoor plot in OIB while Cuddapah plots in CONB. None of the sets plot in the orogenic field, and according to the authors it effectively discriminated only 55% of the data they used. This discriminant diagram brings out effectively the difference in the tectonic settings of Chitaldurg from Shimoga, Newer Dolerite I from Newer Dolerite II and to a good extent the Guntur, Ananthapur–Chittoor from Cuddapah.

The New Ti-Zr diagram (figure 6) (Pearce 1982) gives the fields of the tectonic environment for the basalt in terms of MORB, Arc (IAT) and WPB with some overlap. In figure 6, Chitaldurg plots in MORB but Shimoga plots in Arc (= IAT); Newer Dolerite I plots in MORB but Newer Dolerite II plots in Arc/MORB; Guntur also plots in Arc/MORB; but Ananthapur–Chittoor and Cuddapah plot in WPB/MORB.

4. Interpretation of the Palaeo-tectonic environment

The magma types obtained from the application of chemical discriminants and their plottings in figures 2 to 6 suggest the following tectonic environments for the seven sets of mafic dykes.

The Chitaldurg set of dykes does plot variously and characteristically it plots at the tri-junction of OFB, OIB and CONB in figure 5 and in OFB in figure 6. The high Cr and Ni and Co with a Ni/Co ratio of 2.4 (table 1) is suggestive of ocean floor basalt affinity (Condie 1976). It does plot on the trend of oceanic tholeiite in figure 4. Naqvi *et al* (1972, 1974), based on field and other studies assumed that the Chitaldurg dykes are derived from a single magmatic source beneath them while Shimoga dykes are not. The high Cr also supports MORB affinity. But the high Zr combined with low Y giving Zr/Y of 5.4 for Chitaldurg dykes suggest a continental basalt affinity (Wood *et al* 1979). In view of the overall OFB affinity of Chitaldurg

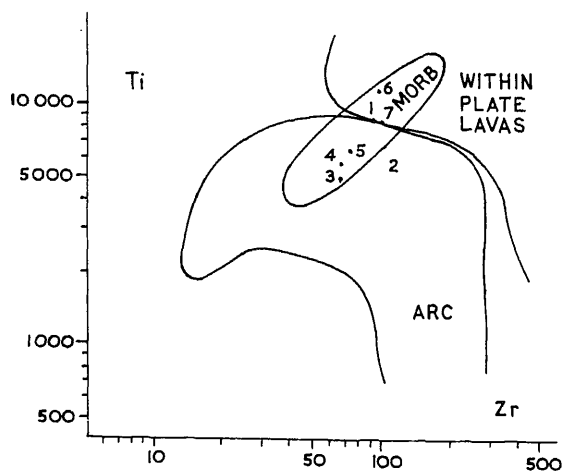


Figure 6. Ti-Zr diagram (after Pearce, 1982).

dykes, it is suggested that these represent an attempt to rift and to generate oceanic crust in the southern peninsular India.

The Shimoga set of dykes shows a consistent affinity to CAB (which is related to Island Arc equated to orogenic basalts) (figures 2, 3, 4 and 6 and on Ti-Zr-Y diagram). Also, the high Co in relation to Ni (table 1) is characteristic of island arc basalts (Condie 1976) to which CAB is related. In view of this affinity it is suggested that the Shimoga dykes represent orogenic intrusive environment following the phase of folding.

Newer Dolerite I and II which differ petrologically (Saha *et al* 1973) show up their differences in magma types. Newer Dolerite I shows affinity to OFB (figures 5 and 6 and on Ti/100-Zr-Sr/2 diagram). Further its high Cr and high Ni/Co (table 1) is suggestive of OFB. The Newer Dolerite II shows affinity to CONB (figure 5) or CAB (figure 4). These differences combined with the postulated period of intrusions from 1700 Ma to 1000 Ma (Saha *et al* 1973) suggest that Newer Dolerite I represents an attempt to rift and to generate oceanic crust at the beginning of Middle Proterozoic and that Newer Dolerite II represents healing and filling of the fractured crust by the end of Middle Proterozoic in the Singhbhum region.

The Guntur and Ananthapur-Chittoor dykes have similarities wherein both show dominant OIB character (figure 5) and the latter has a clear tendency to WPB (figures 2, 6). These characters combined with low Ni/Co and F_1 value less than 0.33 for Ananthapur-Chittoor dykes strongly suggest a 'hot-spot' environment for these dykes. These occur profusely at the Cuddapah-Archaeon boundary (figure 1). Burke and Dewey (1973) proposed a 'hot-spot' in the area of Guntur and intraplate seismicity in the areas is confirmed (Chandra 1977). The idea of Drury and Holt (1980) and Drury (1984) of lithospheric doming due to thermal event before the initiation of Cuddapah Basin fits well into the presently suggested 'hot-spot' environment with intraplate ocean island basalt affinity of the dykes of Ananthapur-Chittoor and also those of Guntur.

The Cuddapah dykes have transitional characters of OFB (MORB) (figures 3, 6), LKT/CAB (figure 2) and CONB (figure 5). Their low Ni/Co negates purely OFB character. The transitional nature of the magma type suggests that the Cuddapah dykes may represent compound events of rifting and of healing of the fractured continental crust.

5. Conclusions

The mafic dykes widespread in parts of peninsular India covering a period from 1700 Ma to 980 Ma give important clues to the nature of thermal events of the mantle during the whole of Middle Proterozoic. The results show that Chitaldurg and Newer Dolerite I have OFB affinity, Shimoga CAB, Guntur, Ananthapur-Chittoor OIB, Newer Dolerite II CONB and Cuddapah transitional to all types. An attempt to rift by Chitaldurg and Newer Dolerite I are inferred. A 'hot-spot' activity leading to lithospheric doming by Guntur, Ananthapur-Chittoor dykes is proposed. Shimoga dykes are orogenic while Newer Dolerite II is fracture filling type. The Cuddapah dykes represent a compound event of rifting and healing the fractured continental crust.

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Lineament analysis of Cannanore district and their implication on the tectonic evolution of the area

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Abstract. The Cannanore district and the adjoining areas mainly comprise of charnockites, gneisses, high and low-grade schists and various types of igneous intrusives. The lineament fabric of the region indicates that the NNW-SSE, NW-SE, ENE-WSW and NE-SW lineament directions are prominent. It is suggested that the area has undergone at least three distinct phases of tectonic activity. The NW-SE and ENE-WSW lineaments appear to have formed during the phase of NW-SE folding. The NE-SW lineaments may be the result of the cross-folding of the earlier folds. The NNW-SSE lineaments have been related to the Precambrian tectonic activity in South India.

Keywords. Lineament analysis; tectonic evolution; tensional fractures; basic magmatism; reactivation.

1. Introduction

Although lineaments are known to provide significant information on the tectonic evolution of a region, systematic study of lineaments in the Kerala region is very limited. Eremenko (1968) identified NW-SE, WNW-ESE, NNW-SSE, N-S and NE-SW lineaments in Kerala. Vardarajan and Nair (1978) and Katz (1982) made a broad study of lineaments in Kerala. Drury and Holt (1980) and Drury *et al* (1984) summarized remote sensing, geological, geochemical and geophysical data and brought out a detailed tectonic interpretation of the South Indian shield. These studies indicate that despite the masking of bed rock by extensive cover of laterites, Quaternary deposits and vegetation, major structures can generally be deciphered.

In the study area three mega-lineaments namely N36°W-S36°E (through Kuthuparamba), N40°W-S40°E (through Manantoddy) and N80°W-S80°E (through Mogral) have been reported (Nair *et al* 1975). The lineament fabric in Kerala was hitherto partly related to the distensional tectonics in response to the rifting of the western continental margin of India (eg. Eremenko 1968; Vardarajan and Nair 1978; Rao 1978; Sinha Roy and Furnes 1981; Furnes *et al* 1983). Here we report new lineament analysis data from North Kerala and suggest that the lineament fabric in the region has developed as a result of Precambrian deformational events.

2. Geology

Figure 1 shows the disposition of various geological units in the area. The area is principally composed of charnockites, gneisses, high- and low-grade schistose rocks.

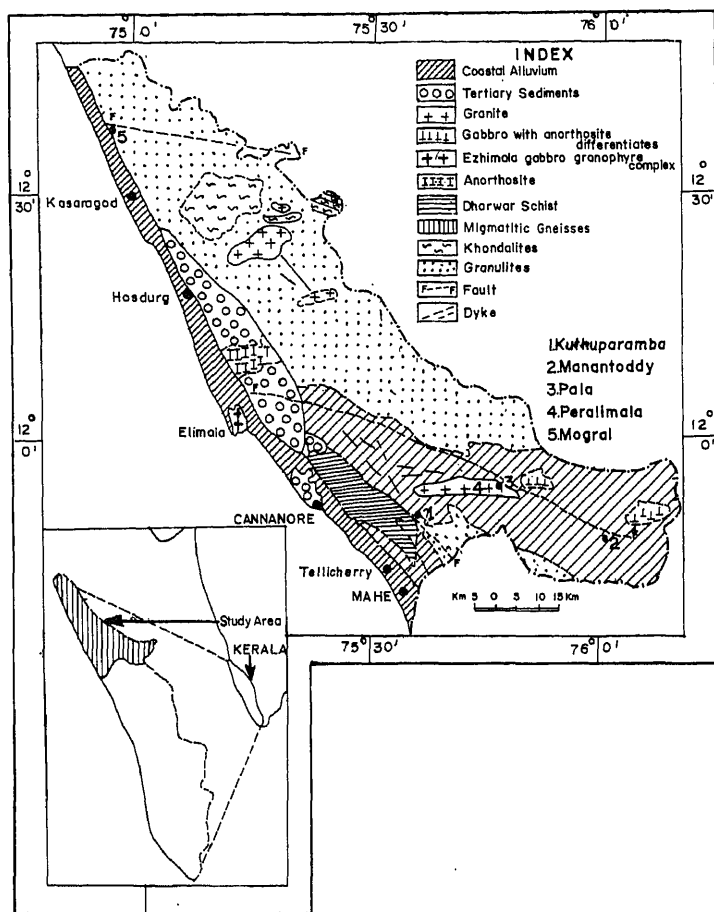


Figure 1. A generalized geological map of Cannanore district (compiled from various sources).

Gabbro, anorthosite, granophyre, granite and syenite occur as intrusive phases. In addition, numerous doleritic dykes trending in the NNW-SSE and ENE-WSW directions cut across all the lithounits. North of Pala, blocks of basic dyke occur as enclaves within the Peralimala granite (figure 2) which has yielded a Rb-Sr isochron age of 760 ± 70 m.y. (Santosh, personal communication). The coastal belt comprises of a narrow discontinuous NNW-SSE trending belt of tertiary sediment and recent alluvial deposits.

The Cannanore region has undergone several episodes of deformation as that

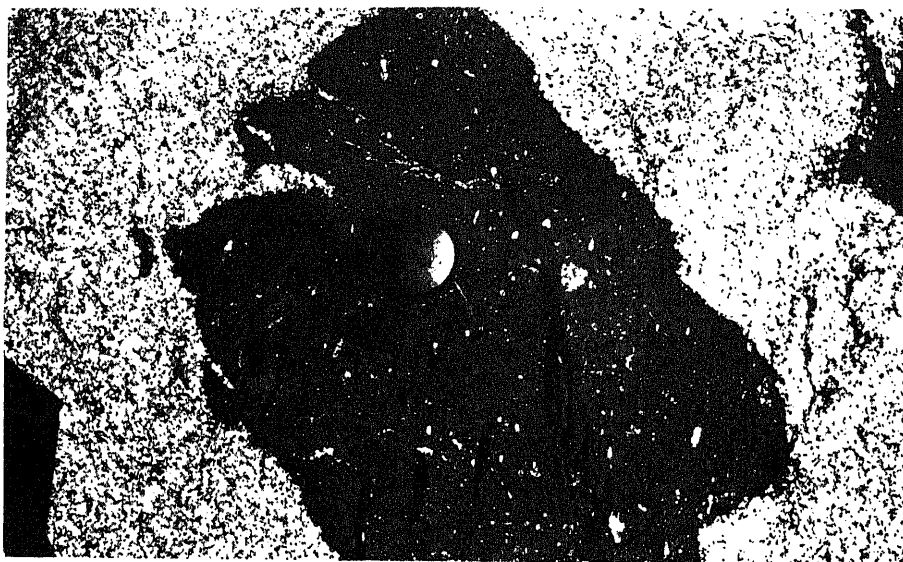


Figure 2. Doleritic enclaves in granite (Pala, Cannanore district).

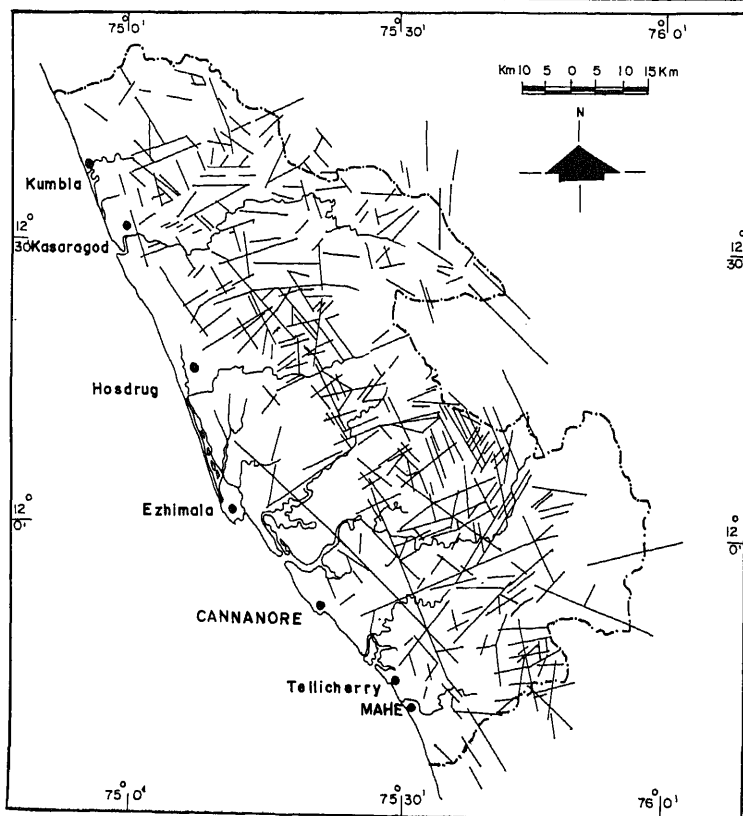
3. Lineament analysis

Landsat images of 12,50,000 scale (Band 5, 7 and false colour composites) were studied using visual interpretation techniques. The lineaments identified using the above technique were transferred on to topographic maps using a Carl Zeiss Sketchmaster and cultural features were filtered out as far as possible. The lineament map of the area is given in figure 3. Limited field checks were carried out to verify the lineaments. The lineaments observed in the area are mainly due to incised fractures/shears of varying dimensions, dykes, differential erosion and vegetation. The highland regions in Cannanore district exhibit lineament pattern more clearly and towards the lowland regions the lineament density decreases due to various obscuring factors which include extensive soil and laterite cover, vegetation and cultivated land.

The orientation and length of lineaments were recorded and lineaments were grouped into 18 azimuth classes of 10° interval. Table 1 gives the mean azimuth, mean observable length and standard deviation for each azimuth class. The number of lineaments and lineament intersections per unit area ($5 \text{ km} \times 5 \text{ km}$ cell size) were recorded and isolineament density and isolineament intersection maps were prepared (figure 4). Topographic maps of 1:250,000 scale were used to demarcate linear drainage channels and their orientation and length were recorded. Rose diagrams and cartesian curves (figures 5 and 6) for lineaments and drainage linears show that NNW-SSE, NW-SE, ENE-WSW and NE-SW linear features are predominant. The E-W drainage linears are quite prominent but are not very conspicuous in the lineament direction detected from Landsat images. Scanlines of Landsat images make an angle of less than 10° with respect to the E-W direction

Table 1. Statistical data for lineament orientation and length in Cannanore district and adjoining areas.

Azimuth class	Total No.	Total length (km)	Number %	Length %	Mean length (km)	Std. deviation (length)	Mean Azimuth	Std. deviation (azimuth)
0-10 NW	12	85	3.26	3.40	7.08	3.80	5.83	2.19
10-20 NW	26	195	7.07	7.52	7.52	4.93	17.00	2.66
20-30 NW	37	287	10.00	11.46	7.76	4.11	24.82	3.03
30-40 NW	24	165	6.52	6.59	6.88	3.90	36.04	2.56
40-50 NW	24	170	6.52	6.79	7.08	3.93	45.67	2.29
50-60 NW	10	77.5	2.72	3.10	7.75	3.94	53.50	2.69
60-70 NW	5	37.5	1.36	1.50	4.00	3.00	68.80	2.49
70-80 NW	10	52.5	2.72	2.10	5.25	2.89	75.60	2.42
80-90 NW	14	102.5	3.80	4.09	7.32	3.47	84.93	2.28
0-10 NE	6	45	1.60	1.80	7.50	2.84	6.33	2.43
10-20 NE	9	50	2.44	1.20	5.56	2.83	14.63	2.74
20-30 NE	14	75	3.80	3.00	5.55	2.65	26.29	2.46
30-40 NE	23	157.5	6.25	6.29	6.56	6.72	35.87	2.74
40-50 NE	30	165	8.15	6.59	5.50	3.19	44.47	2.99
50-60 NE	36	242.5	9.78	9.69	6.74	5.10	55.61	2.76
60-70 NE	40	267.5	10.87	10.69	6.39	6.05	66.50	2.73
70-80 NE	31	223	8.42	8.91	7.19	5.18	74.19	2.53
80-90 NE	17	106	4.62	4.23	7.09	4.20	84.82	2.26



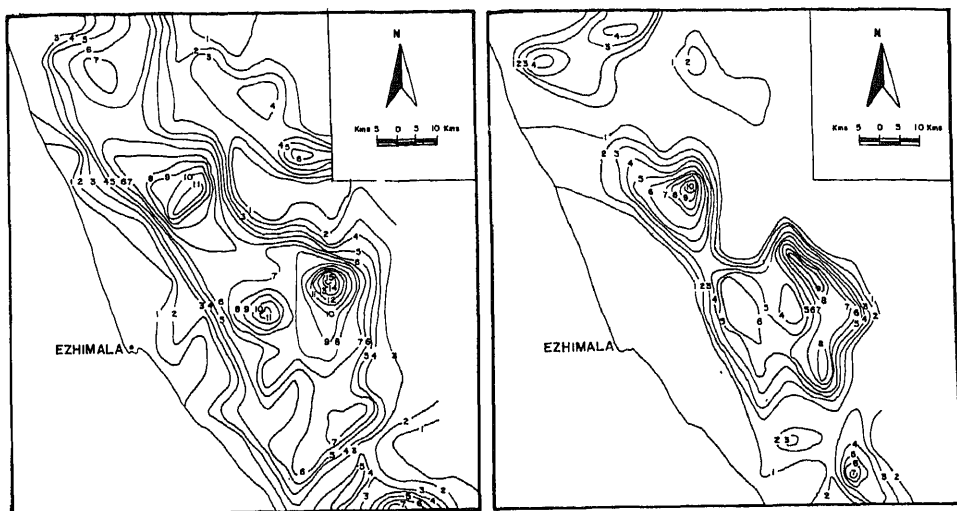


Figure 4. Isolineament density and isolineament intersection maps for Cannanore region.

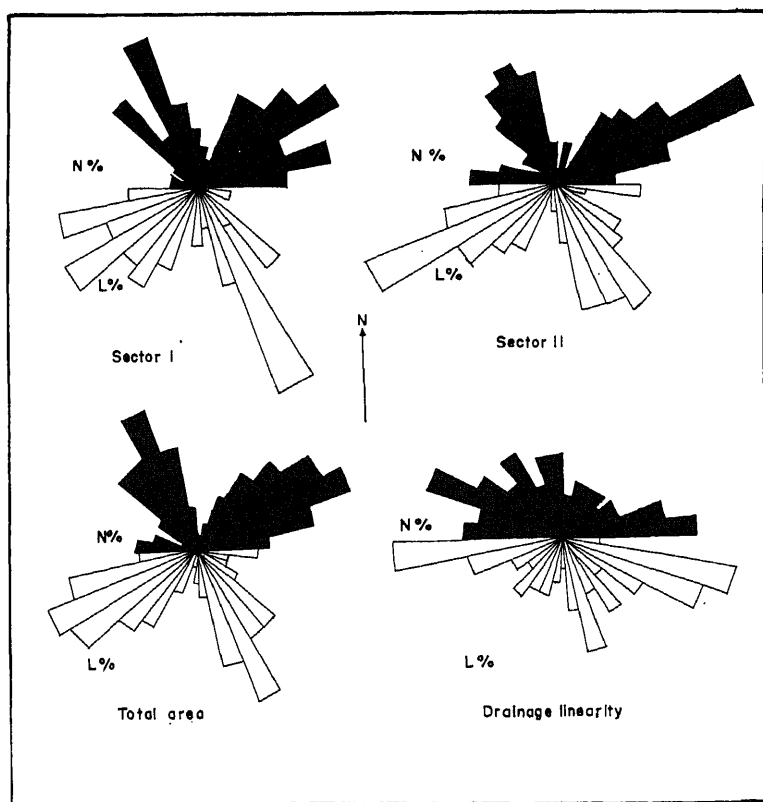


Figure 5. Rose diagram for linear features in Cannanore region (N% = number percent; L% = length percent).

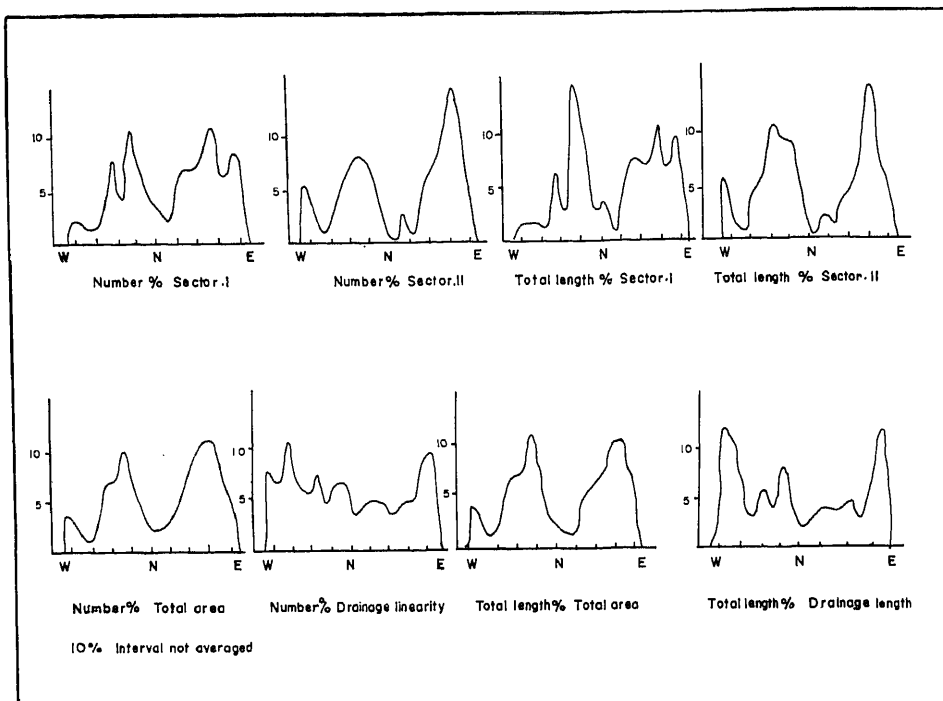


Figure 6. Cartesian curves for linear features in Cannanore region.

E-W trends in some cases may not be related to structural features since these coincide with the general slope direction. However, the relationship between stream order and stream length and bifurcation ratio values for drainage basins (unpublished data of the authors) suggest that the drainage development in the area is structurally controlled.

The lineament density isolines indicate more continuous zones of density anomalies and intersection isolines correlate more with the difference in topography. The lineament density and intersection density maps (figure 4) show two prominent anomalies in the area NE (sector I) and SE (sector II) of Elimala. In sector I the NNW-SSE lineaments are prominent (15%) whereas in sector II the ENE-WSW lineaments are predominant (20%). The NW-SE and NE-SW lineaments are more or less evenly distributed. Distribution of lineaments in major azimuth classes is shown in figure 7.

4. Discussion and Conclusions

The NNW-SSE, NW-SE, ENE-WSW and NE-SW trending lineaments are predominant in the study area. Dyke swarms in the area occur mainly along the NNW-SSE and ENE-WSW trends with minor fluctuations to the NE-SW and NW-SE directions. Dykes can be treated as natural fractures dilated by a combination of crustal extension and magma pressure (Anderson 1972). In case of

pure dilation, dykes get emplaced along the principal stress plane and the least compressive stress acts normal to the dyke (Pollard 1985). Rouchet (1981) indicated that, in folded structures created by compression, the open fractures extend parallel to the maximum stress axis. These fractures would thus be perpendicular to the fold axis. Hence, the NNW–SSE fractures in the Cannanore region are suggested to represent tensional fractures and could have developed due to near N–S compression. The N–S compression may be related to the early deformational episode which resulted in the formation of folded structures with E–W axial trends. The NW–SE to NNW–SSE enechelon fractures and/or dykes (Nair *et al* 1975) also suggest a near N–S compression (Badgely 1965; Wilson 1985).

Similarly the ENE–WSW lineaments can be related to a later deformation event that have produced folds with NW–SE to NNW–SSE axial plane. The ENE–WSW

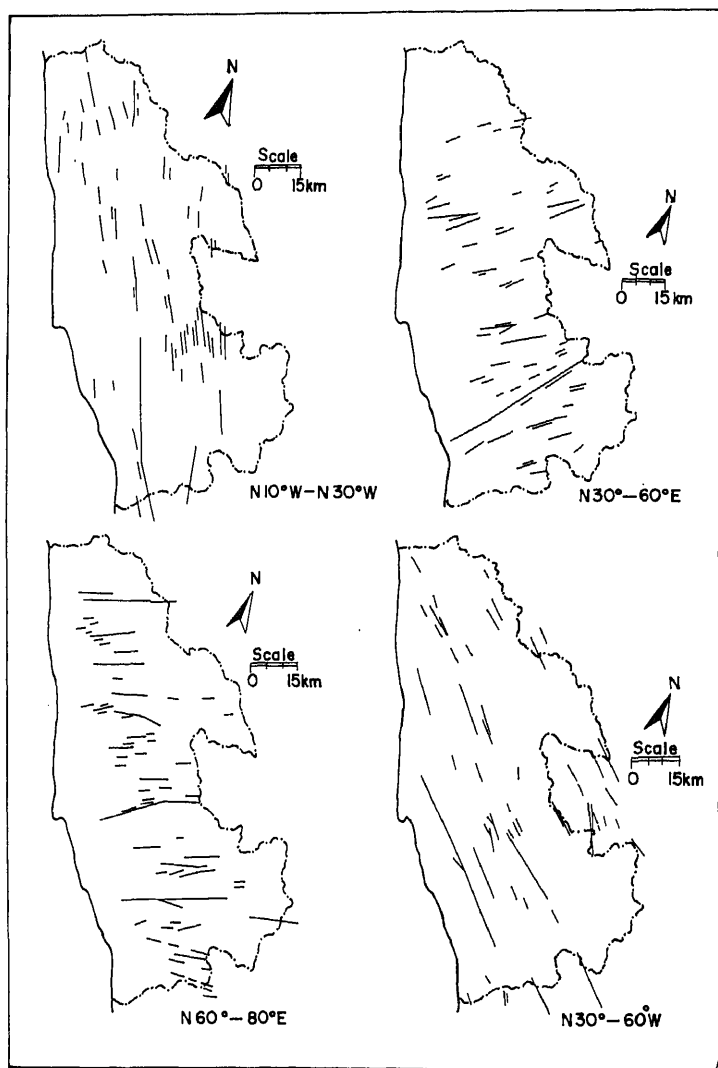


Figure 7. Distribution of lineaments in different azimuth classes.

dyke swarms may also suggest a N-S extension but in the absence of any large scale rifting (Drury *et al* 1984) it seems more probable that the fractures resulted due to near E-W compression. The same deformational event might have resulted in the generation of NW-SE lineaments parallel to the axial plane. The NE-SW lineaments relate to a different tectonic event and are considered to have developed during the cross folding of earlier folds along the NE-SW to NNE-SSW axis (Nair *et al* 1975). The above discussion reveals that NNW-SSE and ENE-WSW lineaments are related to Precambrian deformational event contrary to the present notion that they are responses to the rifting of the western continental margin of India about 110 m.y. ago.

The occurrence of basic dyke enclaves within the 760 ± 70 m.y. granite and the Proterozoic ages of a few dykes (Radhakrishna *et al* 1985, 1986) for which these fractures acted as conduits further supports the contention that the fracturing was initiated during early Proterozoic times. Similarly, Precambrian origin of the ENE-WSW fractures is supported by the occurrence of older igneous bodies along these lineaments and the suggested genetic correlation of the ENE-WSW basic dykes to those of the Cuddapah region (Drury 1984).

In summary, the Precambrian crustal fractures have had a profound influence on the tectonic evolution of the Cannanore region of Kerala. These lineaments have served as conduits for the emplacement of igneous bodies at different stages between the Proterozoic and the Mesozoic times. These fractures are suggested to have been periodically reactivated during which younger igneous bodies were emplaced along them, the basic dyke intrusions of Mesozoic being one of the phases.

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An interpretation of induced magnetic variations near Shillong and Gulmarg

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Abstract. Short-period events such as bays and storm sudden commencements (SSCs) have been analysed to investigate the nature of induced magnetic variations at two Indian magnetic observatories: Shillong and Gulmarg. It seems that near Gulmarg there is obvious connection between the induced magnetic variations and the two large scale features; the main central thrust (MCT) and the main boundary fault (MBF) in the north-west direction. The Dauki fault, an approximately east-west conductor, seems to be responsible for the conductivity anomalies at SHL.

Keywords. Induction; magnetic variations; horizontal disturbance vector.

1. Introduction

Gulmarg (GUL) and Shillong (SHL) are two permanent magnetic observatories situated in the Sub-Himalayan ranges of northern India (Lat: $34^{\circ}01'N$, Long. $74^{\circ}40'E$ for GUL and Lat. $25^{\circ}57'N$, Long. $91^{\circ}88'E$ for SHL). Their locations are shown on the map of India in figure 1a. Many of the deep-seated sub-surface geological features of their neighbouring regions (shown in figure 1b) can be studied through the anomalies in transient geomagnetic variations recorded at these two observatories. At SHL, earlier investigation by Agarwal *et al* (1979) through the analysis of short-time variations showed that Z variations are largely suppressed. They attributed the cause to an eastward induced currents flowing close to the station. Recently Nityananda and Jayakumar (1981) also tried to give a geological interpretation on the basis of induction vectors. However, the path of induced currents could not be delineated. It is important to further investigate the induction in this region through the use of anomalies in horizontal components (H & D) and to infer therefrom the exact nature of the induced current pattern in this geologically complex zone. At GUL, there have been no prior studies of induced magnetic variations. A simultaneous detailed investigation of these two stations (SHL & GUL) can provide not only useful information about the likely behaviour of induced currents, but can also contribute to the fundamental understanding of the Himalayan geology.

To identify the character of sub-surface geological features in the vicinity of SHL & GUL, we have analyzed the short-period events, such as bays and SSCs and compute through them the induction vectors and horizontal disturbance vectors. These parameters are very effective in identifying and locating internal currents causing the anomaly. Results of such analysis and their possible interpretation in terms of known geological features are reported here.

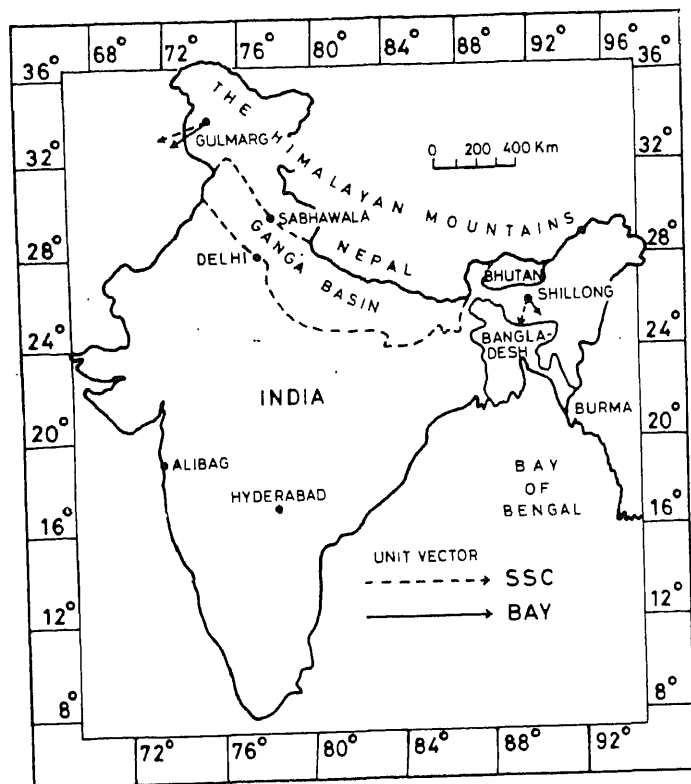
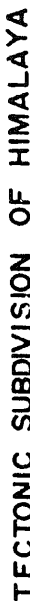


Figure 1 a. The induction vectors at Shillong and Gulmarg for bays and SSC

2. Data and analysis

The magnetic variations during a typical night-time bay at the northern stations GUL & SHL, are shown in figure 2. In the figure the magnetic variations at ABG are also included for comparison. Their features exhibit the following important characteristics in comparison to ABG (i) Both Z & D variations are suppressed at SHL, (ii) Z variations at SHL & GUL are positive in contrast to negative Z variations at ABG which happens to be a coastal station. Enhanced H variations at GUL compared with that at ABG & SHL. The enhancement is more pronounced for short-period night-time events such as bays. The latitudinal dependence of amplitude of D -variations from ABG to GUL does not fit into the trend of uniform amplitude variations of the horizontal components (H & D) over these latitudes (Schmucker 1969).

In order to investigate the nature of induction vectors, about 30 night-time bays and SSCs have been scaled from magnetograms to give amplitudes of ΔH , ΔZ and ΔT . The changes in amplitudes were measured by taking the basevalue as the value of the magnetic field at the corresponding time of the day.



February 26, 1979

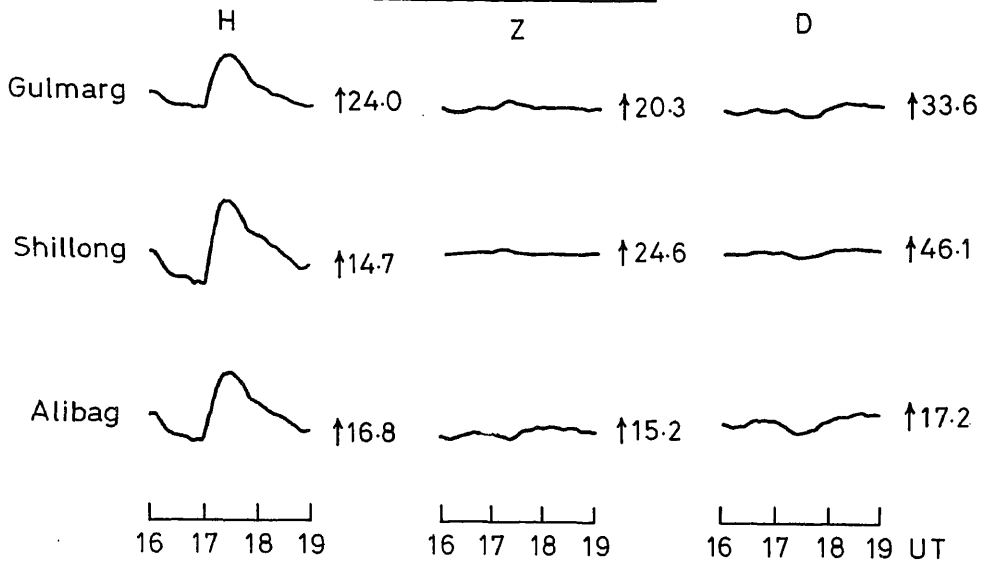


Figure 2. Amplitude variations during a typical magnetic bay observed at low-latitude Indian stations. Units marked are scale values, nT/cm . Time marked on X-axis is in UT.

Table 1. The magnitude ($\bar{S}$) and direction (θ) of induction vectors obtained at Shillong and Gulmarg for both bays and SSCs

Station	Code	Induction Vector			
		Bays		SSCs	
		$\bar{S}$	θ^*	$\bar{S}$	θ^*
Shillong	SHL	0.08	336.0°	0.22	19.3°
Gulmarg	GUL	0.36	65.7°	0.42	78.6°

* measured clockwise from south

in a least square sense to estimate coefficients A and B by minimizing the effect of ε in the following equation:

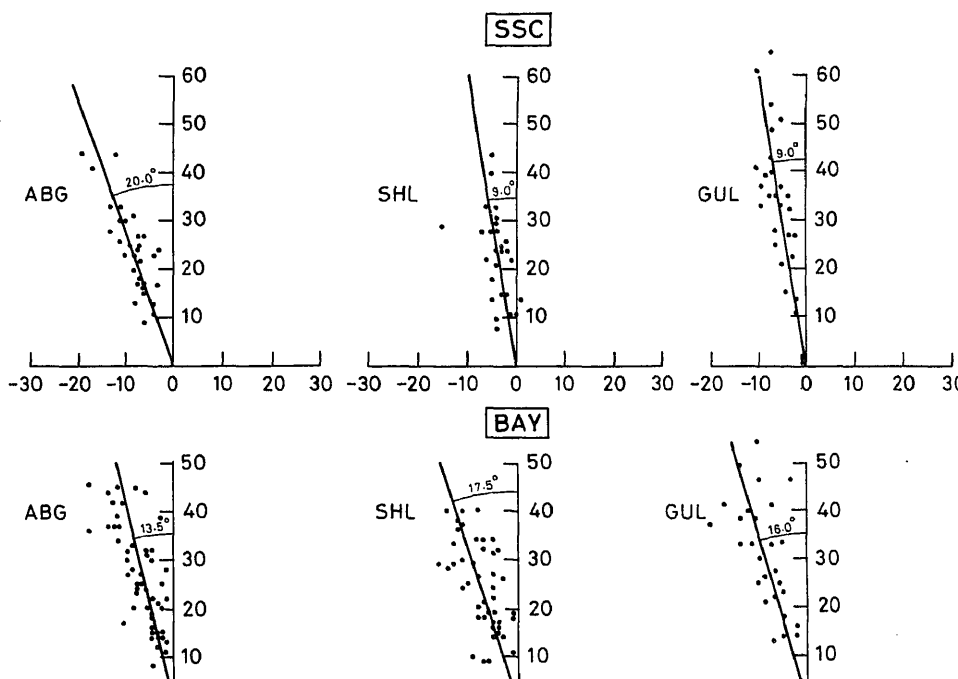
$$\Delta Z = A. \Delta H + B. \Delta D + \varepsilon$$

Here ε denotes the uncorrelated part of ΔZ . The coefficients were then combined to give an induction vector $\bar{S} [= (A^2 + B^2)^{1/2}]$ and its azimuth $\theta = \tan^{-1} B/A$. The induction vectors, so obtained, for bays and SSCs at SHL & GUL are shown on the map of India (figure. 1a) and their values are given in table 1.

To determine whether an anomaly in the horizontal components exists or not, the latitudinal trend of mean directions of horizontal variations at GUL and SHL are compared with the mean direction at ABG, with the assumption that horizontal variations at Alibag are normal (Nityananda *et al* 1977). For computing mean

directions of horizontal variations at the three stations, the amplitude of selected events was plotted with ΔH (in nanoteslas) on the Y -axis (representing North) and ΔD (also in nanoteslas) on the X -axis (representing East). The least square fit of mean direction to scatter of points for all the three stations are shown in figure 3 for bays and SSCs separately. In both class of events, the mean directions at SHL & GUL do not fit in the latitudinal trend of gradual shift towards east, indicating an anomalous behaviour at these stations. At SHL, the change in direction is slightly more eastward for SSCs whereas for bays it is west of ABG. GUL also shows similar changes as that of SHL. These changes in the mean directions indicate the presence of magnetic anomalies in horizontal components at both stations.

For identifying the nature of the anomalies in the horizontal components, we computed horizontal disturbance vectors ($\Delta \vec{B}_A$) following the procedure given in Schmucker (1969). First, the anomalous part in the horizontal components (ΔH_A and ΔD_A) at GUL & SHL were calculated by taking the algebraic difference between the horizontal components as recorded at these stations and the corresponding value recorded at ABG for all possible common events. The anomalous parts ΔH_A and ΔD_A are plotted again taking ΔH_A on the Y -axis and ΔD_A on the X -axis. Figure 4 shows the plots for both stations and for bays and SSCs. The mean direction of the vectors was obtained by the least square fit. The direction of induced currents can be obtained by rotating this vector anticlockwise through 90° (Schmucker 1969).



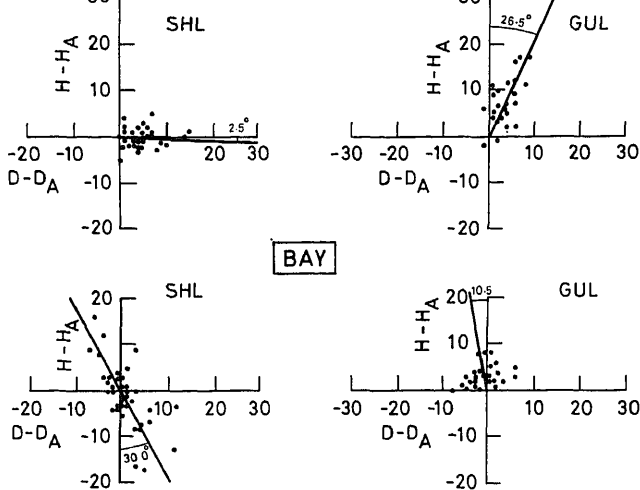


Figure 4. Mean direction of anomalous horizontal variations at Shillong and Gulmarg is obtained after subtraction of the corresponding amplitudes observed at Alibag (H_A and D_A) which is taken to be a normal station. Units marks on the axis are nT.

3. Results and discussion

It is seen from the nature of induction vectors (figure 1a) that the conductor causing the anomaly at GUL lies approximately south-west of that station, and it is towards south or south-east at SHL. The directions of induced electric currents inferred from the anomalous horizontal variations $\Delta \vec{B}_A$ are found to be 79.5° (measured clockwise from the south) for bays and 116.5° for SSCs at GUL whereas it is about 240.0° and 182.5° for bays and SSCs respectively at SHL. It implies that induced current would flow north-west or westward near GUL. At SHL, the direction of induced electric currents inferred would be north-east or eastward for bays whereas for SSCs it would be northward.

The position of GUL in the Himalayan ranges makes an east-west conductor easy to conceive of. The trend of the Himalayan chain is approximately east-west, a rather wide arc swinging from north-west to south-east. All the structural features linked with the Orogeny will also have a dominant east-west trend. Of the large scale features, the two main thrusts: (i) the main central thrust and (ii) the main boundary fault run parallel to each other along the range and their main tectonic trend (figure 1b) is in conformity with the direction of induced currents. These structural features of the Himalayas (possibly an asthenosphere upwelling between the two faults) seem to be responsible for the conductivity anomalies at GUL. It is important to note that between these two faults the lesser Himalayan sedimentaries lie in tangled confusion of structure and stratigraphy (Valdiya 1964). On the south of the main boundary fault extends a vast group of Siwalik and older tertiary sediments whose thickness is about 5 km. These layers of sediments will also affect the short-period fluctuations in the magnetic records of GUL. A schematic

illustration of the tectonic features of Himalayas and Ganga basin is presented in figure 1b after Valdiya (1973).

On the consequence of Himalayan Orogeny, step-like rise of the asthenosphere along the entire range would provide a sharp vertical lateral conductivity contrast. Such a step-like structure can also contribute to the induced magnetic variations at GUL. A similar type of structure has been proposed by us for the conductivity anomalies at Sabhawala (Nityananda *et al* 1981). Recently Srivastava (1981) also studied the night-time bays observed at Sabhawala, Yangi-Bazar (Tashkent) and Alma Ata observatories and proposed the cause of anomaly to be induced currents in a step-structure at the Moho boundary (upper mantle material under the plain juxtaposed against the crustal material beneath the Himalayas) in an east-west direction along the junction of Himalaya foothills. However, it is important to note that the effect of sedimentary basin in Indo-Gangetic plain is not very distinct as the induction vectors for both bays and SSCs point towards south-west instead of towards south or south-east.

To investigate the nature and depth of the conductor, we have utilized horizontal disturbance vector ($\Delta\vec{B}_A$) [where $\Delta B_A = \Delta H_A^2 + \Delta D_A^2$]^{1/2}], we have computed the modulus of ratio $\Delta B_A/\Delta Z_A$ for each event and then averaged them over all the events in each class. The anomalous variation (ΔZ_A) is taken to be the total amplitude of the observed variation in the vertical component. Lilley (1976) has suggested that a response arrow scaled as $\arctan \Delta B_A/\Delta Z_A$ be plotted in the direction of $\Delta\vec{B}_A$ if ΔZ_A is negative, and in the direction of $\Delta\vec{B}_A$ if ΔZ_A is positive thereby obtaining an arrow whose length is maximal and finite directly over the current. The magnitude of these response arrows together with standard deviations is found to be 1.33 ± 0.66 for SSCs and 0.93 ± 0.37 for bays at GUL. The nature of the response arrows indicates that, for periodicities around SSCs, induced currents are close to it and they flow at a shallow depth. At longer periods (i.e. for bays), the currents not only flow at shallower depth (sedimentary layer) but also in the upper mantle thereby smearing the overall induced effects at such periods. This is indicated by the smaller value of the response arrow for the bays.

At SHL, the location of induced currents is very close to the station but slightly towards south is provided by the negligible positive Z-variations (figure 2) and the nature of inductor vectors (figure 1a). The magnitude of the response arrows computed for bays (1.23 ± 0.49) and SSCs (0.94 ± 0.31) suggest that the conductivity contrast existing at much deeper level is much closer to the station than that of the shallow feature. On the basis of known geological features existing here, we find that the prominent Dauki fault (figure 1b) striking E-W, lying south of the Shillong plateau is a deep-seated fault and seems to be responsible for the induced geomagnetic variations at SHL as suggested by Nityananda and Jayakumar (1981). There is also a possibility of contribution from a conducting path in the rise of mantle south of Shillong Plateau. This is because the crustal thinning is much more steep toward Surma Valley than that of Brahmaputra valley (Verma and Gupta

approximately in the north-east or eastward direction (deep seated structure) and another in a northward direction (shallow structure). Since the contribution of induced effects to horizontal variations has been estimated by taking Alibag (ABG) as a normal station which is sufficiently far away in longitude from this station, further confirmation of the results is necessary when some more data from a network of temporary stations are available in this geologically complex zone. The large value of scatter in the magnitude of the response arrows at this station is attributed to the small Z -variations and to the assumption that total ΔZ observed is wholly anomalous.

4. Conclusions

The direction of induced currents inferred from the nature of the horizontal anomalies ($\Delta \vec{B}_A$) indicate that the conductivity contrast, possibly an asthenosphere upwelling between two faults: main boundary fault and the main central thrust, is the cause of conductivity anomalies at GUL. South of the main central thrust, the lesser Himalayan sediments and the Siwalik sediments also affect the short-period fluctuations. At SHL, an approximately east-west conductor, namely Dauki fault, together with dense sediments in the Surma valley seems to be responsible for the conductivity anomalies. The induced effects are more pronounced for shorter periods (i.e. SSCs) than for longer periods (i.e. bays).

It is interesting to note that induced currents in the Himalayan region tend to flow parallel to the main tectonic trend which is more or less NW-SE near GUL and NE-SW in Assam near SHL.

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On the thermal and redox history of the Yamato diogenite Y-74013

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Abstract. The Yamato diogenite, Y-74013, shows a high degree of textural equilibrium with the apparent crystallization sequence: troilite and metal $\rightarrow$ orthopyroxene $\rightarrow$ plagioclase. The position of the large chromite crystals in this sequence is unclear. Except chromite, all other minerals have composition similar to common orthopyroxene achondrites. The chromite is more magnesian than in common diogenites, strongly zoned and, on the whole, intermediate in composition between chromites of diogenites and pallasites. Texture, mineral composition data and an equilibrium thermodynamic analysis of the mineral association strongly indicate that the chromite crystallized earlier than the silicates at a much higher temperature (possibly above 1100°C) and rapidly grew in a medium which was progressively enriched in Mg, Al and Ti. But the chromite failed to reach chemical equilibrium, even at its outermost rim, with the orthopyroxene. The calculated equilibrium $\log f_{O_2}$ of the Yamato diogenite, -20.21 to -11.08 for temperatures between 880°C and 1500°C is well within the normal oxygen fugacity range of pyroxene achondrites.

Keywords. Thermal history; redox history; diogenite; Yamato Y-74013 meteorite.

1. Introduction

Recent work on the Antarctic achondrites has thrown considerable light on the genetic relationships among certain achondrite groups, e.g. eucrites, howardites and diogenites. For instance, Takeda and Yanai (1982) have shown that the eucrite, howardite and diogenite clasts of the polymict Yamato-79 achondrites can be satisfactorily explained by impact processes on a *layered* parent body, in which the layer-sequence from the surface downwards may be: (1) surface-type, unequilibrated eucrite with Fe-Mg-zoned pyroxene, (2) shallow level, cumulate eucrites, with inverted pigeonite and pyroxenes showing exsolution lamellae, (3) polymict eucrite with minor diogenite at intermediate depths, (4) howardites at the deep levels and (5) diogenites at the deepest level. One of the critical tests for such models would be quantitative thermometry and barometry of the different layers to find a reasonable and internally consistent P-T- f_{O_2} profile across the layered parent planet. Possibly the most difficult object for such a test would be the diogenites, the nearly monomineralic orthopyroxene-rocks, to which the commonly available mineralogical P-T-sensors cannot be applied. The diogenites nevertheless play a key role in the evolution of the layered parent body and a quantitative estimate of the pressure and temperature of their equilibration — in conformity with their supposed location at the deepest level — is obviously of great interest.

In this paper, we present detailed petrology and mineralogy of the Yamato-74013

estimation of the pressure and temperature of formation of essentially orthopyroxene-chromite meteorites and apply these methods to the Yamato diogenite.

2. Petrography

A medium-grained orthopyroxenite, Yamato Y-74013, shows xenomorphic orthopyroxene grains interlocked with frequently developed near-120° trijunctions (figure 1a). The orthopyroxene is colourless, non-pleochroic and optically negative with a fairly large optic axial angle. Cleavage traces are only feebly developed and extinction is straight in prismatic section. While some orthopyroxene grains are clear and nearly free from inclusions, many are densely charged with tiny opaque inclusions of troilite and metal (figure 1b). Anhedral to subhedral troilite has profusely grown as single grains or as composite grains with the metal phase. Somewhat larger subhedral troilite and metal grains also cluster at the orthopyroxene grain interfaces. The plagioclase occurs in the form of anhedral, clear pool interstitial to the orthopyroxene (figure 1c). The large chromite euhedra appear to be somewhat isolated from the silicate matrix. No clasts were observed. No evidence of brecciation or shock is also evident, except a moderate degree of fracturing affecting the orthopyroxene. The sequence of crystallization might be metal and troilite first, followed by orthopyroxene with a slight overlapping, and plagioclase crystallizing last. The typically 'interprecipitate' appearance of the plagioclase suggests accumulation (gravitative ?) of orthopyroxene crystals packed with *early* metal and troilite inclusions in a small amount of late, residual, interprecipitate liquid from which the plagioclase crystallized. The position of chromite in this sequence is unclear. On the whole, the texture indicates a high degree of equilibration and grain growth.

3. Mineral composition data

Composition data (tables 1-4) of orthopyroxene, chromite, troilite, metal and plagioclase were obtained from one polished thin section of Yamato Y-74013 diogenite prepared at the National Institute of Polar Research, Japan. The analyses were carried out with the JEOL JXA-35 electronprobe microanalyzer employing a 4-crystal wavelength dispersive spectrometer. Accelerating voltage, specimen current and beam diameter were 15 kv, between 0.20×10^{-7} and 0.75×10^{-7} amp and about 5 μm respectively. All data were corrected for deadtime and background. The final microprobe data reduction was carried out using Bence-Albee method (Bence and Albee 1968) for the silicates and the oxides and the ZAF-based delta method of Hirata and Okumura (1976) for troilite and metal.

The Yamato orthopyroxene (table 1) is on the whole homogeneous and similar in Fe/Mg ratio, Al, Cr and Ca content to the common orthopyroxenes from diogenites. The laboratory orthopyroxene standard (table 1) is similar to the Yamato

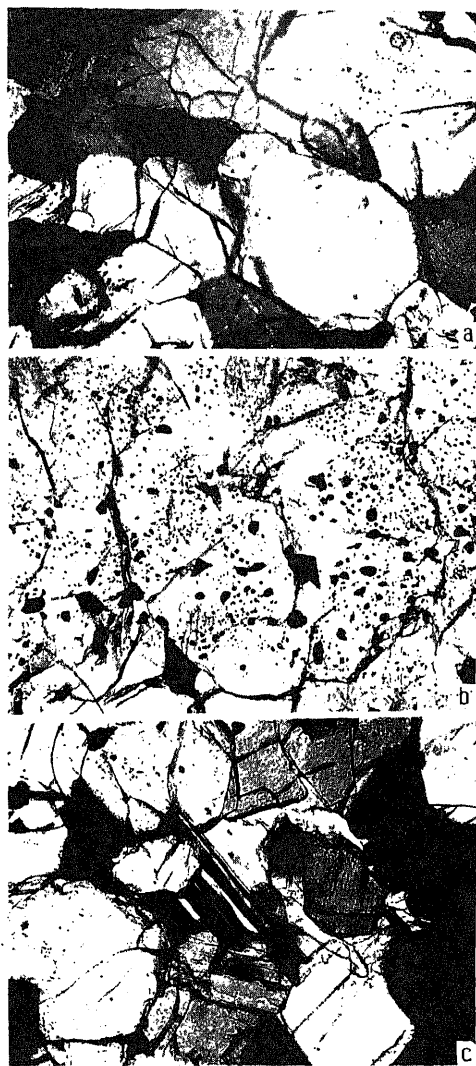


Figure 1. Photomicrograph of Yamato Y-74013 diogenite ($\times 40$) **a.** Coarse, xenomorphic orthopyroxene grains forming near- 120° triple junctions. Polars crossed. **b.** Coarse, xenomorphic orthopyroxene grains densely charged with opaque inclusions of troilite and metal. Single polar. **c.** Clear, anhedral pool of polysynthetically twinned plagioclase interstitial to orthopyroxene. Polars crossed.

confirmed by more data. The Yamato troilite (table 3), like common meteoritic troilite, is nearly stoichiometric with the structural formula $\text{Fe}_{(1.997)}\text{Co}_{(0.001)}\text{S}$. The metal phase (table 3) is nearly pure Fe with small amounts of Co and Ni. Our composition data on the whole agree with the previous reports (Takeda *et al* 1978). Comparison with several well-known diogenites (table 3) brings out the close similarity, especially with Tatahouine diogenite. We did not detect any significant grain-to-grain or within-grain variation in composition of the Yamato metal phase. The Yamato chromite (tables 2 and 5), however, shows significant compositional

Table 1. Orthopyroxene analyses.

	Yamato Y-74013 diogenite Orthopyroxene		Johnstown diogenite Orthopyroxene **
	Rim	Core	
SiO ₂	52.97	53.30	54.10
Al ₂ O ₃	1.66	1.51	1.06
TiO ₂	0.15	0.16	0.13
Cr ₂ O ₃	0.79	0.86	0.81
FeO*	17.37	17.38	15.60
MnO	0.53	0.59	0.48
MgO	25.84	25.46	26.70
CaO	1.74	1.65	1.28
Cations on the basis of 6 oxygens			
Si	1.92	1.94	1.96
Al ^{IV}	0.06	0.06	0.04
Al ^{VI}	0.01	0.01	0.01
Ti	0.004	0.004	0.004
Cr	0.02	0.03	0.02
Fe	0.53	0.53	0.47
Mn	0.02	0.02	0.01
Mg	1.40	1.38	1.44
Ca	0.07	0.06	0.05
Total	4.03	4.03	4.00
Wo	3.5	3.0	2.5
En	70.0	70.0	73.5
Fs	26.5	27.0	24.0
X _{Mg,M1}	0.72	0.69	0.73
X _{Mg,M2}	0.68	0.69	0.71
X _{Fe,M1}	0.27	0.26	0.24
X _{Fe,M2}	0.26	0.27	0.23
X _{MgAl₂SiO₆}	0.0008	0.0008	0.0006

* Total Fe as FeO

** Average of 12 analyses (Floran *et al* 1981).

magnesian, its Mg/Fe²⁺ ratio being almost midway between the corresponding diogenite and pallasite values. Table 5 displays the Mg/Fe²⁺, Y_{Cr}^{Sp} (i.e. Cr/(Cr + Al + Fe³⁺)), Y_{Al}^{Sp} (i.e. Al/(Cr + Al + Fe³⁺)) and Y_{Fe³⁺}^{Sp} (i.e. Fe³⁺/(Cr + Al + Fe³⁺)) values of the chromites of Yamato diogenite, Johnstown diogenite (Floran *et al* 1981), type bronzite achondrite (Bunch and Keil 1971), and pallasite (Bunch and Keil 1971). The core Y_{Cr}^{Sp} and Y_{Al}^{Sp} values of the Yamato chromite are also midway between corresponding typical diogenite and pallasite values. This 'pallasitic' trend in the Yamato chromite composition suggests that the large chromite euhedra (*especially their core sectors*) did not grow in equilibrium with the orthopyroxene.

Table 2. Chromite analyses.

Yamato Y-74013		Johnstown diogenite		Pallasites		Bronzite achondrites		Silicate inclusions in iron meteorites		Mesosiderites		Monomict- pigeonite- plagioclase achondrites		Chassignites		H-group chondrites	
Rim	Core	Johnstown diogenite		Pallasites		Bronzite achondrites		Silicate inclusions in iron meteorites		Mesosiderites		Monomict- pigeonite- plagioclase achondrites		Chassignites		H-group chondrites	
Cr ₂ O ₃	53.72	59.58	53.60	64.00	55.50	71.30	52.00	48.90	46.10	56.90							
Al ₂ O ₃	12.08	7.66	10.60	5.60	10.10	0.84	11.50	9.30	9.80	5.90							
TiO ₂	1.75	1.59	0.73	0.18	1.09	0.46	1.84	4.10	3.7	2.33							
SiO ₂	0.37	0.22	—	—	—	—	—	—	—	—							
FeO*	23.86	23.88	29.60	23.20	28.80	14.00	31.00	35.70	36.50	31.20							
MnO	0.24	0.25	0.56	0.65	0.68	2.51	0.77	0.59	0.54	0.94							
MgO	5.09	4.97	2.92	5.80	3.90	8.90	2.29	0.59	2.86	2.66							
Cations on the basis of 4 oxygens																	
	97.11	98.15	98.01	99.48	100.07	98.01	99.40	99.18	99.50	99.93							
Cr	1.462	1.644	1.500	1.765	1.516												
Al	0.490	0.315	0.442	0.230	0.411												
Fe ³⁺	0.048	0.041	0.058	0.005	0.073												
Total	2.000	2.000	2.000	2.000	2.000												
Fe	0.687	0.697	0.818	0.672	0.759												
Mg	0.261	0.259	0.154	0.301	0.201												
Mn	0.007	0.007	0.017	0.019	0.020												
Ti	0.045	0.041	0.019	0.005	0.028												
Si	0.013	0.008	0.002(Ca)	—	—												
Total	1.013	1.012	1.010	0.997	1.008												

* Total Fe as FeO. Johnstown data include 0.05% CaO and are taken from Floran *et al* (1981). Type meteorite data (columns 4–10) are taken from Bunch and Keil (1971) after deducting small amounts of V₂O₅ and ZnO.

Table 3. Troilite and metal analyses.

	Yamato Y-74013 diogenite troilite	Yamato Y-74013 diogenite metal	Element diogenite metal (clast)*	Shalka diogenite metal (fine)*	Johnstown diogenite metal (clast)*	Tatahouine diogenite metal (clast)*
Fe	63.65	98.59	98.87	98.87	96.19	98.49
Co	0.07	0.85	0.11	1.66	0.71	0.75
Ni	—	0.56	1.02	0.07	3.10	0.76
S	36.64	—	—	—	—	—
	100.36	100.00	100.00	100.00	100.00	100.00

* Data taken from Gooley and Moore (1976). The Co and Ni values represent medians of the corresponding ranges.

Table 4. Plagioclase analyses.

	Yamato Y-74013 diogenite plagioclase		Johnstown diogenite plagioclase
	Centre	Rim	
SiO ₂	48.82	48.11	46.70
Al ₂ O ₃	33.29	33.23	33.80
FeO*	0.27	0.24	0.29
MgO	0.15	0.17	0.46
CaO	15.93	17.28	16.90
Na ₂ O	1.90	1.44	1.39
K ₂ O	0.07	0.04	0.07
SrO	—	—	0.05
	100.43	100.51	99.66
Cations based on 8 oxygens			
Si	2.22	2.19	2.15
Al	1.78	1.79	1.84
	4.00	3.98	3.99
Fe	0.01	0.01	0.01
Mg	0.01	0.01	0.03
Ca	0.78	0.84	0.84
Na	0.17	0.13	0.12
K	0.004	0.002	0.004
Sr	—	—	0.001
	0.97	0.99	1.01
An	81.8	86.4	87.1
Ab	17.8	13.4	12.4
Or	0.4	0.2	0.4

* Total Fe = FeO + Fe₂O₃.

Table 5. Chromite parameters.

	Yamato Y-74013 diogenite		Johnstown diogenite	Bronzite achondrite	Pallasite
	Rim	Core			
Mg/Fe ²⁺	0.380	0.372	0.188	0.265	0.448
Y _{Cr} ^{Sp}	0.731	0.822	0.750	0.758	0.883
Y _{Al} ^{Sp}	0.245	0.158	0.221	0.206	0.115
Y _{Fe³⁺} ^{Sp}	0.024	0.021	0.029	0.037	0.003

4. Compositional zoning in chromite

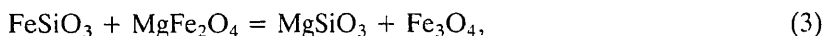
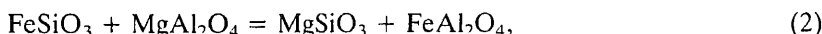
The chromite of the Yamato diogenite shows compositional zoning (table 2). Cr decreases from the core to the border significantly. So does Fe, but to a much smaller extent. Al shows a large increase from the core to the rim and Mg also increases a little from core to rim. Electron microprobe scan shows that all these variations are quite smooth and gradational. Ti and Si also show higher values in the rim than in the core.

Two points are of interest here. Compositional variability in the chromite of equilibrated chondrites and achondrites is uncommon (El Gorseý 1976a). Sharp compositional zoning, especially of Cr and Al in chromite, of the texturally equilibrated Yamato diogenite is therefore a rather remarkable feature. Secondly, the zoning pattern: decrease in Cr/(Cr + Al) (from 0.84 to 0.75) and Fe/(Fe + Mg) (from 0.73 to 0.72) and slight increase in Ti/(Ti + Al + Cr) (from 0.021 to 0.023) from core to rim unmistakably conforms to the zoning trend of the 'early spinel' (El Gorseý 1976b) reported from some relatively coarse-grained lunar basalts. This trend indicates that the chromite grew from a medium, possibly a silicate liquid, which was progressively enriched in Al, Mg and Ti. This is possible only if the chromite had crystallized *before* pyroxene, olivine, plagioclase and any other Ti-rich spinel phase started crystallization.

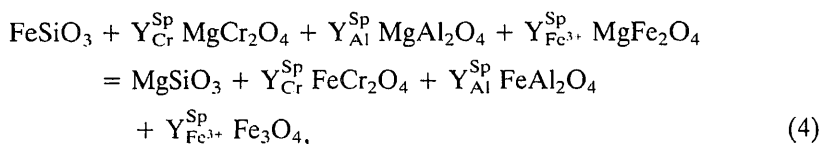
The pallasitic composition and the zoning pattern of the chromite pose intriguing problems of interpretation, especially in view of the strong indications of textural equilibration of the meteorite. While a compositionally zoned mineral must be out of chemical equilibrium with the coexistent phases, its outermost rim's composition may still be chemically equilibrated. In terrestrial rocks, such cases are quite common. Compositionally zoned garnets frequently have their borders equilibrated with coexistent Fe-Mg silicates e.g. biotite and pyroxenes. In terrestrial silicate-chromite associations also, chromite is quite often zoned, but its rim-composition is frequently equilibrated with the coexistent, Fe-Mg silicate (Medaris 1975; Wilson 1982; Ozawa 1983; Viswanath 1983). In all such cases, a meaningful quantitative analysis of the P-T conditions of formation is possible on the basis of

5. Orthopyroxene-chromite-silica thermobarometry

For the thermodynamic analysis of an essentially orthopyroxene-chromite rock, Fe^{2+} -Mg exchange equilibrium between orthopyroxene and chromite must be the major concern. While there are a fairly large number of geothermometric formulations based on Fe^{2+} -Mg exchange between olivine and chromite (Jackson 1969; Evans and Frost 1975; Fabries 1979 etc), there exists no such formulation for an orthopyroxene-chromite association. We therefore developed the general systematics of Fe^{2+} -Mg exchange between orthopyroxene and chromite first. The Mg- Fe^{2+} exchange reactions in an equilibrated orthopyroxene-chromite assemblage are as follows.



where FeSiO_3 and MgSiO_3 represent the ferrosilite and enstatite components of the orthopyroxene and MgCr_2O_4 , MgAl_2O_4 , MgFe_2O_4 , FeCr_2O_4 , FeAl_2O_4 and Fe_3O_4 represent the pirochromite, spinel, magnesioferrite, chromite, hercynite and magnetite components of the chromite. Equations (1), (2) and (3) may be combined to give:



where $Y_{\text{Cr}}^{\text{Sp}} = X_{\text{Cr}}^{\text{Sp}} / (X_{\text{Cr}}^{\text{Sp}} + X_{\text{Al}}^{\text{Sp}} + X_{\text{Fe}^{3+}}^{\text{Sp}})$

$$Y_{\text{Al}}^{\text{Sp}} = X_{\text{Al}}^{\text{Sp}} / (X_{\text{Cr}}^{\text{Sp}} + X_{\text{Al}}^{\text{Sp}} + X_{\text{Fe}^{3+}}^{\text{Sp}})$$

$$Y_{\text{Fe}^{3+}}^{\text{Sp}} = X_{\text{Fe}^{3+}}^{\text{Sp}} / (X_{\text{Cr}}^{\text{Sp}} + X_{\text{Al}}^{\text{Sp}} + X_{\text{Fe}^{3+}}^{\text{Sp}})$$

$$Y_{\text{Cr}}^{\text{Sp}} + Y_{\text{Al}}^{\text{Sp}} + Y_{\text{Fe}^{3+}}^{\text{Sp}} = 1$$

and $X_{\text{Cr}}^{\text{Sp}}$, $X_{\text{Al}}^{\text{Sp}}$ and $X_{\text{Fe}^{3+}}^{\text{Sp}}$ represent the mole fractions of Cr, Al and Fe^{3+} in chromite (chrome spinel).

For reaction (4),

$$K_4 = \frac{a_{\text{MgSiO}_3}^{\text{Opx}} \cdot (a_{\text{FeCr}_2\text{O}_4}^{\text{Sp}})^{Y_{\text{Cr}}^{\text{Sp}}} \cdot (a_{\text{FeAl}_2\text{O}_4}^{\text{Sp}})^{Y_{\text{Al}}^{\text{Sp}}} \cdot (a_{\text{Fe}_3\text{O}_4}^{\text{Sp}})^{Y_{\text{Fe}^{3+}}^{\text{Sp}}}}{a_{\text{FeSiO}_3}^{\text{Opx}} \cdot (a_{\text{MgCr}_2\text{O}_4}^{\text{Sp}})^{Y_{\text{Cr}}^{\text{Sp}}} \cdot (a_{\text{MgAl}_2\text{O}_4}^{\text{Sp}})^{Y_{\text{Al}}^{\text{Sp}}} \cdot (a_{\text{MgFe}_2\text{O}_4}^{\text{Sp}})^{Y_{\text{Fe}^{3+}}^{\text{Sp}}}} \quad (5)$$

where Sp and Opx stand for spinel and orthopyroxene respectively. K and a signify the equilibrium constant and activity respectively.

For equilibrated reactions of the kind (1), (2) and (3)

$$K = \exp(-\Delta G_{f,P,T}^{\circ}/RT), \quad (7)$$

where $\Delta G_{f,P,T}^{\circ}$ represents Gibbs energy (of formation) of the reaction at equilibrium pressure (P) and temperature (T), R is the universal gas constant, and K is the equilibrium constant.

Again, the Gibbs energy of the reactions (1) to (4) are related as follows

$$(\Delta G_{f,P,T}^{\circ})_4 = Y_{Cr}^{Sp} (\Delta G_{f,P,T}^{\circ})_1 + Y_{Al}^{Sp} (\Delta G_{f,P,T}^{\circ})_2 + Y_{Fe^{3+}}^{Sp} (\Delta G_{f,P,T}^{\circ})_3 \quad (8)$$

where the number in the subscripts refer to the numbers of the reaction equations. Therefore,

$$K_4 = \exp[-\{Y_{Cr}^{Sp} (\Delta G_{f,P,T}^{\circ})_1 + Y_{Al}^{Sp} (\Delta G_{f,P,T}^{\circ})_2 + Y_{Fe^{3+}}^{Sp} (\Delta G_{f,P,T}^{\circ})_3\}/RT] \quad (9)$$

Now equating relations (5) and (9), we have the basic activity-free energy relationship between coexisting and equilibrated orthopyroxene and chromite solid solutions:

$$\begin{aligned} & \frac{a_{MgSiO_3}^{Opx} \cdot (a_{FeCr_2O_4}^{Sp})^{Y_{Cr}^{Sp}} \cdot (a_{FeAl_2O_4}^{Sp})^{Y_{Al}^{Sp}} \cdot (a_{Fe_3O_4}^{Sp})^{Y_{Fe^{3+}}^{Sp}}}{a_{FeSiO_3}^{Opx} \cdot (a_{MgCr_2O_4}^{Sp})^{Y_{Cr}^{Sp}} \cdot (a_{MgAl_2O_4}^{Sp})^{Y_{Al}^{Sp}} \cdot (a_{MgFe_2O_4}^{Sp})^{Y_{Fe^{3+}}^{Sp}}} \\ & = \exp[-\{Y_{Cr}^{Sp} (\Delta G_{f,P,T}^{\circ})_1 + Y_{Al}^{Sp} (\Delta G_{f,P,T}^{\circ})_2 + Y_{Fe^{3+}}^{Sp} (\Delta G_{f,P,T}^{\circ})_3\}/RT] \end{aligned} \quad (10)$$

5.1 Orthopyroxene-chromite activity-composition relations

Assuming ideal mixing at each structural site, the activity of orthopyroxene end members may be expressed as follows.

$$a_{MgSiO_3}^{Opx} = X_{MgSiO_3}^{Opx} = (X_{Mg,M1}) (X_{Mg,M2}) (X_{Si,T})^2, \quad (11)$$

and

$$a_{FeSiO_3}^{Opx} = X_{FeSiO_3}^{Opx} = (X_{Fe,M1}) (X_{Fe,M2}) (X_{Si,T})^2, \quad (12)$$

where M1, M2 and T stand for M1 and M2 octahedral sites and the tetrahedral site respectively. For chromite the adopted activity-composition relations are as follows.

$$a_{MgCr_2O_4}^{Sp} = X_{Mg}^{Sp} \cdot (X_{Cr}^{Sp})^2, \quad (13)$$

$$a_{FeCr_2O_4}^{Sp} = X_{Fe}^{Sp} \cdot (X_{Cr}^{Sp})^2, \quad (14)$$

$$a_{\text{MgAl}_2\text{O}_4}^{\text{Sp}} = X_{\text{Mg}}^{\text{Sp}} \cdot (X_{\text{Al}}^{\text{Sp}})^2, \quad (15)$$

$$a_{\text{FeAl}_2\text{O}_4}^{\text{Sp}} = X_{\text{Fe}}^{\text{Sp}} \cdot (X_{\text{Al}}^{\text{Sp}})^2, \quad (16)$$

$$a_{\text{MgFe}_2\text{O}_4}^{\text{Sp}} = X_{\text{Mg}}^{\text{Sp}} \cdot (X_{\text{Fe}^{3+}}^{\text{Sp}})^2, \quad (17)$$

$$a_{\text{Fe}_3\text{O}_4}^{\text{Sp}} = X_{\text{Fe}}^{\text{Sp}} \cdot (X_{\text{Fe}^{3+}}^{\text{Sp}})^2. \quad (18)$$

Substituting the activity-composition relations of the orthopyroxene and chromite end members in (10) we get:

$$\begin{aligned} & \frac{(X_{\text{Mg,M1}}^{\text{Opx}}) (X_{\text{Mg,M2}}^{\text{Opx}})}{(X_{\text{Fe,M1}}^{\text{Opx}}) (X_{\text{Fe,M2}}^{\text{Opx}})} \cdot \frac{X_{\text{Fe}^{2+}}^{\text{Sp}}}{X_{\text{Fe}}^{\text{Sp}}} \\ &= \exp[-\{Y_{\text{Cr}}^{\text{Sp}} (\Delta G_{f,P,T}^\circ)_1 + Y_{\text{Al}}^{\text{Sp}} (\Delta G_{f,P,T}^\circ)_2 \\ & \quad + Y_{\text{Fe}^{3+}}^{\text{Sp}} (\Delta G_{f,P,T}^\circ)_3\}/RT] \end{aligned} \quad (19)$$

Now expanding the $\Delta G_{f,P,T}^\circ$ terms into standard state molar-free energy, entropy and volume of reaction and neglecting the volume terms, because they are exceedingly small for the Mg-Fe²⁺ exchange reactions, we get the final equilibrium equation used in the subsequent calculations:

$$\begin{aligned} & \frac{(X_{\text{Mg,M1}}^{\text{Opx}}) (X_{\text{Mg,M2}}^{\text{Opx}})}{(X_{\text{Fe,M1}}^{\text{Opx}}) (X_{\text{Fe,M2}}^{\text{Opx}})} \cdot \left(\frac{X_{\text{Fe}^{2+}}^{\text{Sp}}}{X_{\text{Mg}}^{\text{Sp}}} \right) \\ &= \exp[-\{Y_{\text{Cr}}^{\text{Sp}} (\Delta G_{f,298\cdot15,1,1}^\circ - \Delta S_{f,298\cdot15,1}^\circ (T - 298\cdot15)) \\ & \quad + Y_{\text{Al}}^{\text{Sp}} (\Delta G_{f,298\cdot15,1,2}^\circ - \Delta S_{f,298\cdot15,2}^\circ (T - 298\cdot15)) \\ & \quad + Y_{\text{Fe}^{3+}}^{\text{Sp}} (\Delta G_{f,298\cdot15,1,3}^\circ - \Delta S_{f,298\cdot15,3}^\circ (T - 298\cdot15))\}/RT] \end{aligned} \quad (20)$$

5.2 Orthopyroxene-chromite thermometry

For the validity of a geothermometric reaction, possibly the most important consideration is the sensitivity of the reaction constant K to a temperature change. For common mineralogical reactions of the kind considered here, the sensitivity can be expressed by the equation: $\ln K/dT = \Delta H/RT^2$ where K , ΔH , R and T stand for reaction constant, heat of reaction, the gas constant and reaction temperature (kelvin) respectively. For many Fe²⁺-Mg exchange reactions in minerals, although ΔH is not generally as large as in the other kind of thermometric reactions, e.g. a breakdown or net-transfer reaction with low volume change, successful thermometric applications have been made in a wide range of petrological problems. We may cite the examples of biotite and garnet (Ferry and Spear 1978; $|\Delta H| = 12454$ cal), garnet and orthopyroxene (Harlay 1984; $|\Delta H| =$

3740 $\pm$ 610 cal), olivine and spinel (Jackson 1969; Evans and Frost 1975; $|\Delta H| = 6053$ cal (Robie *et al* 1979). For our proposed orthopyroxene-chromite thermometer, $|\Delta H|$ ranges from 8332 cal to 163956 cal depending on the spinel end-member chosen e.g. with Cr, Al or Fe^{3+} (Robie *et al* 1979; Helgeson *et al* 1978; Engi 1978). Thus, from the point of view of temperature sensitivity of the reaction constant, the Fe^{2+} -Mg exchange reactions between orthopyroxene and chromite have a considerably better order of validity than most other established Fe^{2+} -Mg exchange thermometers and appear to be significantly better suited than olivine-chrome-spinel for the thermometry of ultramafic mineral assemblages.

We now attempt an empirical calibration by applying equation (20) to a number of equilibrated terrestrial orthopyroxene-chromite assemblages. The standard state thermochemical parameters used in these and subsequent calculations are given in table 6. Applying Wood-Banno (1973) orthopyroxene-clinopyroxene thermometer to an orthopyroxene-chromite rock from the Red Mountain ultramafics, New Zealand (Sinton 1977, Sample no. 37010), Sinton (*ibid*) obtained the equilibration temperature 980°C. We got 620°C for this sample applying equation (20). More recent geothermometric evaluations have shown that Wood-Banno thermometer gives temperatures at least about a 100°C too high (Bohlen and Essene 1979). This brings the two temperatures within about 200°C, putting the uncertainty due to analytical errors at $\pm 50^\circ\text{C}$. Mineral thermometers based on cation exchange generally register lower temperature than thermometers based on reconstructing reactions in the same rock, because of the greater kinetic ease of the former during cooling (Newton 1983). We consider it likely that the orthopyroxene-chromite equilibria in Sinton's sample 37010 were quenched at a somewhat lower temperature than the two-pyroxene equilibrium in the same rock. We have made a few more test applications of the orthopyroxene-chromite thermometer to terrestrial ultramafic rocks. These include 5 assemblages from the Seiad Complex of California (Medaris 1975; samples 1-a, 1-b, 2-a, 21-a and 24-c) and 2 assemblages from the Finero Complex of Italy (Medaris *ibid*, samples 8-a and 8-b). In all these cases, our formulation gives equilibration temperatures reasonably close to independent estimates.

From the foregoing considerations, we take (20) to be a dependable thermometric expression for Fe^{2+} - Mg exchange equilibria in coexisting orthopyroxene and

Table 6. Thermochemical data used in the equilibrium calculations.

Phase	Molar volume cal. bar ⁻¹	Gibbs energy of formation from elements at 298.15°K 1 bar Kcal. mol ⁻¹	Entropy of formation from elements at 298.15°K, 1 bar Gibbs mol ⁻¹
FeSiO ₃	0.75215 (1)	-349.1594 (1)	- 69.6355 (1)
MgSiO ₃	0.78757 (2)	-267.1600 (2)	- 61.9638 (2)
MgCr ₂ O ₄	1.05186 (1)	-326.2001 (1)	- 80.9847 (1)
FeCr ₂ O ₄	1.04111 (1)	-398.9195 (3)	- 91.8356 (1)
MgAl ₂ O ₄	0.94909 (1)	-519.8040 (1)	-100.1554 (1)
MgAl ₂ SiO ₆	1.39242 (4)	-712.8553 (4)	-142.045 (4)
SiO ₂ (Tridymite)	0.63410 (1)	-204.071 (1)	- 43.0292 (1)

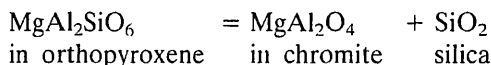
Numbers in parentheses give the source of the data: (1) Robie *et al* (1979) (2) Helgeson *et al* (1978).

chromite, which should be good enough for at least a relative and comparative thermometry of meteorites.

Next we apply the orthopyroxene-chromite thermometer to the Johnstown meteorite, a well-studied, equilibrated diogenite (Floran *et al* 1981). Using table 6 data and Floran *et al*'s data given in tables 1 and 2, we obtain 1093°C as the orthopyroxene-chromite equilibration temperature. This is a probable and reasonable near-solidus temperature in an essentially FeO-MgO-Cr₂O₃-SiO₂ system with small amount of other elements. The same method however gives for Yamato diogenite 1828°C (for chromite rim) and 2142°C (for chromite cores). Comparison of Yamato orthopyroxene-chromite compositions with the Johnstown values shows that the single factor responsible for the improbably high Yamato temperature is the high Mg/Fe²⁺ ratio of its chromite (table 5). For the Yamato orthopyroxene-chromite temperature to be of the same order as that of the Johnstown meteorite, the Yamato chromite's Mg/Fe²⁺ value should be < 0.20. This strongly indicates that the large chromite crystals of the Yamato diogenite over the entire grains from core to rim are out of chemical equilibrium with the orthopyroxene. Considered together with the inference from the zoning pattern of the chromite, this suggests that the Yamato chromite has a very high temperature pre-silicate history and for reasons beyond explanation it did not take part effectively in the subsequent equilibration of the meteorite reflected in its texture.

5.3 Orthopyroxene-chromite-silica barometry

Silica is a commonly observed constituent phase in pyroxene achondrite (Gross *et al* 1976). We formulate here a new barometer based on the equilibrium



MgAl₂SiO₆ being a hitherto unsynthesized component, its ΔG_f° , ΔS_f° and ΔV° are calculated from the data of Wood and Banno (1973). All basic thermochemical data used in the calculations are assembled in table 6. The reaction (4) has a ΔV° and is an excellent pressure-sensor. The derived barometric expression

$$P = \frac{10319.792 - 0.27635 T - RT \ln K}{0.208443}$$

where

$$K = \frac{a_{\text{MgAl}_2\text{O}_4}^{\text{SP}} \cdot a_{\text{SiO}_2}^{\text{Silica}}}{a_{\text{MgAl}_2\text{SiO}_6}^{\text{orthopyroxene}}},$$

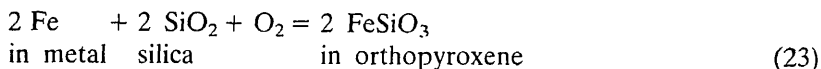
$$a_{\text{MgAl}_2\text{SiO}_6}^{\text{orthopyroxene}} = 4 \cdot X_{\text{Mg},\text{M2}} \cdot X_{\text{Al},\text{M1}} \cdot X_{\text{Al},\text{T}} \cdot X_{\text{Si},\text{T}}$$

$$a_{\text{MgAl}_2\text{O}_4}^{\text{SP}} = X_{\text{Mg}}^{\text{Sp}} (X_{\text{Al}}^{\text{Sp}})^2.$$

The equilibrium pressure of Johnstown diogenite, calculated from equation (22) is 21.8 kbar at 1093°C. For the Yamato diogenite, the same method gives a negative value for the equilibrium pressure. It is obvious that for a realistic pressure value, $a_{\text{MgAl}_2\text{O}_4}^{\text{sp}}$ must be considerably lower than 0.0064 (the figure we get from the Yamato chromite composition) once again, the Mg content of the Yamato chromite both at core and rim seems to be disproportionately higher than what its equilibration with the orthopyroxene would require.

5.4 Orthopyroxene-metal-silica oxygen barometry

Oxygen fugacity can be determined from the equilibrium:



for which we may write

$$\log f_{\text{O}_2} = \frac{\Delta G_f^\circ}{2.3026 RT} - \frac{\Delta S_f^\circ (T - 298.15)}{2.3026 RT} + \frac{\Delta V(P - 1)}{2.3026 RT} + \log K. \quad (24)$$

For the metal phase we have taken Fe activity to be equal to the mole fraction. Our calculations based on (24), standard state thermochemical data given in table 6 and our mineral composition data for Y-74013 diogenite yield $\log f_{\text{O}_2}$ equal to -20.21 for 880°C, -17.72 for 1000°C and -11.08 for 1500°C. $\log f_{\text{O}_2}$ is quite insensitive to pressure. These values are close to those obtained by Gooley *et al* (1976) for common diogenites i.e. -17.5 ± 1.2 at 880°C and -18.5 for the Johnstown diogenite, at the same temperature.

6. Conclusions

Texturally Yamato Y-74013 diogenite appears to have attained a high degree of equilibration. Coarse orthopyroxene grains with inclusions of early formed troilite and metal seem to have accumulated in a late SiO_2 -rich liquid from which a very small amount of plagioclase crystallized last. Orthopyroxene, troilite, metal and plagioclase of the Yamato sample have compositions close to these phases in common orthopyroxenite meteorites. The chromite, however, has significantly higher Mg/Fe^{2+} ratio, compared to common diogenites, and its overall pallasitic composition and the zoning pattern point to a high temperature, pre-silicate history. The chromite may represent cognate xenocrysts from an earlier and higher temperature crystallization regime.

For quantitative temperature estimate of orthopyroxene-chromite rocks, a new

barometer based on Mg-tschermak (in orthopyroxene) – spinel (in chromite) – silica equilibrium, yields an equilibrium pressure of 21.8 kbar for the same material at 1093°C. These values are well in accord with the possible location of diogenites in the deep mantle of a layered eucrite-howardite-diogenite parent planet.

The same P-T sensors, however, give quite improbable pressure and temperature for the Yamato diogenite even when the composition of the outermost rim of the zoned chromite crystals is considered. We conclude that the large chromite grains of the Yamato diogenite crystallized much earlier than the silicates at very high temperatures (possibly higher than 1100°C) and rapidly grew in size with a continuous compositional zoning from core to rim, reflecting a progressive enrichment of the parent liquid in Mg, Al and Ti. But for reasons unclear at the moment, these early chromites did not reach chemical equilibrium, even at their outermost rims, with the latter orthopyroxenes displaying an equilibrated, xenomorphic-granular texture. The equilibrium log fO_2 of the Yamato diogenite, calculated from the orthopyroxene – metal – tridymite equilibrium, is between –20.21 and –11.08 for the temperature range 880–1500°C. This is a normal oxygen fugacity range for pyroxene achondrites.

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Natural radioactivity distribution studies in Trivandrum district, Kerala, India

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Abstract. The distribution pattern of radioactive minerals in Trivandrum district of Kerala has been investigated. The surface radioactivity was measured employing a jeep-mounted four-channel gamma-ray spectrometer coupled to a high volume Na I (Tl) crystal detector. The distribution of radioactivity has been correlated with the corresponding litho-units and major structures of the study area. The total countrate from sedimentaries including beach sands and pegmatite rich zones (~ 3,500 CPS) are significantly higher than that of the laterites and gneisses which are substantiated by laboratory studies.

Keywords. Radiometric survey; jeep mounted gamma ray spectrometer; radioactive elements; Compton stripping coefficients; monazite, tectonic lineaments.

1. Introduction

The southwest coast of India is known to be one of the five major natural radioactive zones in the world. Several localities rich in radioactive minerals have been identified along the coastal regions of India, particularly in Kerala and Tamil Nadu (Brown and Dey 1975). The beach sands contain monazite, ilmenite, rutile, sillimanite, garnet and zircon abundantly. These heavy minerals are known to have been transported from the high ranges of Western Ghats through rivers to the coast (Mahadevan *et al* 1958; Aswathanarayana 1964). However, detailed work on these placer deposits and their bearing on the geology is lacking. The radiation dose rate and the effect on the environment have been studied in this area (Bharatwal and Vaze 1958; Ganguly *et al* 1964; Sunta *et al* 1971; Kulkarni *et al* 1974). In view of the geological significance a systematic radiometric study coupled with geological investigations was carried out in Trivandrum district covering the coastal plains, the midland and the highland areas (Western Ghats). These studies are aimed at identifying anomalous radioactive pockets and source regions and their bearing on the geological problems. Radiometric surveys in the polyphasedly deformed gneiss-granulite terrain of Kerala with enclaves of metasediments, numerous belts of pegmatite and extensive fault/shear zones are of utmost interest. The radiometric response can show characters attributable to recognizable events such as potash metasomatism which give an empirical pattern correlatable with lithology (Sherrington 1977) and delineate zones rich in radioactive minerals (Lovborg *et al* 1976, 1979; Wormald and Clayton 1976). Although airborne radiometric investigations have been successfully employed elsewhere for geological mapping and

and Humphery 1976) jeep-borne radiometric surveys have not been carried out in many regions. This paper presents the natural surface radioactivity measurements in Trivandrum district, Kerala (figure 1) employing a jeep-borne gamma-ray spectrometer. Laboratory analyses of the representative samples from the anomalous zones have also been carried out to substantiate the field measurements. The distribution pattern of radioactivity and the possible correlation with geological features are discussed.

2. Instruments and methods

A Scintrex type four-channel gamma-ray spectrometer (GAD-6), a high volume sensor of 1853 cc Na I (Tl) crystal (GSA-61S) and a strip-chart recorder (RCM-4)

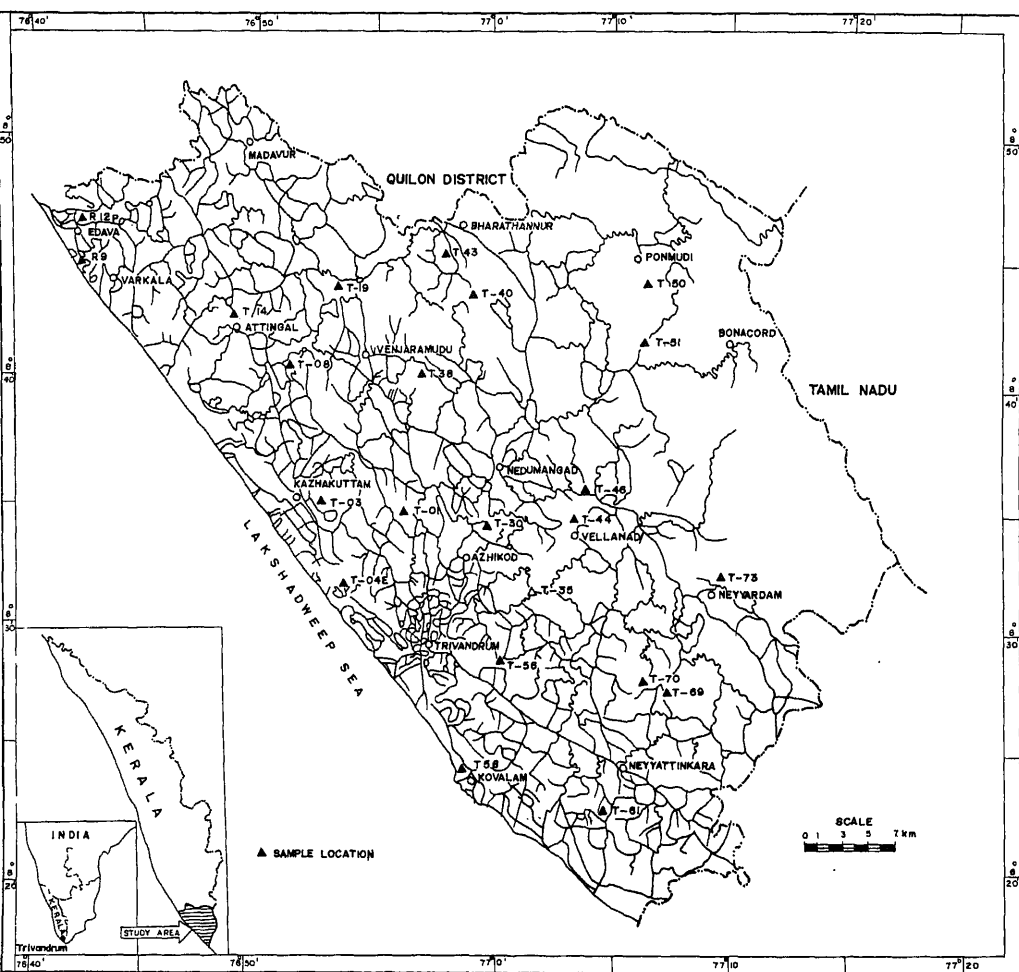


Figure 1. Map showing the study area, jeep traverse network, and locations of the samples.

were used for the survey operations. These instruments were specially designed for radiometric survey to measure individual elemental distribution in soils and rocks (Lovborg *et al* 1979; IAEA 1979). The sensor, mounted on top of the Landrover (jeep) at a height of about 3 m from the ground, enabled scanning an area of about 15 to 20 m radius of the ground effectively. However, since the detector was not shielded, it may be sensitive to the radioactivity from all directions up to a few hundred meters. Precautionary measures were taken to protect the sensor from jerks of the vehicle, temperature variation and moisture. Energy was calibrated once in a week using a ThO_2 calibration source of intensity 2.02 microcuries to ensure accurate measurements.

The spectrometer is a battery-operated spectral stabilized gamma-ray analyser which analyzes the responses from the sensor according to the four calibrated energy windows at 2.62 MeV line for thorium, 1.76 MeV line for uranium, 1.46 MeV line for potassium and 0.4 MeV to 2.77 MeV for total radioactivity. The output from these energy windows was fed to the corresponding pointers of the strip-chart recorder. The movement of the chart was synchronized with vehicle speed to obtain an authentic record of the radiometric data with respect to the distance. The pointers of the chart were calibrated by adjusting the amplifier gain of the chart-recorder using the digital display of the spectrometer. The countrate obtained in the field was corrected for Compton effect and recorded on the chart. The Compton correction factors were calculated in the laboratory conditions and the spectrometer is set accordingly. To eliminate the Compton effect on the energy spectra of the natural gamma radiation, stripping coefficients α , β and γ were determined using pure series (equilibrium sources) of thorium and uranium standards and set to the inbuilt stripping ratio switches of the spectrometer. For this purpose, a thorium source of 2.02 microcuries and uranium source of 1.15 microcuries of 5.5 cm dia each are used to calculate the stripping coefficients. As these standards were small in size, they were kept close to the detector to achieve reasonable counts for calculation of such coefficients. The stripping factors obtained in this way were compared with the standard values given in the manual. The relation between element concentration and countrate is given in table 1. Since the survey operations were confined to motorable roads, all the road junctions were numbered on the map as well as on the chart while recording the radiometric data. The vehicle was run at a constant speed of about 20 km/hr and every 100 m distance was marked on to the chart by an autopuncher. When in operation the accumulated counts for every 3 sec were recorded on the chart through pointers coupled to the output of the spectrometer. The recorded data on the chart was cross-checked with digital display from time to time and also in places

Table 1. Element concentration — Jeep-borne countrate correlation.

Sensor (volume in cc)	Thorium (ppm/count/ sec)	Uranium (ppm/count/ sec)	Potassium (ppm/count/ sec)	Stripping Coefficients.		
				α	β	γ
GSA-61S (1853)	2.7	1.37	0.13	0.5	0.49	0.67

where higher countrate is observed. The topographical features are noted to relate their influence on the countrate. Since the measurements are confined to a relatively thin surface layer of about 30–35 cm (Darnley 1975) the recorded radioactivity in each channel for thorium, uranium, potassium and total counts were alluvium or drift. Hence no correlation gradation was attempted with underlying bed rock in areas covered by transported material.

The data recorded in graphical form in the charts were decoded by averaging the counts for every 100 m distance. For easy reduction of the data, levels of radioactivity in each channel for thorium, uranium, potassium and total counts were chosen. Their values were plotted on the map and contoured. While contouring, several parameters such as field radioactivity, surrounding rock units, soil condition geometrical factors etc were considered. Representative samples from the anomalous radioactive zones were collected with the help of a portable 5 × 5 cm NaI(Tl) crystal detector. These samples were analyzed in laboratory to confirm the field data. The samples were powdered to 100 mesh size and analyzed using a gamma-ray spectrometer (GRS 23, ECIL) in the laboratory. The samples weighing 200 g were sealed in plastic containers and analyzed with counting time varying between 1000 and 2000 sec depending on the activity level present. The counts recorded for each element were converted to their respective concentration levels using standards of equal amount (Adams and Gasparini 1979). The radioelement concentrations are given in table 2. The results are comparable with field observations.

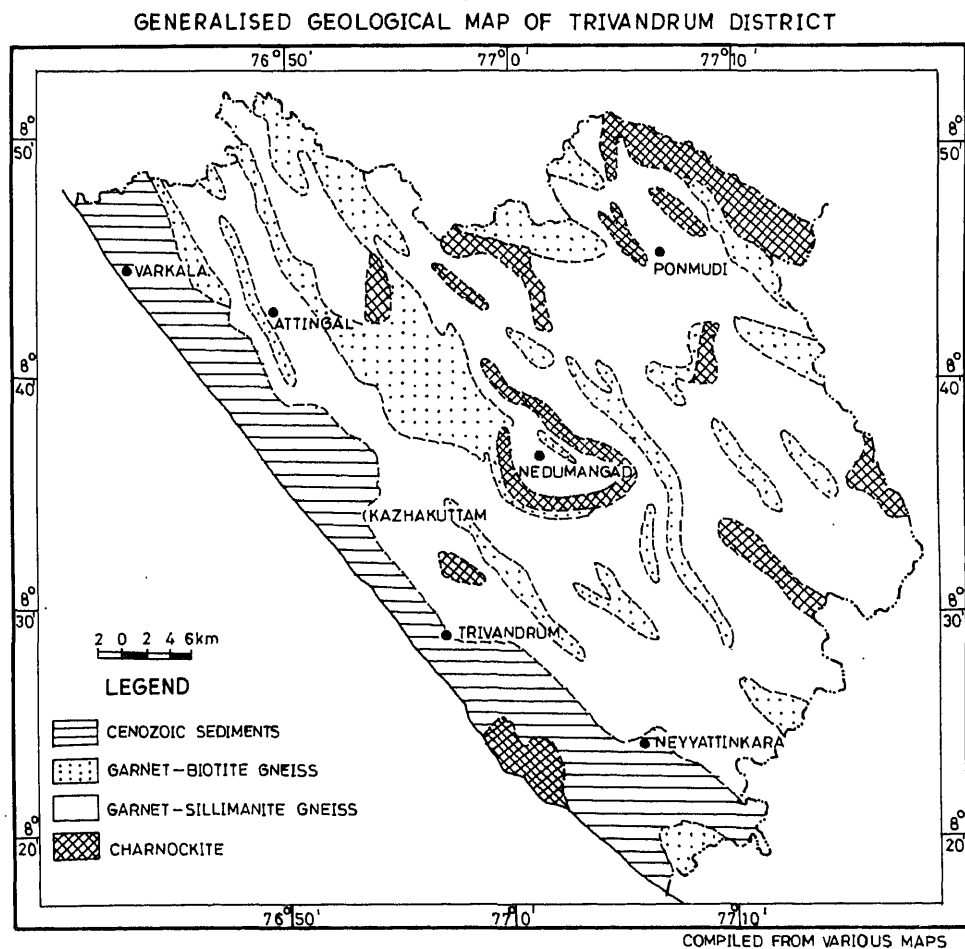
Table 2. Spectrometric analyses of radioelement concentrations of selected samples.

Sample No.	Sample description	K%	e ^U (ppm)	e Th (ppm)
T-30	Quartzo feldspathicgneiss	1.1	3	82
T-38	Garnet biotitegneiss	1.5	4	80
T-43	Quartzo feldspathicgneiss	1.4	8	79
T-44	Graphite schist	–	256	1282
T-50	Garnet sillimanitegneiss	0.6	19	25
T-69	Garnet sillimanitegneiss	0.9	11	23
R12-P	Sandstone rich in heavy minerals	–	362	8266
T-03	Surface soil rich in heavy minerals	–	658	3984
T-12	Soil upto 1 m depth	0.7	28	182
T-61	Surface soil	–	792	7922
T-01	Stream sediment	–	17	117
T-04E	Sediment rich in heavy minerals	–	1268	6085
T-08	Stream sediment	1.7	10	68
T-14	Stream sediment	–	109	397
T-19	Stream sediment	–	121	456
T-35	Stream sediment	–	101	451
T-40	Stream sediment	–	667	2947
T-46	Stream sediment	–	86	267
T-51	Stream sediment	–	52	263
T-56	Stream sediment	–	61	276
T-70	Stream sediment	–	265	1961
T-73	Stream sediment	1.2	297	977
R-09	Beach placer	–	1412	21999
T-58	Beach placer	–	507	3049

3. Geological set-up

The study area viz Trivandrum district is the southern most part of Kerala having an area of over 2000 km². The area can be divided into three distinct physiographic units, highlands, midlands and lowlands. The highlands (> 100 m) characterized by hill ranges with diverse drainage pattern are covered with thick vegetation. The midlands (10–100 m) characterized by flat topped hills and valleys have lateritic soil cover (5 to 10 m). The road density in the lowlands is fairly high while in the highlands it is rather poor. The route network related to the present jeep-borne survey is depicted in figure 1.

The area is predominantly constituted by rocks of Precambrian age which include charnockites, garnet-sillimanite gneisses, garnet-biotite gneisses and their migmatitic variants (figure 2). The charnockites contain patches of gneisses with diffused contacts. The gneisses abounding the south and southeast of Trivandrum are highly migmatised. All these rock types are intensely folded and sheared. The pegmatites



which traverse the gneisses at places have fairly high content of monazite veins and disseminations of graphite. Gabbros and dolerites of late Mesozoic to early Cenozoic age traverse the gneisses and granulites.

Tertiary and recent sediments are found mainly in the coastal tracts of this region as narrow NNW–SSE trending bands. The tertiary sediments unconformably overlie the Precambrians which tend to assume an E–W trend as one approaches south towards Trivandrum, and extend into Tamil Nadu with the same trend. These bands are mostly formed of clays and sandstones with older 'teri' sands. Laterites occur in low flat topped ridges and hills, covering the tertiaries and the crystalline rocks between the Western Ghats and the coastal area.

4. Results and Discussion

The distribution of the total radioactivity and the individual radioactive elements Th, U and K obtained in counts per sec are presented in figures 3 to 6. Unlike potassium the distribution pattern of uranium and thorium is similar to that of total radioactivity. The levels of radioactivity over the sedimentary formations vary from low to high, whereas over the crystalline formations, the radioactivity is generally maintained at low levels barring a few anomalous zones. Thus, there is a general relationship between radioactivity and the geology of the area. Based on the radio-elemental distribution, the crystalline-sedimentary contact zones can be inferred. The demarcation of the contact zone can be clearly seen in the total radioactivity map (figure 3). Anomalous zones within the sedimentary formations (around Edavai, Kazhakuttam and Kovalam) as well as in the crystalline formations (around Vellanad, Azhikode and Venjaramud) are identified where peak values > 5,000 CPS in total radioactivity are observed as against the normal background of 1,200 CPS. The laboratory analyses of the samples collected from these anomalous zones are given in table 2. The Precambrian charnockites, gneisses and their migmatitic derivatives are reported to be the parent rocks for radioactive minerals in Kerala and Tamil Nadu (Pichamuthu 1959; Poulouse 1956). In the present study, the charnockites and high grade gneisses generally indicate depleted Th and U values with respect to potash values. Similar feature was reported from the granulites of other parts of the Indian Peninsula (Narayanaswamy and Venkatasubramanian 1969; Atal *et al* 1978) and from other parts of the world (Iyer *et al* 1984). All the anomalous zones identified within the crystalline formations lie along the sheared flanks of the major plunging antiform. When shearing and crystallization affect the late crystallising minerals, much of the Th is readily released (Constable and Hubbard 1981) and gets concentrated in the acidic intrusives. In addition, intrusive phases of granites and granitic pegmatites are the main sources for monazite (Soman and Nair 1983) in the study area. Hence, monazite is the cause of higher values of thorium. Slightly higher concentrations are observed around Neyyar dam, Neyyattinkara, Ponnudi, Madavur etc. where the monazite distribution in the rock is unevenly scattered.

In the midland regions the charnockites and gneisses are lateritized to a considerable extent and high thorium values are observed occasionally at these exposures. These laterites are easily eroded and the disintegrates are sorted and carried by the streams resulting in the concentration of heavy minerals. A slight

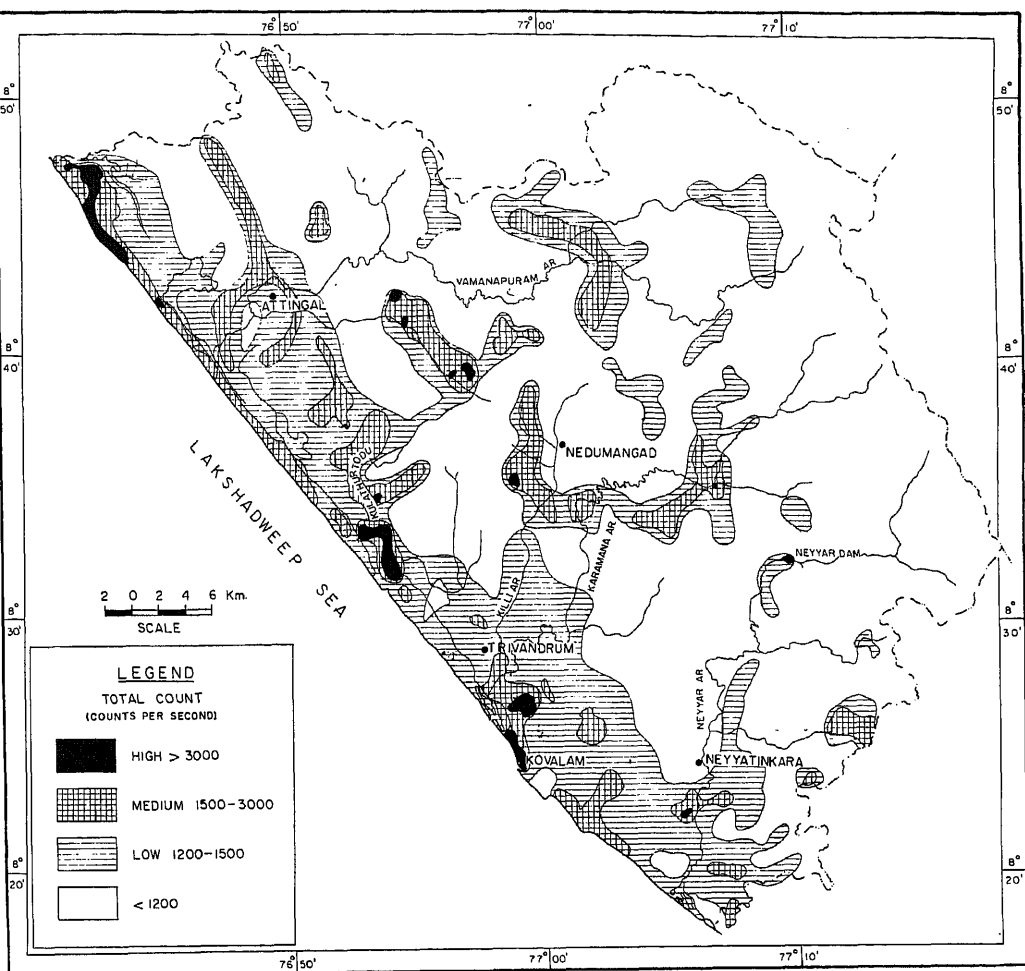


Figure 3. Radiometric map of Trivandrum district showing the distribution of total radio activity (measured with 1853 cc NaI(Tl) detector).

increase in the countrate is also one of the features at several places along the terraces of certain rivers and their major tributaries. The laterite cappings are usually enriched in thorium relative to uranium (IAEA 1979) as heavy minerals like zircon and monazite which are resistant to weathering get concentrated in them.

The coastal lands represent a depositional surface in contrast to the midlands and high lands. Enormous amount of material is brought from the higher altitudes by rivers and torrents and the heavy minerals get separated due to the continuous wave action of the sea forming beach placers along the coast. The Quilon and Warkalli formations exposed to the north around Varkala-Edavai, and to the south around Kovalam have thin bands enriched in heavy minerals. The areas around Kazhakuttom, northwest of Trivandrum, is mainly of sand loams with a zone rich in black sand containing monazite. The higher values recorded along the coastal belt

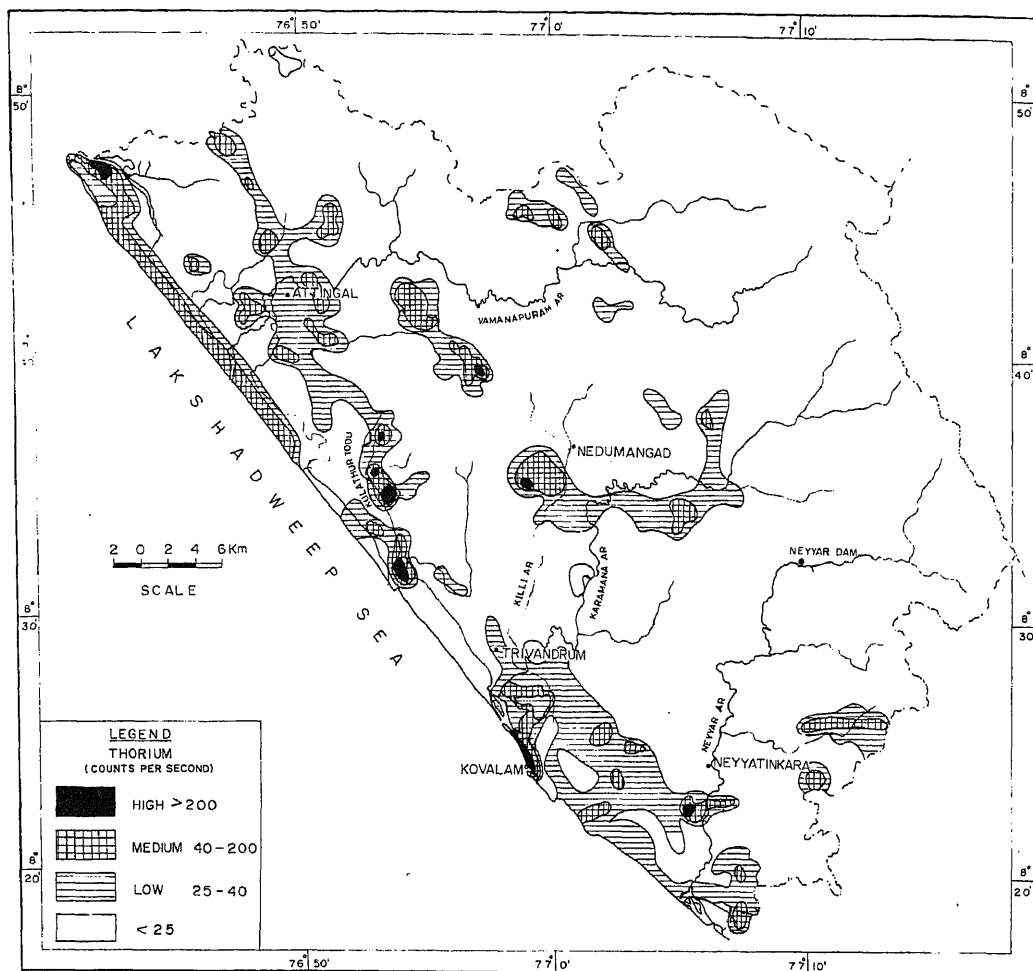


Figure 4. Distribution of radioactivity due to thorium concentration (measured with 1853 cc NaI(Tl) detector).

and minor cheralite. These minerals are derivatives of the litho-units forming the high ranges of the region.

Physiography also seems to have played a dominant role in controlling the radioactive mineral distribution. The study area is drained by three major river systems namely Neyyar, Karamana and Vamanapuram Ar. The west flowing drainage system of these rivers had developed in response to coastal upwarping and faulting related to development of continental margin. Subsequently, the drainage pattern and the physiography are largely controlled by the fault/fracture systems. Various evidences suggest that the course of rivers follow the tectonic lineaments. These lineaments also control the occurrence of both acid and basic intrusives which are potential locales of mineralization. The bulk of the sediment load carried down by these rivers and streams might have originated from areas close to these lineaments.

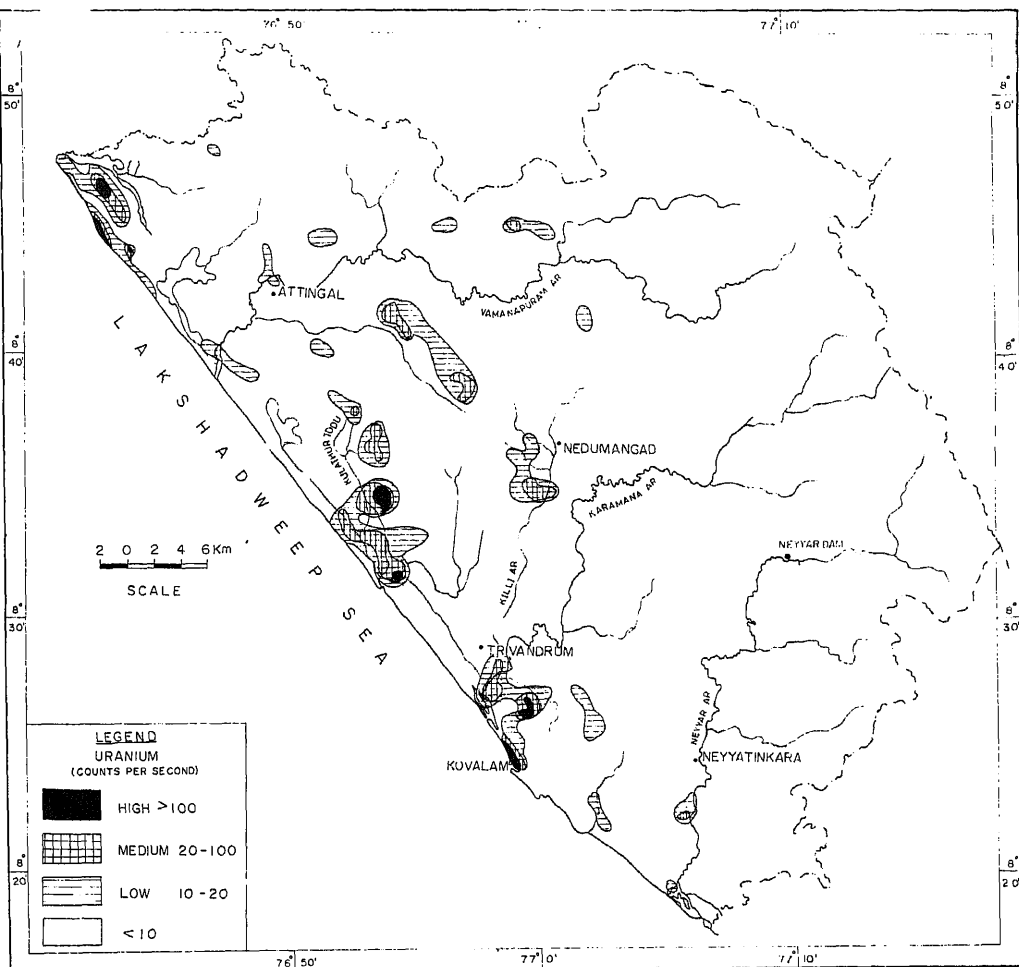


Figure 5. Distribution of radioactivity due to uranium countrate (measured with 1853 cc NaI(Tl) detector).

which have influenced the localization of these radioactive minerals in the coastal belt.

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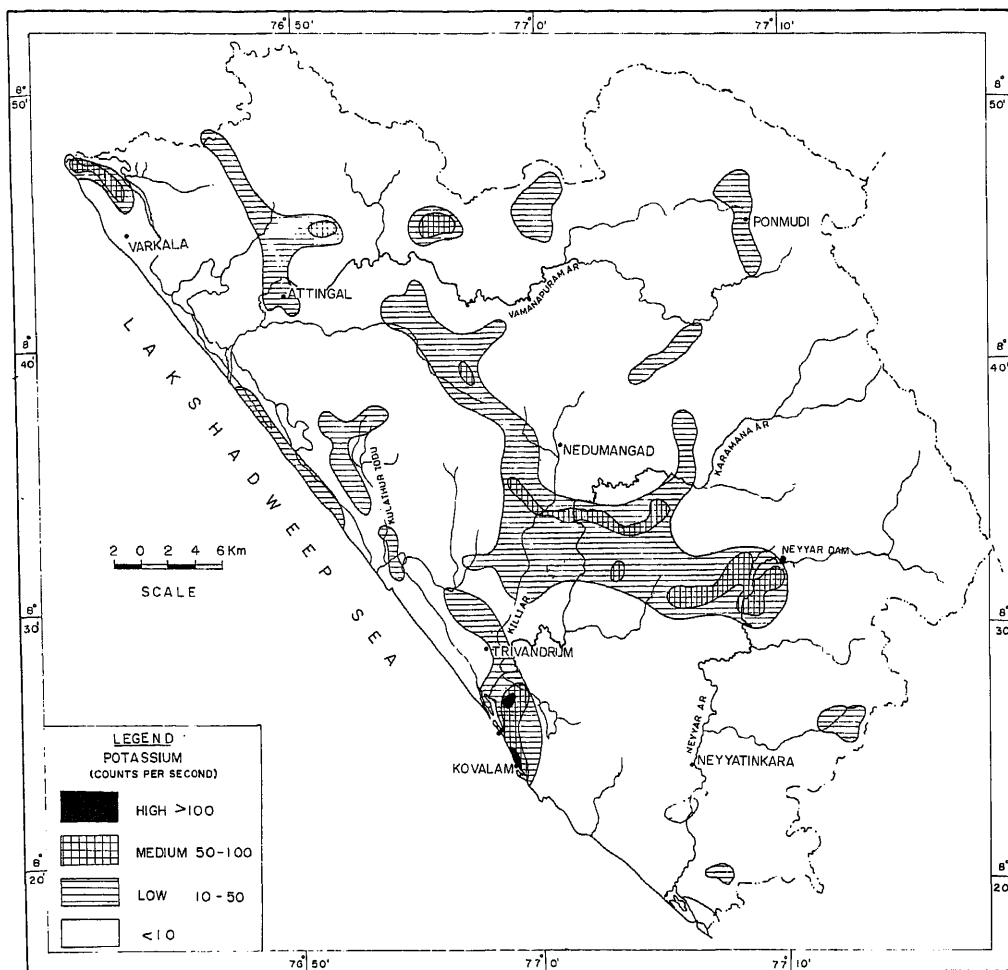


Figure 6. Distribution of radioactivity due to potassium count rate (measured with 1853 cc NaI(Tl) detector).

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Seasonal and solar cycle variations of night-time anomalous enhancements in total electron content

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Abstract. Seasonal and solar cycle variations of the various characteristics of night-time anomalous enhancements in total electron content (TEC) of the ionosphere are presented for a low latitude station, Hawaii by considering TEC data for a full solar cycle. All the characteristics of the TEC enhancements have seasonal and solar cycle dependence. TEC enhancement characteristics such as frequency of occurrence, amplitude and duration are positively correlated with solar activity. The possible source mechanisms for the observed enhancements are also discussed.

Keywords. Seasonal variations; solar cycle variations; anomalous enhancement; total electron content.

1. Introduction

It is well known that the electron density N_e and total electron content (TEC) of the ionosphere do not decrease throughout the night as predicted by a simple theory. Anomalous enhancements in these parameters have often been observed under a wide range of geophysical conditions (eg: Evans 1965; Klobucher *et al* 1968; Young *et al* 1970; Janve *et al* 1979; Davies *et al* 1979; Essex and Klobuchar, 1980; Leitingner *et al* 1982). The characteristics of these enhancements have an important bearing on the source mechanisms responsible for the maintenance of night-time ionosphere (Titheridge 1968a).

Although detailed studies have been made on various aspects of the night-time TEC enhancements, one aspect that has not yet received any attention is the effect of solar cycle variation on the various characteristics of the phenomenon. Some mechanisms have been proposed which might be responsible for the observed TEC enhancements but none could be identified with any degree of certainty. In order to understand the importance of the various source mechanisms, it is essential to build a fairly comprehensive picture on all observational aspects of the phenomenon. In this paper, we present the seasonal and solar cycle variations of the various characteristics of the night-time TEC enhancements for a low latitude station, Hawaii, by considering TEC data for nearly a full solar cycle (September 1964-December 1973). The possible source mechanisms for the observed TEC enhancements are also discussed.

2. Experimental data and analysis

The TEC data derived from Faraday rotation measurements of the VHF telemetry transmissions of Syncom-3 and ATS-1 geostationary satellites by Yuen and Roelofs (1971) and Roelofs (1973) at Hawaii ($21^{\circ} 15' \text{N}$, $157^{\circ} 46' \text{W}$) from September 1964 to December 1973 from the data base for the study. Syncom-3 data is used for the period September 1964–December 1966 and ATS-1 data for the remaining period. The subionospheric point of ATS-1 corresponding to an altitude of 400 km is (19.7°N , 157.2°W) and for Syncom-3 the corresponding coordinates are (19.7°N , 159.3°W) in 1964 and (19.1°N , 161.0°W) in 1966.

In characterizing a night-time TEC enhancement, the same criterion as adopted by Young *et al* (1970) is applied. Accordingly, an anomalous TEC enhancement is defined as the excess content. ΔTEC which remains after the exponentially decaying background part of the diurnal content is subtracted from the total content. For the present study, as done by Young *et al* (1970), only those enhancements which have an amplitude $\geq 20\%$ of the background content have been considered. The majority of the enhancements are found to have a single peak either in pre-midnight or in post-midnight hours whereas a few have peaks in both halves of the night. In the case of enhancements having double peaks, only the prominent peak is considered for statistical study. The enhancements are divided into pre-midnight and post-midnight enhancements depending on the local time at which the prominent peak of the enhancements occurs. As the TEC enhancements at low latitudes are independent of geomagnetic activity (Tilferidge 1968b and

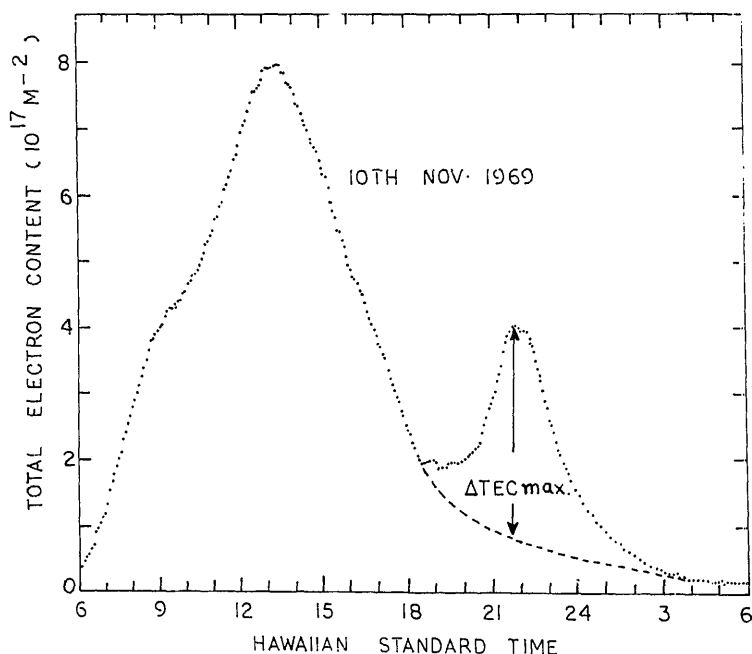


Figure 1. Diurnal variation of TEC at Hawaii on a solar maximum day illustrating the

Young *et al* 1970), no distinction is made between the quiet night and disturbed night enhancements. An example of the diurnal variation of TEC at Hawaii on a solar maximum day illustrating the occurrence of a strong night-time anomalous enhancement is shown in figure 1. As indicated in the figure, the amplitude of the TEC enhancement can be as much as three times the night-time background content.

3. Results

3.1 Frequency and time of occurrence

During the period under study, covering nearly a full solar cycle, a total of about 1670 night-time TEC enhancements have been observed at Hawaii. The histograms of the percentage of occurrence in each month for pre-midnight and post-midnight enhancements and the monthly mean Zurich relative sun spot numbers for the whole period of study are shown in figure 2. It is seen that the night-time TEC

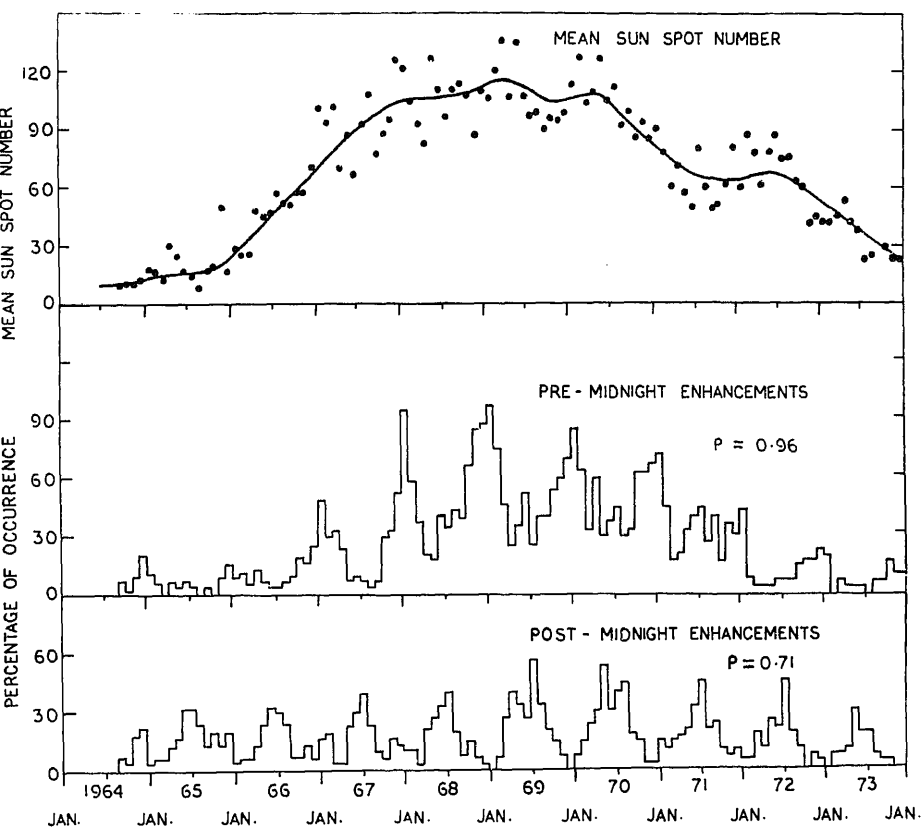


Figure 2. Dependence of the frequency of occurrence(%) of pre-midnight and post-midnight TEC enhancements on solar activity during the period 1964-73 at Hawaii. Top portion of the figure gives the variation of monthly mean sun spot number.

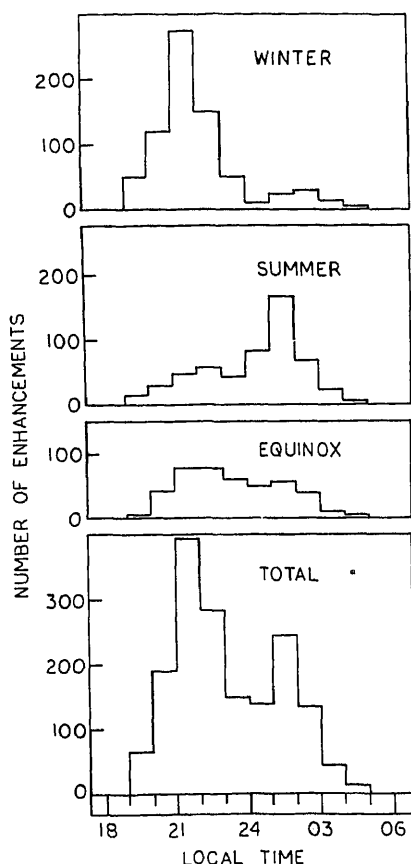


Figure 3. Distribution of the occurrence time of the peak of TEC enhancement at Hawaii for each season and for all the data combined.

enhancements occur in all seasons, more frequently in winter and less so in summer and equinox. Out of the total number of 1670 enhancements, 675 occur in winter, 570 in summer and 425 in equinox. Frequency of occurrence of pre-midnight and post-midnight enhancements, are well correlated with solar activity with correlation coefficients of 0.96 and 0.71 respectively. During the solar maximum year (1969), 77% of the nights have TEC enhancements of which 55% occur during pre-midnight hours and 22% during post-midnight hours. During the solar minimum year (1973), only 16% of the nights have TEC enhancements of which 7% occur during pre-midnight hours and 9% during post-midnight hours.

TEC enhancements can maximize anywhere from 1900 to 0500 LT. Figure 3 shows the distributions of the occurrence times of the enhancements peak in winter, summer, equinox and for all the data taken together. In winter, more than 80% of the enhancements maximize during pre-midnight hours and the most probable time of maximization is 2100–2200 LT. In summer, more than 65% of the enhancements maximize during post-midnight hours, the most probable time being 0100–0200 LT. During equinox, there is no definite probable time for content maximization although the frequency of occurrence is slightly greater during the

plot, two peaks are seen naturally, one at 2100–2200 LT and the other at 0100–0200 LT, which correspond to the winter and summer enhancements respectively.

3.2 Time duration and amplitude

Histograms showing the distributions of the half-amplitude duration (τ) and the amplitude ($\Delta\text{TEC}_{\text{max}}$) of pre-midnight and post-midnight TEC enhancements for all the data are shown in figures 4(a) and (b) respectively. The most probable value of τ for pre-midnight and post-midnight enhancements are about the same with a value of about 100 minutes. However, a few cases of TEC enhancements lasting throughout night have been observed especially during summer. In figure 4(b), the number of enhancements varies sharply with the amplitude, $\Delta\text{TEC}_{\text{max}}$. Pre-midnight and post-midnight enhancements can have values of $\Delta\text{TEC}_{\text{max}}$ in the ranges $0.2 \times 10^{17} - 3 \times 10^{17} \text{ m}^{-2}$ and $0.1 \times 10^{17} - 2 \times 10^{17} \text{ m}^{-2}$ respectively. The most probable values for the two cases are $0.6 \times 10^{17} \text{ m}^{-2}$ and $0.4 \times 10^{17} \text{ m}^{-2}$. Normalized (with respect to the starting level of the enhancement) $\Delta\text{TEC}_{\text{max}}$ has been found to range from 15% to 220% and the most probable value is around 50%. Enhancements which are greater than 100% are about 1/4 of the total number.

Seasonal variations of τ of pre-midnight and post-midnight enhancements presented in figure 5 (dashed curves) averaged for all the years, show a broad

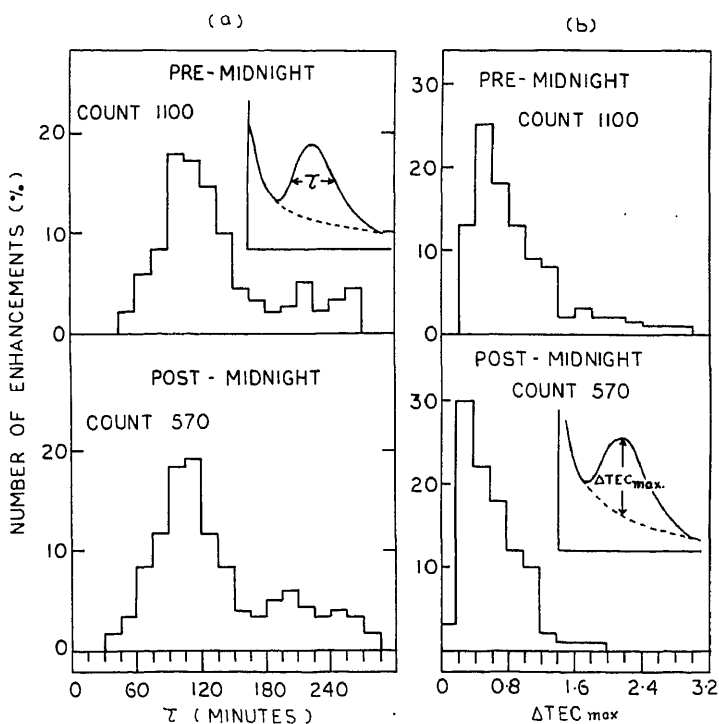


Figure 4. Distribution of (a) the half-amplitude duration (τ) and (b) the amplitude ($\Delta\text{TEC}_{\text{max}}$) of pre-midnight and post-midnight TEC enhancements at Hawaii.

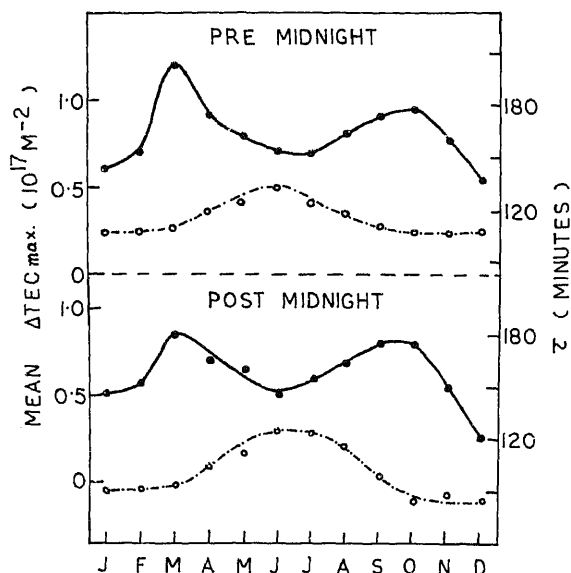


Figure 5. Monthly mean variation of τ (○---○) and $\Delta\text{TEC}_{\text{max}}$ (●---●) of pre-midnight and post-midnight TEC enhancements at Hawaii.

maximum around summer months. The variation for the post-midnight enhancements is a bit more pronounced than that for pre-midnight. Seasonal variations of $\Delta\text{TEC}_{\text{max}}$ of pre-midnight and post-midnight enhancements, shown in the same figure (solid curves), reveal peak values during the equinoctial months.

The half-amplitude duration τ and the amplitude $\Delta\text{TEC}_{\text{max}}$ of TEC enhancements show very good correlation with solar activity. Mean values of τ and $\Delta\text{TEC}_{\text{max}}$ for both pre- and post-midnight enhancements vary essentially in the same manner as the sun spot number in all seasons as shown in figure 6. In the case of $\Delta\text{TEC}_{\text{max}}$, however, the variation is sharper for pre-midnight than for post-midnight enhancements. Further, if one considers the enhancements in terms of the percentage of the daytime peak TEC, the variation would appear to be much less pronounced. While the daytime peak TEC and post-midnight $\Delta\text{TEC}_{\text{max}}$ vary by about the same factor over a solar cycle, the corresponding factor is about 1.5 times as great for the pre-midnight $\Delta\text{TEC}_{\text{max}}$.

4. Discussion

The results presented above provide a fairly comprehensive picture of the seasonal and solar cycle variations of anomalous night-time TEC enhancements at a low latitude. Seasonal variations of TEC enhancements presented here agree in general with those reported earlier (Young *et al* 1970; Janve *et al* 1979). However,

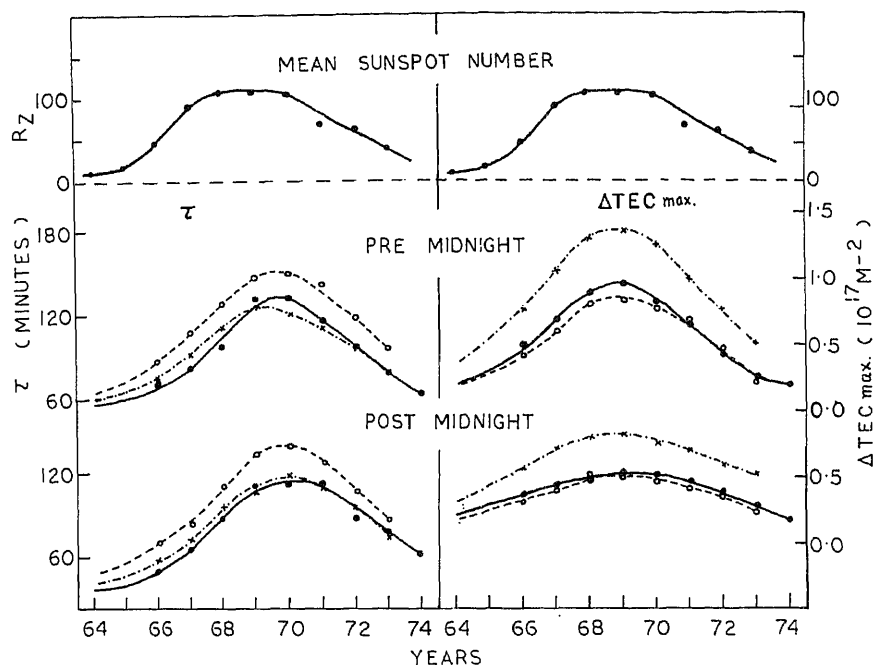


Figure 6. Solar cycle variation of yearly mean sunspot number and seasonal mean τ and $\Delta\text{TEC}_{\text{max}}$ of pre-midnight and post-midnight TEC enhancements for the three seasons (●—● winter, ○—○ summer and ×—× equinox) at Hawaii.

TEC enhancements on solar activity is studied here by considering TEC data for nearly a full solar cycle.

The potential source mechanisms for the observed night-time TEC enhancements at low latitudes are believed to be (a) plasma transfer from conjugate ionosphere and (b) electrodynamic drifts and neutral winds. There are a few other mechanisms linked with the phenomena of night-time TEC enhancements such as plasma diffusion from protonosphere (Titheridge, 1968a), Cross-L plasmaspheric compression and consequent enhancement in plasmasphere-ionosphere plasma flow (Davies *et al*, 1979) and corpuscular ionization (Titheridge 1968a) which are important only at mid and high latitudes.

According to conjugate point transfer mechanism of ionization (Rishbeth 1968; Wickwar 1974), the plasma transfer will occur only when there exists a large density or temperature gradient between two magnetically conjugate points. During winter, this large gradient is present because sun sets later at the conjugate summer hemisphere. The hypothesis of conjugate ionosphere as a source seems to be favoured by the following observational features as well: (a) the maximum frequency of occurrence of night-time electron content enhancements during

the observed enhancements is clear from the enhancements which also occur during post-midnight hours when there is no solar EUV radiation at the conjugate ionosphere.

Neutral winds and electrodynamic drifts may also act as important source mechanism, especially to account for the post-midnight TEC enhancements. The pattern of the neutral winds are such that it has a strong N-S component directed towards the equator during 0100–0300 LT (Kohl and King 1967). The component shows a sharp gradient at low latitudes and this leads to a strong convergence of plasma with peak occurring at about the same latitude as the equatorial anomaly peak. The flux calculations show that the observed peak enhancement can be realized in about an hour. In addition to the neutral winds, electrodynamic drifts also can be of some significance for the low latitude enhancements. There is a strong eastward drift which drops rapidly with time after midnight in the latitude range of 0–30°N (Maeda and Kato 1966). This, when coupled to the E–W gradients in the total electron content, can lead to a convergence of the plasma during post-midnight hours. Its contribution is about 1/3 of that of the neutral winds. Thus, while no single process could account for the observed TEC enhancements, it does seem possible to meet the requirements, atleast qualitatively, on the basis of a combination of the processes as outlined above.

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Estimation of evaporation rates over the Arabian Sea from satellite data

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Abstract. Utilizing both the SAMIR brightness temperatures of Bhaskara II and GOSSTCOMP charts of NOAA satellite series, the evaporation rates over the Arabian Sea for June 1982 are estimated through the bulk aerodynamic method. The spatial distribution of evaporation rates estimated from the satellite data sets coincides well with those obtained from ship observations as well as from climatological data. The accuracy in the estimation of evaporation rates has considerably been improved after the removal of bias in sea surface temperature and is about ± 0.8 mm/day.

Keywords. Satellite microwave radiometer; Bhaskara II; evaporation; GOSSTCOMP charts; air-sea exchange.

1. Introduction

The ocean-atmospheric coupled system exchanges heat, momentum and energy between the two fluids. A larger part of the heat is supplied by the tropical oceans, through evaporation for development of large-scale weather systems. Monsoon rainfall pattern over India has been related to the air-sea interaction parameters over the Arabian Sea (Shukla 1975; Shukla and Mishra 1977; Bhumralkar 1978; Rao *et al* 1981; Gadgil *et al* 1984 and Babu *et al* 1985). The knowledge of evaporation rates over the ocean is essential for parametrizing ocean-atmospheric coupled predictive models. For real-time acquisition of evaporation fields over wider oceanic areas, Liu (1984) suggested the possibility of using satellite measurements of surface wind speed (SW), sea surface temperature (SST) and integrated water vapour (WV) content data derived from satellite multichannel microwave radiometer (SMMR) in the bulk parametrization method in which the WV content was related with surface humidity. He further suggested the possibility of combining observations from different sensors such as infrared sensors — AVHRR (advanced very high resolution radiometer), HIRS (high resolution infrared sensor), VAS (VISSR atmospheric sounder) and microwave radiometers on board individual satellites for the estimation of evaporation rates. Taylor (1984) also mentioned that satellite microwave radiometers provide good estimates of parameters for use with the bulk aerodynamic formula. Several investigations have been reported in the literature relating WV content in the atmosphere with humidity fields in the lower levels over different regions (Reitan 1963; Bolsenga 1965; Smith 1966 and Zillman 1972). Similar studies have not been documented so far over the northern Indian Ocean.

In the present paper, the feasibility of deriving evaporation rates from SAMIR data of Indian satellite Bhaskara II and AVHRR data of NOAA series is studied. An empirical relation between surface dew point temperature and WV content is derived for monsoon season based on observations made over both the Arabian Sea and the Bay of Bengal where seasonal variability of weather is well pronounced. A typical mean monthly distribution (June 1982) of evaporation rate over the Arabian Sea estimated from satellite data is compared with that obtained from ship's data and also with climatological pattern.

2. Data and methods

The evaporation rate E (cm/day) from the sea surface can be estimated from the following bulk-aerodynamic formula

$$E = 9.0284 \times 10^{-3} (e_s - e_a) U_a, \quad (1)$$

where e_s and e_a are the saturated vapour pressure of air (mb) at SST and vapour pressure of air (mb) at anemometer height respectively and U_a is the wind speed (m/sec) at anemometer level. The constant (9.0284×10^{-3}) has been obtained after considering ρ (density of air) as 1.2×10^{-3} (g/cm³) and C_D (drag coefficient) as 1.4×10^{-3} (Rao *et al* 1981) though C_D varies with wind speed and low level stability. The assumption of constant ρ and C_D , if applied uniformly in both sources of data (satellite and ship), may not matter as far as relative comparison of results from these two sources is concerned.

SAMIR (satellite microwave radiometer) data analyzed for this study are limited to 4° to 24°N and 60° to 80°E over the Arabian Sea. The values of surface dew point temperature and wind speeds derived from SAMIR and SST from AVHRR (NOAA) averaged over each 4° square grid in the study area of the Arabian Sea during June 1982 have been used in (1). In the analysis of evaporation rates derived from conventional ship's data obtained from Indian Daily Weather Reports (IDWRs), linear interpolation has been done for some grids for which data are not available.

2.1 Retrieval of WV content and surface wind speeds from SAMIR

The SAMIR on board Bhaskara II, an Indian experimental satellite for earth observations, consists of three Dicke type microwave radiometers which measure the brightness temperature at 31.4, 19.35 and 22.235 GHz with a spatial resolution of 125 km and a temperature resolution of about 1°K. The details of SAMIR sensor have been given by Pandey *et al* (1984).

SAMIR data for Bhaskara II orbits in 1982 over the Arabian Sea are obtained from Space Applications Centre, Ahmedabad. For each grid the average brightness temperatures at 19.35 and 22.235 GHz frequencies have been computed and then WV content (gm/cm²) and wind speed (m/sec) are estimated from the following regression equations given by Pandey *et al* (1984) and Pathak *et al* (1983).

$$\text{WV} = 0.3178 - 8.9963 F, \quad (2)$$

$$\text{SW} = -14.43 - 0.1187 T_B(19) + 0.1833 T_B(22), \quad (3)$$

where $T_B(19)$ and $T_B(22)$ are the brightness temperatures at 19.35 and 22.235 GHz frequencies respectively and $F = \ln(280 - T_B(22)) - \ln(280 - T_B(19))$.

Relation between WV content and surface dew point temperature

In general, the moisture content in the atmosphere is more in the near surface layer and decreases with height. Smith (1966) originally developed the following general empirical relation between WV and surface dew point temperature $t_d(^{\circ}\text{C})$

$$\text{WV} = \frac{\varepsilon E_0}{g(\lambda + 1)} \times 10^{(\alpha t_d - \beta)/(t_d + \gamma)}, \quad (4)$$

where $E_0 = 6108 \text{ dynes/cm}^2$; $\alpha = 7.5$; $\beta = 178.5^{\circ}\text{C}$; $\gamma = 148.4^{\circ}\text{C}$; $\varepsilon = 0.622$ and λ is a correction function which is dependent on both season and latitude. In order to simplify the $(t_d + \gamma)$ term is taken as 160.2°C after considering normal t_d value (11.8°C) for the northern Indian Ocean. Minimum errors are expected if climatological t_d is used since t_d is one order less than γ . For the northern Indian Ocean region, λ is taken from Smith's (1966) table. Finally (4) is simplified as following

$$t_d = -0.3632 + 15.0661 \ln(\text{WV}) \quad (5)$$

In order to compare the r.m.s error in the estimation of t_d from the above equation with that obtained from the actual data, a direct regression equation is also developed using the radiosonde data collected on board various research ships over different parts of the northern Indian Ocean during MONSOON 77. Figure 1 shows the location of research ships along with the period of observations. The number of observations at each location is given in table 1. A low correlation (correlation coefficient: 0.26; r.m.s error: $\pm 0.87^{\circ}\text{C}$) is obtained between t_d and WV, using all the data points as mentioned in table 1. This correlation is considerably improved (correlation coefficient: 0.94; r.m.s error: $\pm 0.24^{\circ}\text{C}$) when

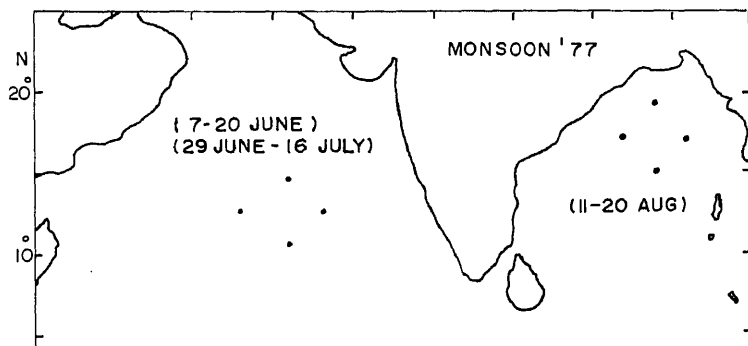


Table 1. Number of radiosonde observations used for development of regression relation between dew point temperature and WV content.

Location	June	July	August 1977
10.5°N & 66.0°E	60	68	—
12.5°N & 63.9°E	52	68	—
12.5°N & 68.0°E	56	72	—
14.5°N & 66.0°E	44	72	—
15.2°N & 89.0°E	—	—	36
17.2°N & 91.0°E	—	—	32
17.3°N & 87.0°E	—	—	36
19.3°N & 89.0°E	—	—	40

averaged monthly values at each location – total 12 points (4 ships $\times$ 3 months) – instead of individual values, are considered giving rise to the following equation:

$$t_d = 13.93 + 6.628 \ln (\text{WV}). \quad (6)$$

It is to be pointed out here that even though daily t_d , WV values do not correlate well due to small-scale fluctuations, monthly averages agree better.

The scatter diagram of $\ln (\text{WV})$ versus dew point temperatures and the regression lines obtained separately by (5) and (6) are shown in figure 2. Equation (6) is finally used to estimate surface dew point temperatures from the SAMIR derived WV content since r.m.s. error ($\pm 0.92^\circ\text{C}$) obtained by (5) is high.

3. Results and discussion

To examine the validity of the empirical relation between WV content and t_d over the northern Indian Ocean during the other seasons, the t_d values estimated from SAMIR WV content during Bhaskara II orbits in 1982 are compared with ship's data (within 125 km, since spatial resolution of SAMIR is 125 km) (table 2). Dew point temperature estimates from SAMIR in general agree fairly well with the ship data when WV content is more than 4 gm/cm^2 . It is further seen that the SAMIR estimates of dew point temperatures are negatively biased (mean difference between satellite and ship data being -1.27°C) during January whereas they agree with ship's data during June (bias 0.12°C). The seasonal variation in the t_d bias may be due to two sources: (i) wind effect in SAMIR brightness temperatures (T_B) and (ii) scale height effect in WV. Lower winds in January result in lower T_B which leads to lower/underestimated WV (t_d) values. Figure 3 shows that vertical distribution of WV content in January over the Arabian Sea (For example: Minicoy Island; Data source: Ramage and Raman 1972) is lesser exponential when compared to that in June. This lesser scale height effect gives rise to smaller WV for same t_d and hence underestimates t_d for a given WV.

The satellite-derived SST over the Arabian Sea for June 1982, obtained from NOAA-GOSSTCOMP Charts are found to be negatively biased (-1.4°C) with respect to ship SST data. This is comparable with the bias (-0.9°C) obtained from

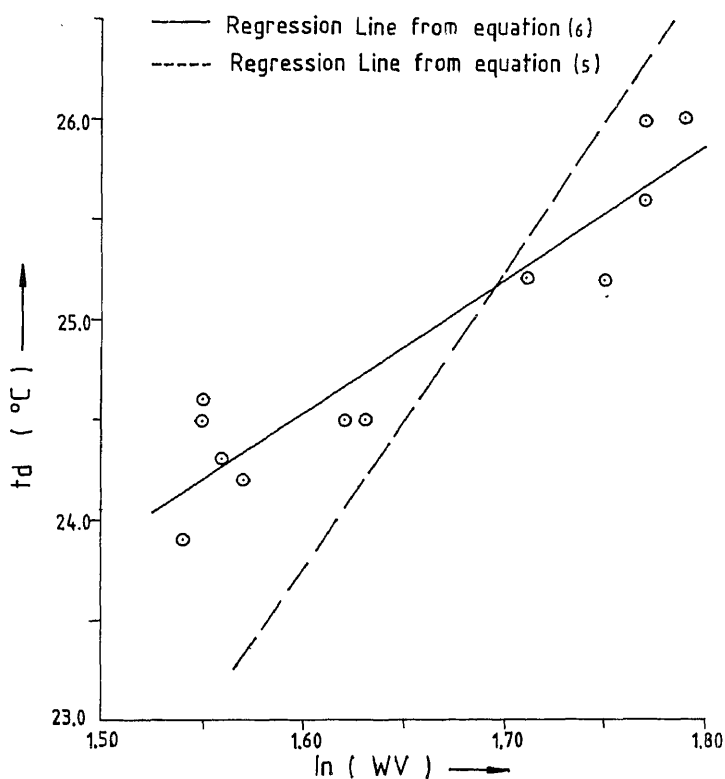


Figure 2. Scatter diagram of $\ln(WV)$ versus t_d with regression lines.

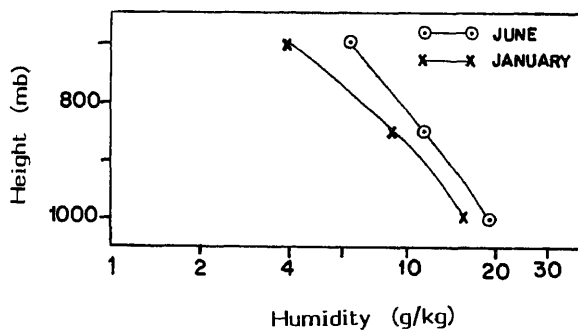


Figure 3. Vertical distribution of humidity at Minicoy Island in the Arabian Sea.

Table 2. Comparison of dew point temperatures estimated from SAMIR data with the *in situ* observations.

Date (1)	Orbit No. (2)	Time of (IST) Observation		SAMIR Position (5)	WV content (g/cm ²) SAMIR (6)	Dew point (°C)	
		SAMIR (3)	Ship (4)			SAMIR (7)	Ship (8)
13.1.1982	821	1908	1730	18°09 N 69°43 E	2.06	15.2	17.9
24.1.1982	985	1517	1730	09°10 N 72°57 E	3.30	20.1	25.0
28.1.1982	1038	0242	0530	19°41 N 85°57 E	3.56	20.8	24.0
29.1.1982	1053	0227	2330	10°11 N 75°43 E	3.76	21.4	22.0
24.2.1982	1447	0351	0530	16°11 N 62°12 E	3.87	21.7	19.7
19.3.1982	1778	0734	0530	11°28 N 74°29 E	4.14	22.4	22.0
20.3.1982	1813	0728	0530	13°42 N 73°50 E	4.30	22.8	24.0
22.3.1982	1842	0523	0530	06°33 N 87°48 E	5.96	26.1	26.5
30.3.1982	1962	0338	0530	13°05 N 74°01 E	4.59	23.4	23.4
07.4.1982	2081	0013	2330	08°58 N 76°16 E	3.56	20.8	25.1
19.4.1982	2275	1940	1730	09°19 N 70°52 E	4.70	23.7	23.4
23.4.1982	2326	0508	0530	16°32 N 67°34 E	3.63	21.0	21.8
26.4.1982	2370	0252	0530	15°22 N 85°53 E	5.40	25.3	25.0
04.5.1982	2498	1301	1130	05°07 N 82°27 E	4.37	22.8	23.0
23.5.1982	2788	1715	1730	06°41 N 76°02 E	6.55	27.1	26.8
04.6.1982	2961	0241	0530	16°13 N 69°59 E	4.85	24.0	23.8
16.6.1982	3154	2025	1730	08°59 N 84°04 E	6.39	26.8	26.9
28.6.1982	3265	0454	0530	19°12 N 70°24 E	5.88	26.0	26.0

error: ± 0.94 m/sec). A scatter diagram between observed (ship data) and estimated (from SAMIR T_B) wind speeds is presented in figure 4.

The mean distribution of evaporation rates over the Arabian Sea for June 1982 estimated from Bhaskara II – SAMIR and NOAA–AVHRR data and from ship observations are presented in figures 5a, b respectively. It is found that the spatial trends of evaporation rates derived from satellite data fairly agree with those obtained from ship observations. They are also comparable with climatological trends given by Hastenrath and Lamb (1979) (figure 5c). However there is a consistent underestimation of evaporation rates (shown by solid line contours in

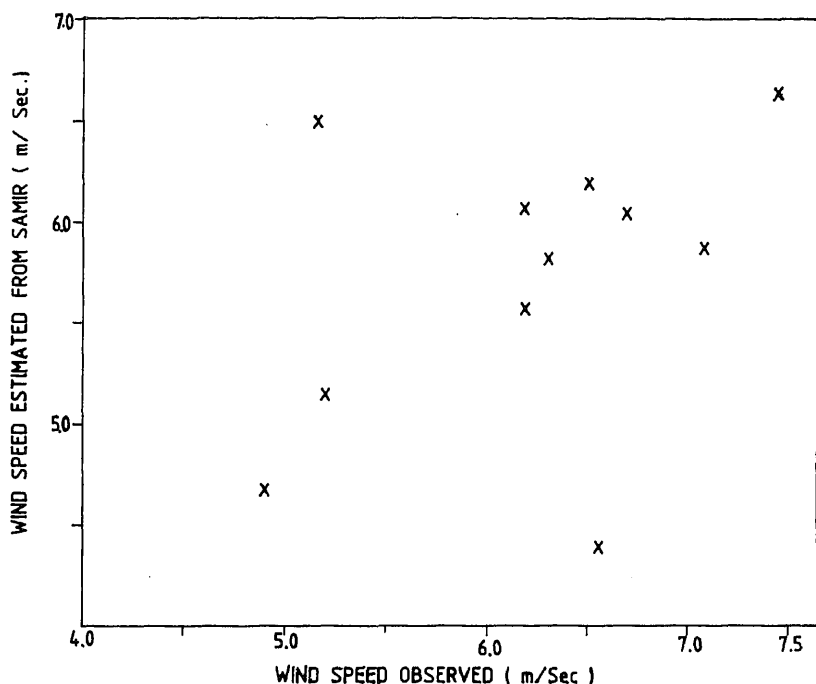


Figure 4. Scatter diagram of wind speeds observed versus estimated.

figure 5a) from satellite observations which could be mainly due to the effect of negative bias of SST from AVHRR data as discussed earlier. When the bias (-1.4°C) in SST from NOAA-AVHRR data set is removed, the accuracy in the evaporation rates improves to $\pm 0.8 \text{ mm/day}$ (error 16%). The dashed line contours in figure 5a represent the evaporation rates after the removal of bias-correction in SST. This error estimate is based on monthly averages obtained in 11 four-degree grids where both satellite and ship observation coverages are reasonably good when compared to those in other grids in the area of study. Table 3 shows the number of data points considered in the averaging process for each grid. The present result is encouraging compared to the 30% error reported by Simon and Desai (1986) for the equatorial Indian Ocean based on GOES winds and TIROS temperature and humidity data. Further studies over the northern Indian Ocean are planned using variables derived from a single sensor (SMMR) to achieve evaporation estimations with better accuracy.

4. Conclusions

(i) An empirical equation between mean monthly WV content and the surface dew point temperature fields over the northern Indian Ocean is developed for the summer monsoon season with a correlation coefficient (0.94) and r.m.s error of $\pm 0.24^{\circ}\text{C}$.

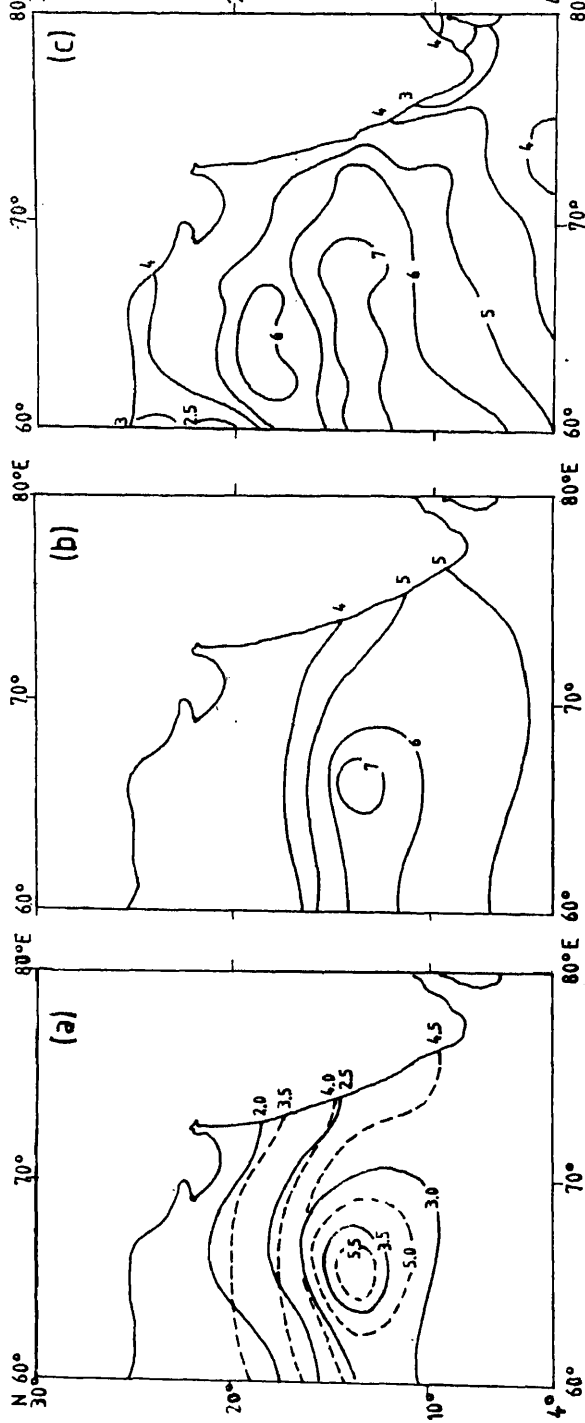


Figure 5. Distribution of evaporation rates (mm/day) over the Arabian Sea during June 1982. (a) from satellite data before (—) and after (---) removing bias in SST. (b) from ship observations. (c) climatological mean June distribution (after Hastenrath and Lamb, 1979).

Table 3. Number of observations used in the averaging process

Grid No.	Area	Satellite	Ship
1.	4-8°N; 60-64°E	—	2
2.	4-8°N; 64-68°E	1	1
3.	4-8°N; 68-72°E	3	1
4.	4-8°N; 72-76°E	30	11
5.	4-8°N; 76-80°E	—	25
6.	8-12°N; 60-64°E	—	9
7.	8-12°N; 64-68°E	3	8
8.	8-12°N; 68-72°E	20	20
9.	8-12°N; 72-76°E	23	23
10.	8-12°N; 76-80°E	—	10
11.	12-16°N; 60-64°E	4	3
12.	12-16°N; 64-68°E	16	5
13.	12-16°N; 68-72°E	32	10
14.	12-16°N; 72-76°E	24	30
15.	16-20°N; 60-64°E	18	5
16.	16-20°N; 64-68°E	29	12
17.	16-20°N; 68-72°E	39	16
18.	16-20°N; 72-76°E	—	6
19.	20-24°N; 60-64°E	11	3
20.	20-24°N; 64-68°E	22	7
21.	20-24°N; 68-72°E	4	3

(ii) The evaporation rates can be estimated with a reasonable accuracy of ± 0.8 mm/day using different satellite data sets (SAMIR, AVHRR) when the bias in SST from NOAA-AVHRR data is removed.

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Wind gradients at meteor heights over mid-latitude stations

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Abstract. The gradients of amplitude and phase of diurnal and semidiurnal components of zonal and meridional winds in the height intervals of 80–90 km and 90–100 km are studied using the meteor wind radar data collected at Atlanta (34°N, 84°W) during the period August 1974–March 1978. The results are compared with those at Adelaide (35°S, 139°E). It is found that the gradients vary in an opposite manner between the two height intervals.

Keywords. Meteor winds; amplitude and phase gradients; diurnal and semidiurnal components; mid-latitudes.

1. Introduction

Radar tracking of ionized trails of meteors yields accurate and systematic information on upper atmospheric winds in the height region 80–100 km. The use of this method for investigating various phenomena such as turbulence, tides and diffusion in the upper atmosphere at these heights was pioneered by Greenhow and Neufeld (1956, 1959) and the results obtained so far at various places in the world have been reviewed by Hocking (1985) and Forbes (1985). Recently, the meteor radar technique has been made more powerful and sophisticated to achieve maximum height resolution of order 1 km to study the height variation of neutral wind parameters in detail (e.g., Revah *et al* 1967). Observations carried out at many places indicate that gradients in wind amplitude and phase could be as high as 4 msec⁻¹ km⁻¹ and 12° km⁻¹ respectively over a height range 85–95 km (Elford and Robertson 1953; Greenhow and Neufeld 1955). The seasonal and annual variation of wind gradients has been extensively studied by Greenhow and Neufeld (1955, 1956) at Jodrell Bank, England. They report that the magnitude of wind gradient varies between 1.5 and 0.4 msec⁻¹ km⁻¹ during winter and summer months in the height region 85–95 km. In a comparative study between meteor winds and ionospheric drifts, Muller (1968) observed the height gradients of wind amplitude and phase of 1.06 msec⁻¹ km⁻¹ and 4.7° km⁻¹ respectively. By analyzing the meteor wind data for Adelaide for the period 1967–69, Ahmed and Devara (1979) reported that gradients of amplitude and phase of semidiurnal wind component varied markedly between 80 and 90 km, and 90 and 100 km height intervals.

An attempt has been made in this paper to study the variation of wind gradients over mid-latitude stations in the altitude range 80–100 km which contributes the information on propagational characteristics of wave phenomena prevailing at those altitudes. For this purpose, diurnal and semidiurnal components of zonal

(EW) and meridional (NS) winds between 80 and 100 km over Atlanta (34°N, 84°W) obtained using the Georgia Tech radio meteor wind facility during the period August 1974–March 1978 are analysed to obtain information on gradients of amplitude and phase in the height intervals of 80–90 km and 90–100 km. The results are compared with those at Adelaide (35°S, 139°E), almost geographically symmetric station. The details of the analysis, results achieved and conclusions drawn are presented below.

2. Data and analysis

The data of monthly mean amplitude and phase of diurnal (24 hour) and semidiurnal (12 hour) components of EW and NS winds at 80, 90 and 100 km for Atlanta for the period August 1974–March 1978 are available. The details of the radar facility used and the mathematical procedure followed for estimating the amplitude and phase of diurnal and semidiurnal components from the hourly mean meteor wind values have been described by Roper (1975, 1978). The differences in amplitude and in phase were computed between the heights 80 and 90 km, and 90 and 100 km for diurnal and semidiurnal components of EW and NS winds. The gradients so obtained for each month were averaged for respective months and mean values for all January months, February months etc over the 44-month period (August 1974–March 1978) are utilized for studying the behaviour of wind gradients of amplitude and phase of EW and NS winds in the 80–90 km and 90–100 km height intervals over a mid-latitude station.

3. Variation of height gradients over different months

3.1 *Diurnal amplitude*

The monthly mean variations in the gradients of amplitude of diurnal component of EW and NS winds in the 80–90 km and 90–100 km height intervals are shown in figure 1 for Atlanta (A,B,C,D) and Adelaide (a,b,c,d). It is clear from figure 1 that the gradients of EW and NS winds are positive in the 90–100 km height interval and negative in the 80–90 km height interval throughout the year at Atlanta. In the case of Adelaide (figure 1a & b) although the EW wind gradient in both the height intervals appears to be swinging between positive and negative values, it is negative during most of the months in the 80–90 km height interval. Also, the variations in the gradients at Atlanta and Adelaide for the height interval 80–90 km follow each other for most of the months. The results of Atlanta clearly show that the diurnal oscillation is damped (negative amplitude gradient) between the levels 80 and 90 km, and amplified (positive amplitude gradient) between the levels 90 and 100 km.

3.2 *Semidiurnal amplitude*

The monthly mean variations in the gradients of amplitude of semidiurnal component of EW and NS winds in the 80–90 km and 90–100 km height intervals

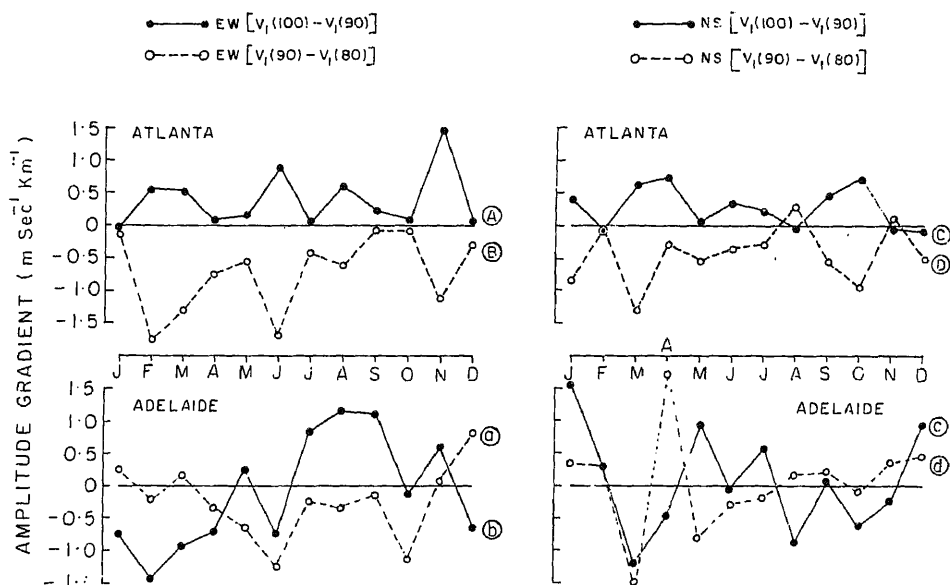


Figure 1. Monthly variations in gradients of amplitude of diurnal component of zonal and meridional winds for Atlanta (A,B,C,D) and Adelaide (a,b,c,d).

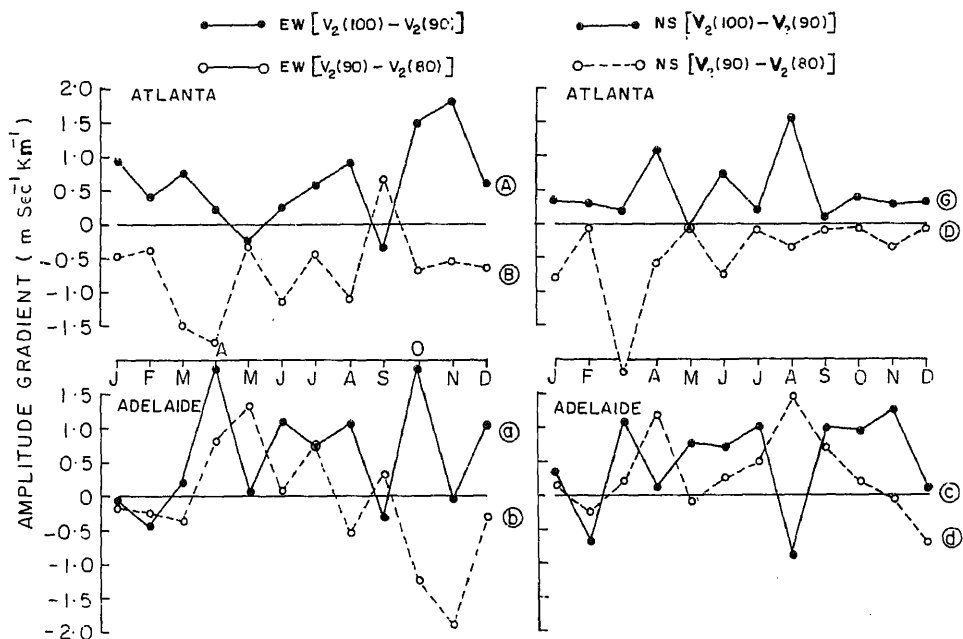
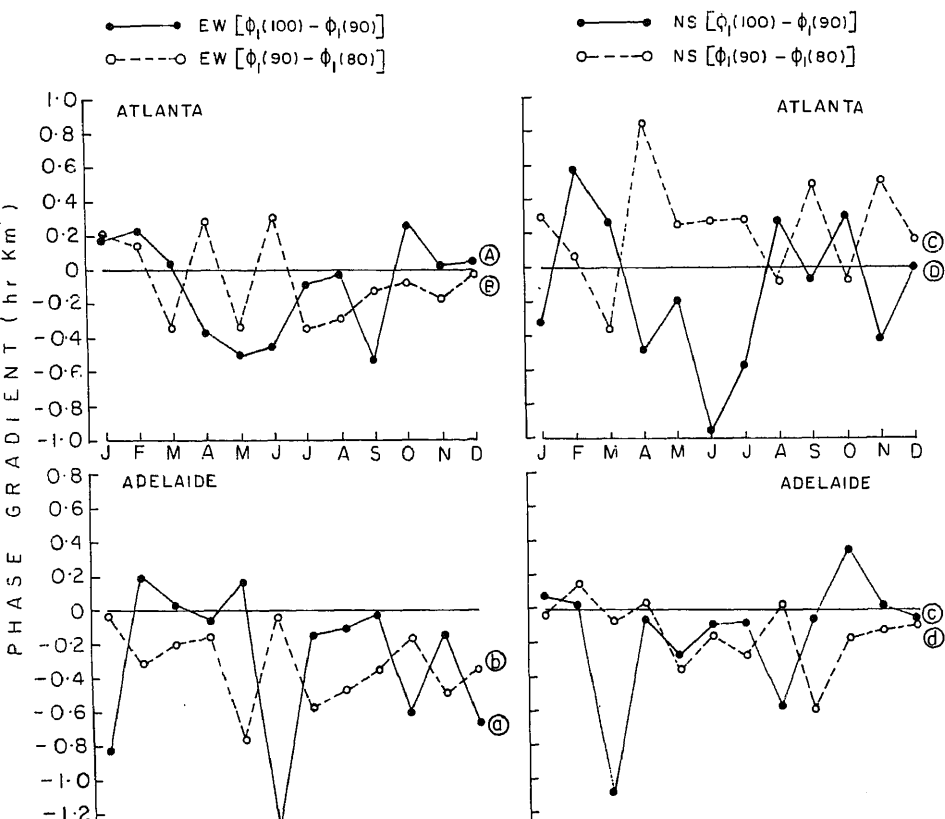


Figure 2. Same as figure 1 for semidiurnal component.

January, February and September at Adelaide. The same gradient is negative in the 80–90 km height interval in all the months except September at Atlanta while it is swinging between positive and negative values at Adelaide. Figure 2 (C & D) points out that the gradient of semidiurnal component of NS wind at Atlanta is positive in the 90–100 km height interval and negative in the 80–90 km height interval throughout the year. At Adelaide, this gradient is positive for both the height intervals except during February and August in the 90–100 km height interval; February and December in the 80–90 km height interval.

3 Diurnal phase

The month-to-month mean variations in the gradients of phase of diurnal component of EW and NS winds in the 80–90 km and 90–100 km height intervals are displayed in figure 3 for Atlanta (A,B,C,D) and Adelaide (a,b,c,d). It can be seen that the variations in the gradients of phase of EW and NS winds at Atlanta and Adelaide are erratic and at times follow each other in both the height intervals.



Also, it is evident from figure 3 (a,b,c,d) that the nature of variation of gradient in the 80–90 km interval is almost opposite to that in the 90–100 km interval.

3.4 Semidiurnal phase

Figure 4 depicts the monthly mean variations in the gradients of semidiurnal component of EW and NS winds in the 90–100 km and 80–90 km height intervals for Atlanta (A,B,C,D) and Adelaide (a,b,c,d). The magnitudes of gradients at Atlanta are small compared to those at Adelaide. Also, it appears that there is an appreciable change in the gradients of phase for semidiurnal components of EW and NS winds at Adelaide.

3.5 Seasonal and annual variation of height gradients

The seasonal and annual mean gradients of amplitude and phase of diurnal and semidiurnal components of EW and NS winds at Atlanta for the 90–100 km and 80–90 km intervals are presented in table 1. For purposes of comparison, results of

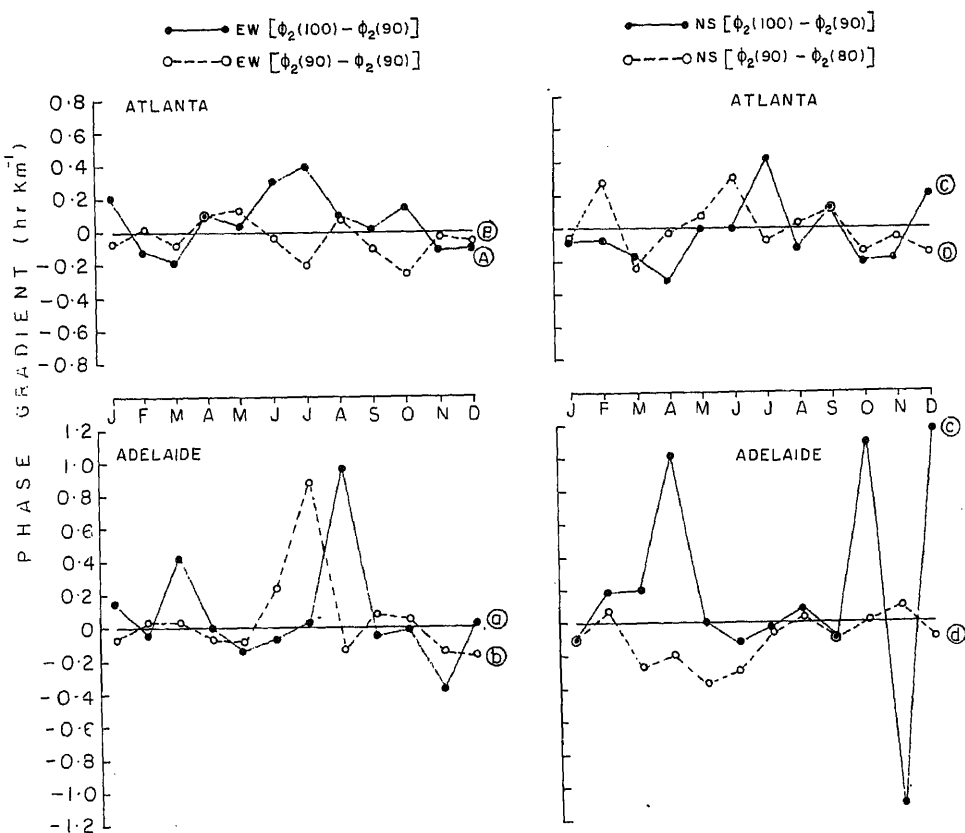


Figure 4. Same as figure 3 for semidiurnal component.

Table 1. Seasonal and annual mean variation in gradients of amplitude ($\text{m sec}^{-1} \text{ km}^{-1}$) and phase (hr km^{-1}) of diurnal (V_1, θ_1) and semidiurnal (V_2, θ_2) components of zonal (EW) and meridional (NS) winds for the height intervals 80–90 km and 90–100 km.

Gradient	Spring	Summer	Autumn	Winter	Annual
EW [V_1 (90)– V_1 (80)]	–1.29 (–0.11)	–0.90 (–0.70)	–0.62 (0.25)	–0.54 (0.37)	–0.84 (–0.05)
EW [V_1 (100)– V_1 (90)]	0.39 (–1.40)	0.35 (0.10)	0.32 (0.70)	0.49 (–0.28)	0.39 (–0.12)
NS [V_1 (90)– V_1 (80)]	–0.57 (0.27)	–0.44 (–0.40)	–0.40 (0.08)	–0.42 (0.35)	–0.46 (0.08)
NS [V_1 (100)– V_1 (90)]	0.44 (–0.49)	0.22 (0.47)	–0.38 (–0.47)	0.09 (0.72)	0.28 (0.06)
EW [V_2 (90)– V_2 (80)]	–1.24 (0.05)	–0.67 (0.72)	–0.40 (–0.51)	–0.59 (–0.71)	–0.72 (–0.11)
EW [V_2 (100)– V_2 (90)]	0.49 (0.56)	0.22 (0.64)	0.53 (0.99)	1.16 (0.35)	0.60 (0.64)
NS [V_2 (90)– V_2 (80)]	–0.96 (0.38)	–0.33 (0.23)	–0.15 (0.79)	–0.37 (–0.20)	–0.45 (0.30)
NS [V_2 (100)– V_2 (90)]	0.55 (0.18)	0.33 (0.86)	0.71 (0.36)	0.39 (0.57)	0.49 (0.49)
EW [ϕ_1 (90)– ϕ_1 (80)]	0.03 (–0.22)	–0.13 (–0.44)	–0.18 (–0.32)	0.001 (–0.28)	–0.07 (–0.32)
EW [ϕ_1 (100)– ϕ_1 (90)]	–0.03 (0.05)	–0.36 (–0.43)	0.10 (–0.25)	0.09 (–0.54)	–0.10 (–0.29)
NS [ϕ_1 (90)– ϕ_1 (80)]	0.18 (0.03)	0.27 (–0.27)	0.11 (–0.24)	0.32 (–0.09)	0.22 (–0.14)
NS [ϕ_1 (100)– ϕ_1 (90)]	0.13 (–0.36)	–0.58 (–0.15)	0.18 (–0.09)	–0.24 (0.01)	–0.13 (–0.15)
EW [ϕ_2 (90)– ϕ_2 (80)]	0.02 (–0.007)	–0.04 (0.35)	–0.10 (0.06)	–0.01 (–0.11)	–0.03 (0.07)
EW [ϕ_2 (100)– ϕ_2 (90)]	–0.07 (0.13)	0.25 (–0.05)	0.09 (0.30)	–0.008 (–0.06)	0.07 (0.08)
NS [ϕ_2 (90)– ϕ_2 (80)]	0.003 (–0.14)	0.09 (–0.22)	–0.01 (–0.0003)	–0.05 (0.01)	0.01 (–0.09)
NS [ϕ_2 (100)– ϕ_2 (90)]	–0.18 (0.47)	–0.14 (–0.06)	–0.01 (0.38)	–0.02 (0.01)	–0.09 (0.20)

Adelaide (with paranthesis) are also presented in table 1 and the most striking feature is that at Atlanta, the seasonal mean height gradients of amplitude and phase of both diurnal and semidiurnal components in the 80–90 km and 90–100 km height intervals are opposite to each other except for diurnal component of EW wind during summer and winter; semidiurnal components of NS and EW winds during summer and winter. The annual mean height gradient also showed similar behaviour except for phase of diurnal component of EW wind. Any such systematic variation is not seen at Adelaide.

Discussion

The variations observed in the height gradients of amplitude and phase of the zonal wind components over the height intervals of 80–90 km and 90–100 km are discussed in relation to the atmospheric oscillations. In general, the height variations of wind amplitude and phase at meteor heights can be described by means of the propagational aspects of tidal energy from troposphere to upper atmosphere. The seasonal variations in the height gradients of amplitude and phase observed in the present study are considered to be due to the variations in vertical wavelength and corresponding dominant modes of diurnal and semidiurnal tides at Atlanta and Adelaide as explained by Ahmed and Roper (1983). The opposite trends of variation observed in the gradients of amplitude and phase of diurnal and semidiurnal components of EW and NS winds in the 80–90 km and 90–100 km height intervals suggest the formation of amplitude node at 90 km which indicates depletion of tidal energy associated with a phase reversal at 90 km.

Conclusions

Analysis of the meteor wind data for Atlanta and Adelaide showed that the trends of amplitude and phase of diurnal and semidiurnal components of EW and NS winds are positive in the 90–100 km height interval and negative in the 80–90 km height interval at Atlanta whereas at Adelaide they swing between positive and negative values. This suggests that the formation of amplitude node at 90 km is a more regular feature at Atlanta than at Adelaide, a southern hemisphere station with very nearly the same latitude as that of Atlanta. Inter-comparison of available mid-latitude meteor wind radar results at closer height intervals may provide further understanding of the problem.

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Sea surface temperature variability over North Indian Ocean — A study of two contrasting monsoon seasons

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Abstract. Using the satellite derived sea surface temperature (SST) data for 1979 (bad monsoon) and 1983 (good monsoon), the SST variability for two contrasting monsoon seasons is studied. The study indicates that large negative anomalies off the Somali and Arabian coasts are associated with good monsoon rainfall over India. The strong monsoonal cooling in these regions can be attributed to strong low level winds and intense upwelling. The reappearance of 27°C isotherm off Somali coast in May/June coincides with the onset of southwest monsoon over India. Further, the influence of zonal anomaly of SST off Somalia Coast (SCZASST) and Central Indian Ocean Zonal Anomaly of SST (CIOZASST) with monsoon rainfall over India is brought out. The former is negatively related to the monsoon rainfall over western and central parts of India, whilst CIOZASST is positively related.

Keywords. North Indian Ocean; zonal anomaly; sea surface temperature; south equatorial trough.

1. Introduction

In recent years, considerable attention has been paid to the relationships between SST distribution over the Indian Ocean and the monsoon activity over India and adjoining areas (Ellis 1952; Saha 1970; Shukla 1975; Shukla and Misra 1977; Washington *et al* 1977; Raghavan *et al* 1978; Weare 1979; Pisharoty 1981; Mishra 1981; Joseph 1981; Joseph and Pillai 1984). However, considerable uncertainty still exists as to the exact nature of the relationship between SST patterns and monsoon activity. Different hypotheses have been put forth regarding these interactions by Shukla (1975), Saha (1970, 1974), Washington *et al* (1977) and Joseph and Pillai (1984).

These theoretical and empirical models need verification by extensive testing with observed data. Conventional ship observations over the Indian Ocean are sparse, and as such, not ideally suited for the above purpose.

It is now possible to get SST data on a routine basis through satellites. Though uncertainties still exist as to the accuracies of this data, especially for tropical areas (Pichel and Banks 1983), it has the advantages of large area coverage and real time accessibility, which are important attributes from the prediction point of view. Mishra (1981) studied SST variability over the North Indian Ocean during the southwest monsoon season for the years 1977, 1978 and 1979, using the satellite derived SST data. His study revealed that the beginning of cooling of surface waters over the Arabian Sea is associated with the advance of the southwest monsoon, and that the formation of the Somali and Monsoon currents and

In this paper, we have analyzed satellite-derived SST data over the Indian Ocean during the pre-monsoon and monsoon period for 1979 and 1983. The results are discussed in the light of various hypotheses regarding the ocean-atmosphere interaction and its possible effect on monsoon rainfall over India.

2. Data and methodology

The years 1979 and 1983 were chosen because of contrasting features of monsoon rainfall over the Indian subcontinent. Figure 1 shows the departures of seasonal rainfall (1 June to 30 September) from the normal for 1979 and 1983 for various meteorological sub-divisions (Weekly Weather Report, India Meteorological Department). In general, over most parts of the country the rainfall was deficient in 1979, and normal to excess during 1983. In both the years the advance of monsoon over Kerala was delayed by about two weeks (the normal onset data being 1st June).

SST data for 1979 and 1983 were taken from the Global Operational Sea Surface Temperature Computation (GOSSTCOMP) charts (derived from NOAA satellites), for the period May to September. These charts give temperature contours at 1°C interval for the area from 30°N to 30°S and 40°E to 100°E at weekly intervals. Pichal and Banks (1983) found that the root mean square (RMS) differences between satellite and *in situ* data were generally less than 1°C . However, in the latitude belt from 10°N to 30°N , RMS differences of 1.6°C and biases of -1°C to -3°C were observed for 1982. These lower quality observations were attributed to the effect of El-Chichon volcanic eruption in late March 1982. We have assumed that the effect of this contamination would have decreased substantially by May 1983, and that the data would have an accuracy of 1°C , which corresponds to the contouring interval on the charts. The climatological SST data were taken from Hastenrath and Lamb (1979).

The SST values for every two degree latitude-longitude grid points were estimated by visual interpolation (for both GOSSTCOMP as well as climatological charts). The SST anomalies (T_z) were then calculated as follows.

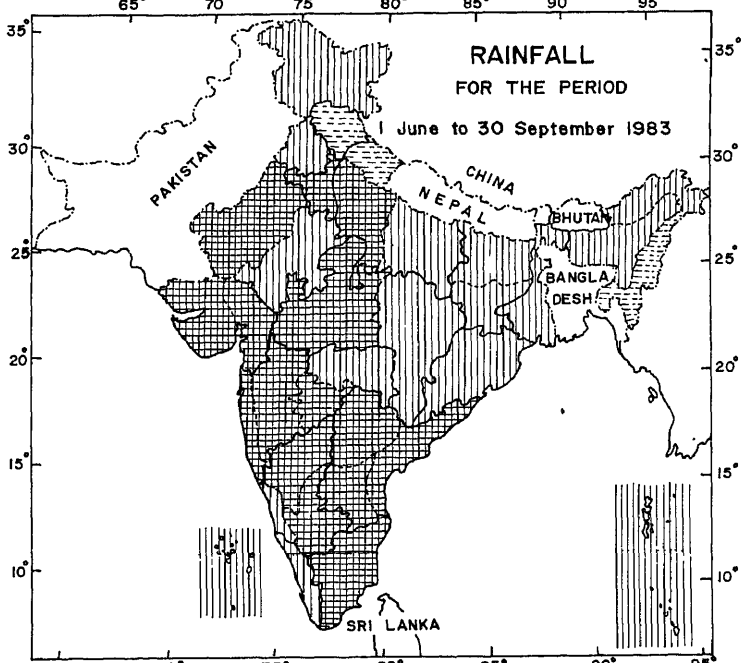
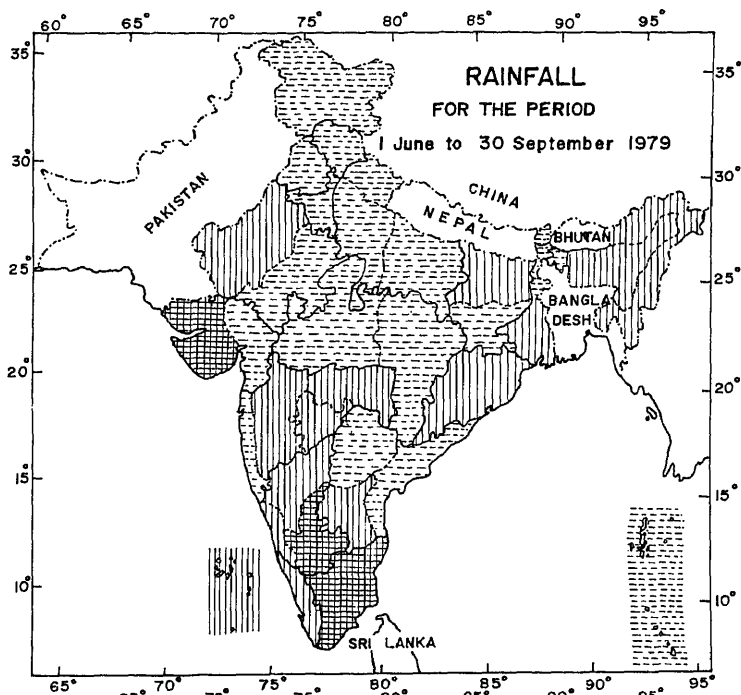
$$T_z = T - \bar{T},$$

where T and $\bar{T}$ are the observed grid point and zonal mean SSTs respectively.

3. Results and discussion

3.1 Monthly variation of SST

The weekly averaged SSTs obtained from the GOSSTCOMP charts have been further averaged to give the monthly mean SST. Figure 2 gives the monthly variation of mean SST for three representative stations A, B and C in the sea off the Somali coast (10°N and 52°E), South Central Arabian Sea (10°N and 60°E) and South Central Bay of Bengal (10°N and 90°E) respectively, from May to September for the years 1979 and 1983.



At station A, the climatological values of SST drop sharply from May onwards, whereas in 1979 and 1983, the decrease of SST from May to June is comparatively less. Correspondingly, the Indian Daily Weather Report (IDWR) data show that the monsoon onset was delayed by about 2 weeks in both 1979 and 1983 from the normal onset date. However, the May to June cooling is more (less) pronounced in 1983 (1979), the year of excess (deficit) rainfall. The rate of cooling can be taken as indicative of the intensity of upwelling in this area. The climatological values as well as the 1979 values show increasing temperatures from August to September, while in 1983, the temperature continues to decrease during this period. This may be due to the late withdrawal of southwest monsoon in 1983 and the persistence of stronger winds off Somali coast and consequent upwelling for a longer period. However, the climatological temperatures in this area are lower than the corresponding values for both 1979 and 1983.

At station B there is little difference in the monthly mean SST variation between 1979 and 1983 and the values for both the years compare well with the climatological data, though here also climatological values are generally lower. However, the differences are not as high as at station A.

At station C, in 1983 the monthly mean SST values were greater than normal from May to July, and less than normal from August to September. The values were nearly normal in 1979.

3.2 Reappearance of 27°C isotherm

An analysis of the behaviour of the 27°C isotherm in the Arabian sea for the years 1979, 1981, 1982 and 1983 shows that this isotherm was absent throughout the premonsoon month of May off Somalia coast and appeared with the onset of southwest monsoon over Kerala coast (table 1). Geostationary satellite wind data (Young *et al* 1980) for 1979 showed that the appearance of the 27°C isotherm coincides with the establishment of the low level jet over the central Arabian Sea.

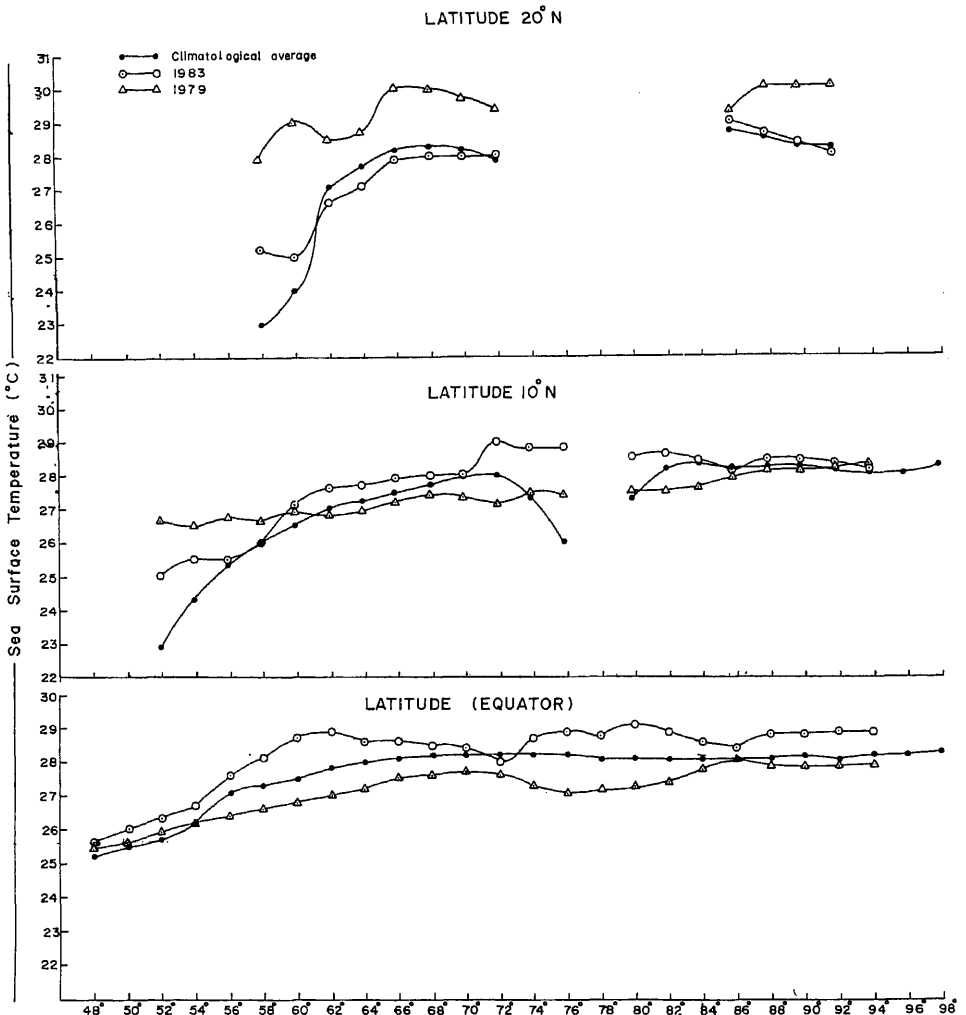
Table 1. Relationship between the appearance of the 27°C isotherm off the Somali coast and the advance of the Northern Limit of Monsoon/Southwest monsoon over Kerala.

Year	Week in which the 27°C isotherm appears off Somali coast in GOSSTCOMP chart	Advance of Northern Limit of Monsoon/Southwest Monsoon over Kerala
1979	12th June	11th June
1981	2nd June	30th May
1982	1st June	30th May
1983	14th June	13th June

The date referred to in the GOSSTCOMP chart is the middle day of that particular week.

3.3 Longitudinal variation of SST in July

Figure 3 gives the longitudinal variation of SSTs in July for the years 1979, 1983 along with the climatological mean values for the latitudes 20°N , 10°N and Equator. The figure clearly shows that there is larger temperature difference between the western and eastern portions of the Indian ocean during a good monsoon season (1983) than in a poor monsoon season (1979). This east-west temperature gradient is greater at 20°N and 10°N as compared to the equator. At 20°N the maximum SSTs are observed around $66\text{--}70^{\circ}\text{E}$ and the minimum values are observed at 58°E near the Arabian coast. The range of SST is 3°C (2°C) in 1983 (1979). At 10°N the maximum SSTs are observed from $68\text{--}72^{\circ}\text{E}$ and the minimum around 52°E near the



Somalia coast. The range of SST is 3°C (2°C) in 1983 (1979). At the equator, in the region west of 60°E , there is strong (weak) temperature gradient during the good (bad) monsoon year. The temperature gradient in the western part of the equator during 1983 may be due to the strong winds (ORV Sagar Kanya 1st Cruise data) and strong cross equatorial flow. The weak temperature gradient during 1979 was due to weak surface winds and weak cross equatorial flow (Geostationary satellite data).

Saha (1970) showed that at the equator, pressure and SST are negatively correlated. We assume that this relationship holds good for the Arabian Sea also. Thus the large east-west temperature gradient observed during 1983 was favourable for strong low level westerly winds and intense evaporation over Arabian Sea.

3.4 Zonal anomaly of SST

Weekly zonal anomaly of SST charts were prepared for the period May to September for both the years and representative charts depicting the onset and active conditions for 1979 and 1983 are presented in figures 4A to 4D.

3.4a *Zonal anomalies during 1979:* Figure 4A represents actual onset conditions during 1979 and no significant SST anomaly features are present. The weak zonal

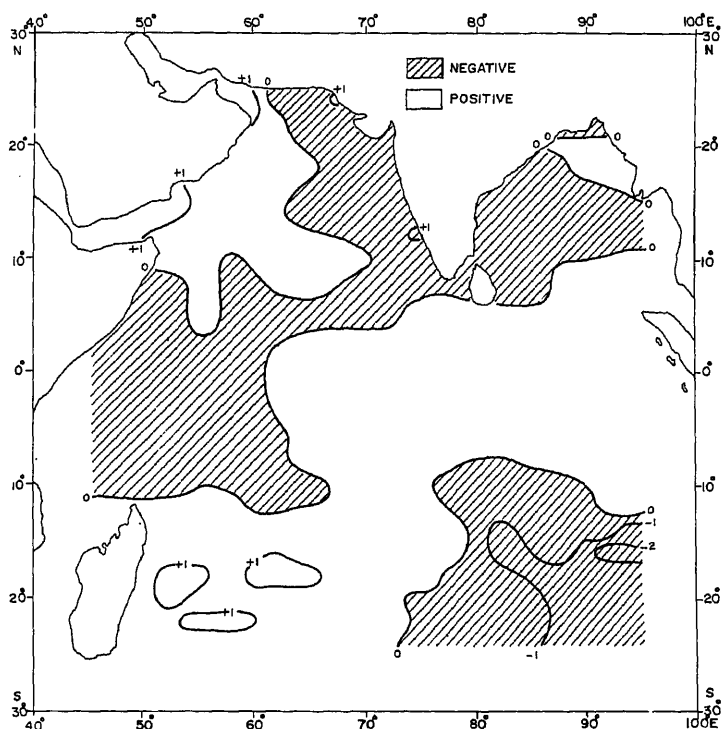


Figure 4A. Zonal anomaly chart of SST depicting the onset conditions (12 June 1979).

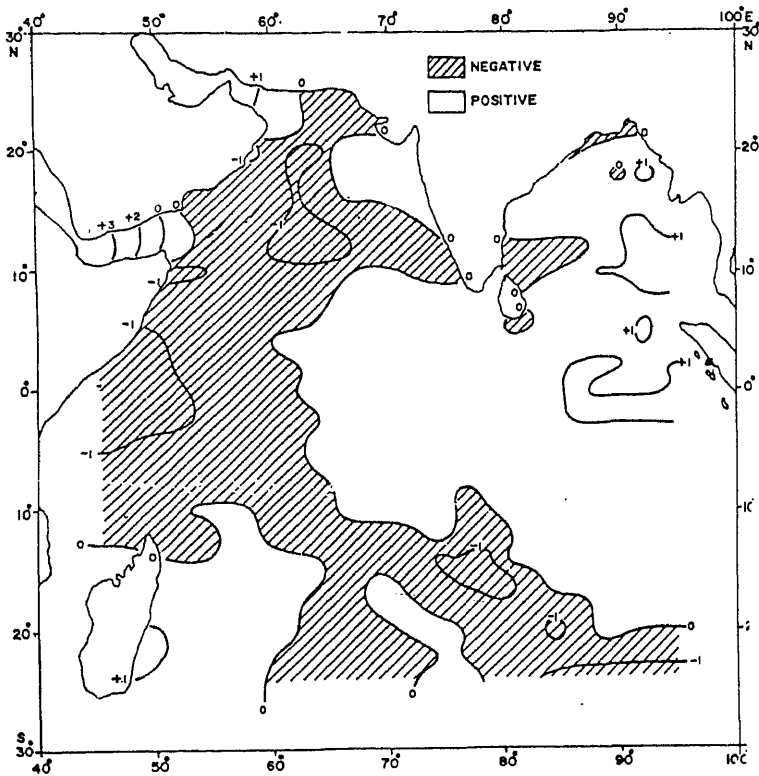


Figure 4B. Zonal anomaly chart of SST depicting the active monsoon conditions (24 July 1979).

anomaly observed during June appears to be associated with weak surface winds (Geostationary satellite data) and weak rainfall activity over India. The July conditions (active monsoon conditions, figure 4B) clearly reveal the beginning of upwelling along Somali and Arabian coasts by negative zonal anomalies in this region, and these features get more pronounced by August. By the beginning of September, these features have already begun to disappear. The maximum negative zonal anomalies of the order of 2°C were observed during August in the western Arabian Sea.

3.4b Zonal anomalies during 1983. Figure 4C shows the actual onset conditions during 1983. Strong negative anomalies indicative of intense upwelling off Somali coast can be seen. The negative anomalies become more pronounced by July (active monsoon conditions, figure 4D) and the features persist throughout August and September. The maximum negative anomalies of the order of 4°C were observed during July off Somali. Upwelling off Arabian coast is also considerably stronger in 1983 than in 1979.

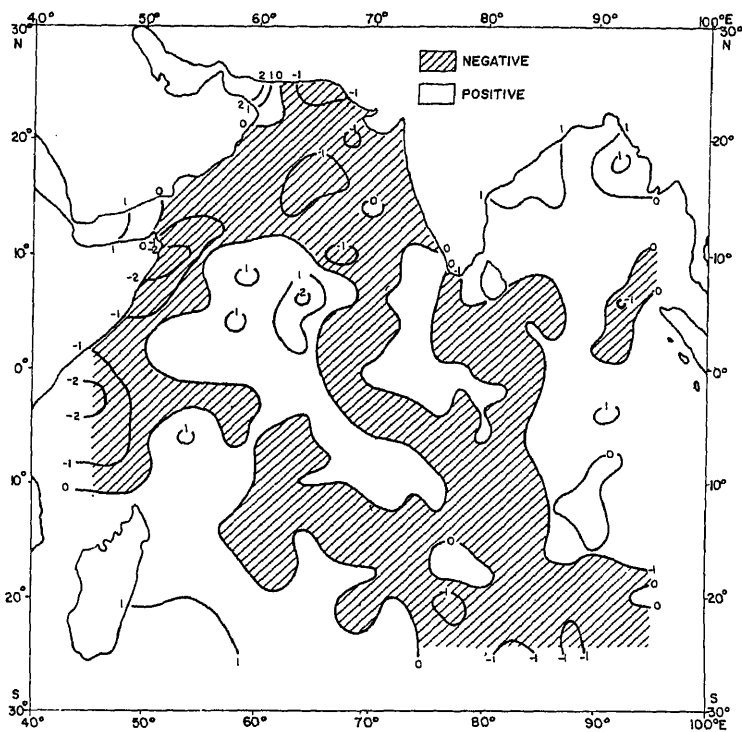


Figure 4C. Zonal anomaly chart of SST depicting the onset conditions (14 June 1983).

3.5 Relationships between SCZASST, CIOZASST and the monsoon rainfall over India

Studies by Shukla and Misra (1977), Goswami (1983) and Ramesh Babu *et al* (1985) have shown a positive correlation between SST and monsoon rainfall over different parts of India. In the present study we have chosen two regions which were not studied earlier namely, Somalia coast (4–10°N and 48–52°E) and Central Indian Ocean area (4°S–4°N and 64–84°E) for correlating with the monsoon rainfall of about 15 meteorological subdivisions representing various parts of the country.

We have chosen the SCZASST as the average zonal anomaly between 4°N and 10°N, 48°E and 52°E, for correlating with the monsoon rainfall for the following reasons: It is believed that strong low level winds known as Somali Jet, transport significant amount of moisture to the Indian subcontinent. Thus a relationship between the intensity of Somali Jet and monsoon activity over India can be inferred to exist (Raghavan *et al* 1975). Shukla and Misra (1977) obtained a negative correlation between SST and pressure. Thus the existence of strong negative zonal winds are associated with low SSTs. Saha (1970) had shown the negative correlation between SST and pressure. Thus the existence of strong negative zonal anomalies can be linked to strong surface winds and intense upwelling off Somalia coast. These strong surface winds cause intense evaporation over the Arabian Sea. The net effect of the above mentioned factors would be to produce good monsoon over the Indian subcontinent.

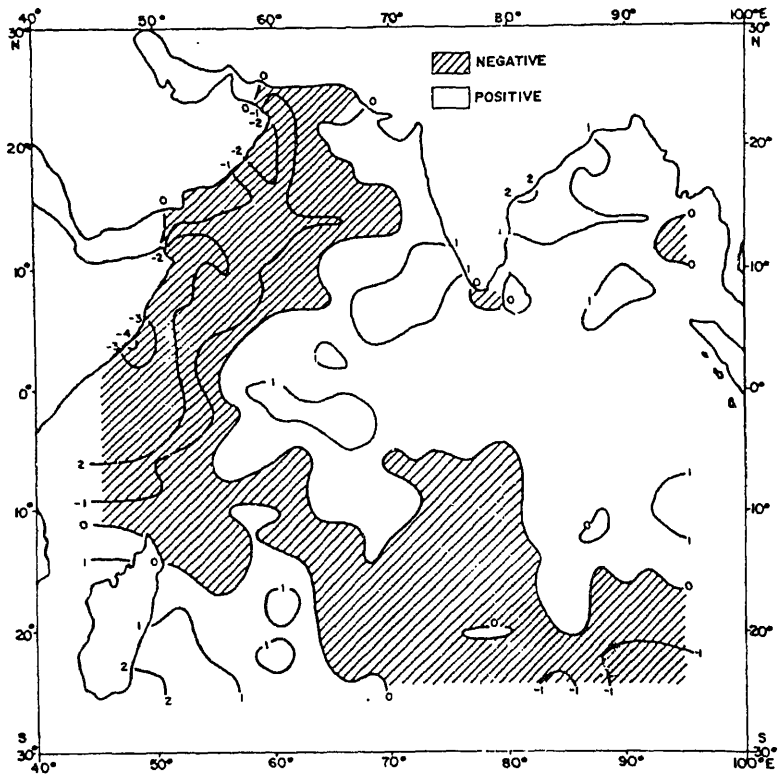


Figure 4D. Zonal anomaly chart of SST depicting the active monsoon conditions (19 July 1983). The chart also indicates pronounced upwelling off Somali and Arabian coasts as compared to the year 1979.

We have found out the correlation coefficients between weekly values of SCZASST and the weekly monsoon rainfall of various meteorological subdivisions for the years 1979, 1983 and the combination of both years. Those values which are statistically significant at 95 and 99% level for the combined data (1979 + 1983) are presented in table 2. It is observed that the SCZASST is negatively and significantly correlated with rainfall over western and central parts of India for the same week. The correlation coefficients in general were found to decrease when analyzed for lags upto three weeks. However, they were still significant for west and east Madhya Pradesh upto lag 1 and lag 3 respectively (95% level).

The CIOZASST is taken as the average of the zonal anomalies between 4°S and 4°N, 64°E to 84°E. The reason for choosing this area for correlating with monsoon rainfall is as follows. Washington *et al* (1977) have shown that the SST anomalies in this region are under the influence of South Equatorial Trough (SET). Further, studies of Cadet and Olory-Togbe (1981) and Prasad *et al* (1983) have shown that there exists a negative correlation between the strengthening of SET activity and monsoon rainfall over India. Table 2 gives the statistically significant correlation coefficients between CIOZASST and monsoon rainfall over different meteorological subdivisions.

Table 2. Statistically significant correlation coefficients (95 and 99% level) between SCZASST and weekly subdivisional monsoon rainfall over India for 1979 and 1983 (combined analysis).

Subdivision	1979 + 1983			
	0	1	2	3
Kerala	—	—	—	—0.565*
Konkan & Goa	—0.551*	—	—	—
West Rajasthan	—0.421*	—	—	—
East Rajasthan	—0.559*	—	—	—
West Madhya Pradesh	—0.602*	—0.454	—	—
East Madhya Pradesh	—0.635*	—0.545	—0.477*	—0.381
Marathwada	—	—	—0.444	—
Vidarbha	—0.537*	—	—	—
Coastal Andhra Pradesh	—	—	—0.440	—

* Refers to the correlation coefficients which are significant at 99% confidence level. There are 15 data samples each in 1979 and 1983.

Table 3. Statistically significant correlation coefficients (95 and 99% level) between CIOZASST and weekly subdivisional monsoon rainfall over India for 1979 and 1983 (combined analysis).

Subdivision	0	1	2	3
Kerala	0.431	—	—	—
Konkan & Goa	0.543*	0.378	—	—
East Rajasthan	0.435	—	—	—
West Madhya Pradesh	0.634*	0.486*	—	—
East Madhya Pradesh	0.660*	0.519*	—	—
Marathwada	0.507*	0.472*	—	—
Vidarbha	0.558*	0.521*	—	—
Coastal Andhra Pradesh	—	—	—	—0.538*

* Refers to the correlation coefficients which are significant at 99% confidence level. There are 15 data samples each in 1979 and 1983.

logical subdivisions. It is observed that the CIOZASST is positively and significantly correlated with the monsoon rainfall over most of the subdivisions for which SCZASST showed negative correlation.

4. Conclusions

(a) The decrease in SST in western Arabian Sea (off Somali and Arabian coasts) during the monsoon months (June to September) is more pronounced during a good monsoon year (1983) than during a bad monsoon year (1979).

(b) Large negative zonal SST anomalies exist off the Arabian and Somalia coasts during good monsoon year. This may be due to the combined effect of strong low level winds and intense upwelling.

(c) The reappearance of 27°C isotherm near the Somalia coast area in May–June coincides with the onset of southwest monsoon over India.

(d) The SCZASST is negatively and significantly correlated with the monsoon rainfall over western and central parts of India. The correlation is maximum for the same week (zero lag) and falls off with time lag from 0 to 3 weeks.

(e) The CIOZASST is positively and significantly correlated with the monsoon rainfall over the western and central parts of the country, and the correlation is maximum with zero lag.

These preliminary results, which are encouraging need to be verified with sufficiently large data set consisting of good, bad and normal monsoon years.

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overview of climate models

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Abstract. Climate models and results, especially for paleo-climatic scale are reviewed. It is concluded that the climatic system is more stable than it was thought to be when Budyko-Sellers type models first came into existence. Even a 2% decrease in solar radiation may not result in White Earth due to negative feed-backs. A decrease in CO_2 to about 200 ppm can result in White Earth. A doubling of CO_2 increases the surface temperature by an order of 1°C . The total climatic system which is nonlinear can exhibit very long period internal oscillations, even of the order of 1,00,000 years though none of the time constants involved are in that range individually.

Keywords. Climate model; paleo-climatic scale; white earth.

Introduction

Climate on earth can be defined as the statistical average state of the total earth-atmospheric system. The parameter (or parameters) describing such a state must be stable. A conservative quantity in a closed system cannot obviously be such a parameter. The length of time to be considered for averaging depends on the time scales that exist and are observable in reality. Two time scales, one seasonal and the other being a very long paleoclimatic one (about a few centuries) are of great concern to us, because on these time scales the variations are quite significant according to observations. Seasonal climate fluctuations are quite common and well known for years without rains and without summers. On the paleoclimatic scale, there is irrefutable evidence that this earth has experienced an extensive variation in the distant past. The climate during such ice ages was crucial to the continuation of life on earth. An attempt is made here to survey the existing theories of climate, especially on the paleoclimatic scale. The literature on this subject is so vast that it is almost impossible to review or refer to all the works. Therefore, attention will be focussed on a few quantitative theories typifying the approach. First, we shall refer to some observations very briefly and then describe the theoretical approach. Later, we discuss various advanced theories and finally, try to indicate the possible future course. The notations are mostly given in appendix A.

Climatic observations

Regarding ancient climates, a variety of observational methods such as geological, paleobiological and physical methods are followed. All these methods examine ancient material to study the climate existing then. Briefly the idea behind each method can be stated as follows:

Geomorphological features such as striated surfaces over wide areas of low relief are generally taken as evidences of extensive glaciation in the past. Existence of different kinds of biota in the ancient samples is taken as the evidence of suitable environment for these types of life forms in the period concerned. The oxygen isotope ratio (O_{18}/O_{16}) or the ratios of the isotopes of other suitable elements, in the samples is taken as an indication of temperature because according to Urey's (1947) theoretical analysis carbonates precipitated from a single aqueous solution at different temperatures will have different O_{18} contents (Emilliani 1955).

Figure 1 gives the ice age history of the earth as reconstructed by following all the methods described above. One can see that ice ages are rare phenomena. However, they do not seem to be periodic and hence not easily predicted in advance. Therefore it is necessary to understand the possible causes of such catastrophic climatic events in order to be able to predict them and to know the future of the planet at least till the last stages of the sun.

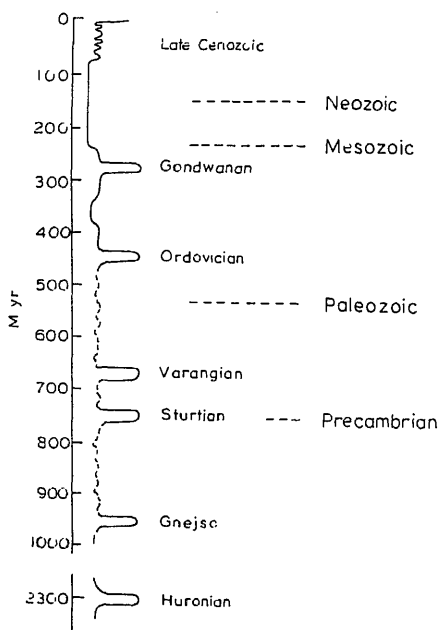


Figure 1. Ice ages through geological time (in millions of years before present) (from Gibbon J 1978).

3. The physical basis of climate change

Figure 2 gives a deceptively simple picture of the earth atmospheric system. One can easily see that the climate is a state of the earth atmospheric system as a result of complex interaction of the atmosphere with its lower boundary and the incoming solar radiation. It is theorized that climatic perturbations can occur either due to external changes such as changes in solar radiation or due to internal changes in albedo, seasurface temperature, cloudcover, atmospheric composition etc. To solve such questions while complex feedback mechanisms are operating, one must take recourse to mathematical modelling. The atmosphere-earth system is replete with many feedback loops. A positive feedback is destabilizing while a negative one is stabilizing. Ice albedo feedback is an example of positive feedback. The greater the ice cover the greater the planetary albedo, and lesser the energy input, lesser the temperature and hence greater the ice cover. Infrared cooling is an example of negative feedback. The greater the temperature, the greater the infrared radiation into space and the cooling, and hence, lesser the temperature. With many such competing and conflicting mechanisms it is obvious that mathematical modelling is

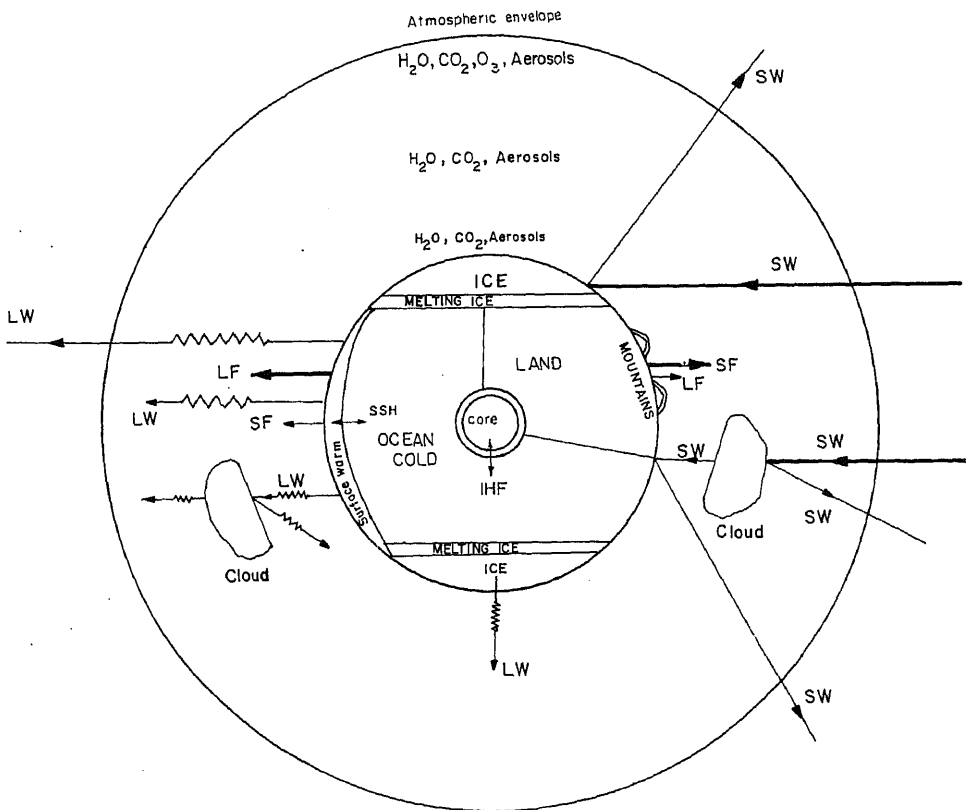


Figure 2. Earth atmosphere radiative system (LW: Long Wave; SW: Short wave radiations; LF: Latent heat flux; SF: Sensible heat flux; SSH: Subsurface heat flux; IHF: Interior heat flux—thick and thin lines represent high and low values respectively).

volcanic dust, S_0 = normal solar constant, and α = Albedo of the planet. ΔS_D was scaled such that there was 0.5% decrease in S_R during 1963 when Mt. Agung erupted in Bali. For ΔS_S an empirical formula was used:

$$S_S = 0.011 \sqrt{N} - 0.0006 N,$$

where N is the sunspot number.

This model could predict the temperature time series since 1600 AD for the northern hemisphere reasonably well.

(ii) Bryson and Dittberner model—To test the effect of anthropogenic dust and increase in CO_2 , these authors proposed another zero-dimensional model. The model is for global surface temperature of the earth. But to calculate the energy reaching the ground, a three-layer atmosphere model having, (a) lower troposphere with dust, (b) an upper troposphere which is dust-free but which has clouds near the base and (c) a stratosphere which has variable amount of volcanic dust, was considered.

The transmissivities of these three layers are assumed to be a , $c(1-N)$ and b respectively, where N is equivalent opaque cloud amount. The fractional forward scattering diffuse operators for these layers were taken to be f_a , f_b and f_c respectively. The albedo of the surface was taken as α_0 . Thus following a cascading computation Bryson and Dittberner (1976) expressed direct and diffuse net shortwave radiation reaching the ground as follows:

$$S_R = S_0(1 - \alpha_0)(1 - N) \cdot \{a + f_a(1 - a)\} \{b + f_b(1 - b)\} \{c + f_c(1 - c)\}.$$

To calculate the net longwave radiation the atmospheric back radiation was assumed to be proportional to the outgoing radiation from the surface. Thus R_g is ground radiation $= \epsilon_e \sigma T_0^4$ and R_a is atmospheric radiation $= \epsilon_e \sigma T_0^4 (1 - \epsilon_a)$, where ϵ_e = surface emissivity, T_0 = screen temperature, and ϵ_a = atmospheric emissivity.

The atmospheric emissivity ϵ_a was taken to be due to that of water vapour (ϵ_w), and of CO_2 and O_3 (ϵ_c), a correction emissivity (ϵ_v) and emissivity due to all other constituents such as dust etc (ϵ_0). Thus

$$\epsilon_a = \epsilon_w + \epsilon_c + \epsilon_0 - \epsilon_v,$$

where $\epsilon_w = 0.60$; $\epsilon_c = 0.0235 \log(\text{CO}_2) + 0.0537 \text{ CO}_2$ in ppm; $\epsilon_v = 0.12$; and $\epsilon_0 = 0.06$.

Taking the green house effect of clouds, the authors finally arrive at an expression for the net long wave radiation at the surface.

$$I_R = (1 - N) \epsilon_e \sigma T_0^4 (1 - \epsilon_w - \epsilon_c + \epsilon_v - \epsilon_0) + N \epsilon_a (\sigma T_0^4 - \sigma T_c^4) (1 - \epsilon_{sc}),$$

where ϵ_{sc} = net cloud emissivity, and T_c = cloud base temperature.

The sum of the sensible and latent heat fluxes L at the surface was given a constant value of 0.6 to 1.0 W m^{-2} . Thus finally Bryson and Dittberner write the zero-dimensional model equation corresponding to (9) as

$$\rho C_m \frac{\partial \bar{T}_0^{xyz}}{\partial t} = S_R + I_R - L. \quad (11)$$

In (11), surface flux is neglected as the model is annual-hemispheric. In this model the transmissivities a , b , c are all of the order of 0.9. This model was utilized to explain the surface temperature variation from 1890 to 1960. Some results of the sensitivity analysis of this model are given in table 1.

We notice that this model has no other feedback except negative longwave radiation feedback. Though the sensitivity results show changes in the right direction, in this type of model, the variations in temperatures can be expected to have smooth and continuous dependence on the controlling parameters.

Weare and Snell (1974) treated the atmosphere as a diffuse thin cloud and considered a zero-dimensional model based on (9). Their sensitivity results, summarized comparatively by them and reproduced below in table 2, show greater stability due to the diffuse thin cloud feedback.

While the sensitivity of Bryson and Dittberner model for 1% change in solar constant is 2.0 K, it is only 0.7 K for diffuse thin cloud model of Weare and Snell. This type of differences, which may seem small but which are indeed quite large considering what can happen to the earth atmospheric system by reduction of global temperature by 2 K, highlight the importance of considering various feedbacks physically correctly.

When one considers other feedback mechanisms, interesting changes can occur. For instance if the albedo is made a function of temperature itself (ice albedo effect) the model can be expected to exhibit dramatic changes at some critical initial and boundary conditions as we shall see later. It is quite difficult to include such a

Table 1. Sensitivity results of Bryson & Dittberner model.

Symbol	Value	Sensitivity for 1% change
S	340.0 W m ⁻²	± 2.00 K
L	94.1 W m ⁻²	± 1.33 K
ϵ_0	0.98	± 0.73 K
N	0.40	± 0.40 K
α_0	0.11	± 0.11

Table 2. Temperature change (in K) for a given environmental change by various models (Weare and Snell 1974).

Variation	Diffuse cloud model	Budyko (1969)	Sellers (1973)	Rasool & Schneider (1971)	Weatherald and Manabe (1975)
Doubling CO ₂	0.7	4.0	10.1	0.8	2.3
Doubling aerosol optical depth	-1.1	—	< -5.0	1.8	—
1% decrease in solar constant	-0.7	-5.0	-5.0	—	-1.3
1% increase in solar constant	0.7	—	—	0.9	1.3 K

$$F_{\text{HVO}}^5 = F_{\text{SSH}} = \text{sub-surface heat flux.}$$

Each of these components depends on highly complex physical processes. F_{SR} and F_{LW} depend on absorption and scattering properties of the earth and the atmosphere which in turn depend on the composition. Similarly F_{SH} , F_{LH} and F_{SSH} depend on dynamical conditions such as wind velocity, wind shear and static stability. Though we have some fundamental knowledge of these properties, it is by no means adequate yet. Our theories are still developing and as such the fluxes we calculate from the existing theories are most probably approximate. It is outside our scope here to discuss the computational details of these flux calculations.

We have given the broad mathematical framework of these energy balance models. In all these models, or in any model for that matter we have the following components in general:

- (a) Predictands, or parameters that are being predicted such as temperature, ice line etc. For these parameters one has to explicitly prescribe from observations and physical reasoning, the initial and boundary conditions.
- (b) Internal structural parameters such as the large scale conductivity, emissivities etc which are generally empirically parametrized. These can be variable or fixed, but have some uncertainty about their value because of lack of knowledge.
- (c) External parameters such as solar constant, angular velocity of the earth etc, which are determined by nature only, but which are also variable.
- (d) Parameters that are invariable such as specific heats, Steffan-Boltzman constant etc.
- (e) Finally, unknown stochastic parameters.

The types of questions asked about any system concern the sensitivity and stability of the system defined by the model equations. For example:

- (i) For fixed structural and external parameters, say corresponding to the present observed state, what is the nature and sensitivity of the model prediction over a period of time, for infinitesimally small perturbations in the initial state which is treated as the initial control state? This question involves a number of questions regarding prediction, stability and predictability raised and examined by Lorenz (1963).

If, depending on the initial magnitude of the 'small' perturbation, the system remains proportionally close to the control system evolution for all times, the system is considered stable. If it differs from the control climate state by greater and greater 'distance in phase space' as time progresses, it is termed as unstable and hence not easily predictable. In such a case or even in the case of bounded 'distance' the predictability has a limiting time which is the time at which the 'distance' between control state and the perturbed state is equal to the natural distance between any two random states of the control evolution. The questions asked are generally: (a) What is the stability property of the system? (b) What is the predictability of the system? There are many sub-questions like the predictability of different parameters, different scales etc here. But we do not go into such details which are outside our scope.

- (ii) For fixed structural and external parameters, what is the nature of evolution

conditions? There is no point in introducing perturbations so large that under those conditions even model equations do not hold. For instance if we start with very cold subzero temperatures over the earth, evidently it will be initially ice-covered (perhaps with solidified air), and will be ice-covered for ever. In this type of investigation it is of great interest to see what final states are reached by the system for different initial conditions and whether they are stable for small perturbations? If the system always comes to a single steady statistical state, it is termed as transitive system. If it has no single final steady state it is called an intransitive system. Depending critically on initial conditions if the system has more than one steady state solution, it is termed almost intransitive. Some simple nonlinear systems have this property of multiple equilibria. This classification is given by Lorenz. Nonlinear systems can be almost intransitive and there can be sudden changes from one weather or climate regime to another weather or climate regime, critically depending on the initial conditions. Our interest again will be the free final states within the physically meaningful range.

(iii) For the same external parameters, how does the nature of the system with regard to stability, predictability and final states, change with differing structural parameters? It must be expected that the nature of the system depends very much on the structural parameters. Sometimes some of the physically possible solutions disappear from the physically meaningful range if the structural parameters are changed.

(iv) For the same structural parameters how does the nature of the system change with changing external parameters? Here again perturbations in external parameters should be within physically meaningful limits of interest. There is no interest left if the sun is shut off or made a red giant.

(v) For the same external and structural parameters, how does the nature of the system change for various amplitudes of stochastic forcings? Here the stochastic forcings again must be within physically plausible range. They must be determined from observations.

With this digression into the general structure of mathematical models and the questions of interest we shall now turn our attention to the energy balance models.

3.2 Zero-dimensional energy balance models

(i) Schneider and Mass model: To determine whether volcanic dust and sun spot associated variation of incoming solar radiation can explain global temperature variations since 1600 AD, Schneider and Mass (1975) applied the earth atmospheric system model given by

$$\frac{\partial}{\partial t} \rho C_m \theta^{xyz} = -\bar{F}_{HVT}^{xyz} = S_R - I_R, \quad (10)$$

which corresponds to (9). Here S_R is the solar radiation flux into system, and I_R is the long wave radiation flux out of the system. The solar radiative flux was further divided into its parts as follows:

$$S_R = \frac{1}{4} (\Delta S_S + \Delta S_D + S_0) (1 - \alpha),$$

the only rational method for the study of such systems. A number of such models have already been proposed and are being tested. We shall first deal with paleoclimatic models. These models can be categorised as (1) zero and one-dimensional energy balance models, (2) vertically or zonally averaged one-dimensional dynamical models, and (3) three-dimensional dynamical models. We shall now discuss these types of models in that order.

3.1 The energy balance models

The basis of all these models is the time-averaged thermodynamic energy equation for the earth (ocean and land) or the atmosphere or for the earth-atmosphere system as a whole. It is written as

$$C_m \left(\frac{\partial}{\partial t} \bar{\rho}' \bar{\theta}' + \nabla \cdot \overline{\rho \nu \theta'} + \frac{\partial}{\partial z} \overline{\rho w \theta'} \right) = - \frac{\partial}{\partial z} \bar{F}'_{\text{HV}} - \nabla \cdot \bar{F}'_{\text{HH}}.$$

This is the basic equation for three-dimensional models.

The zonal average form of (1) gives one type of two-dimensional model equation as

$$C_m \frac{\partial}{\partial t} \bar{\rho}^{ix} \bar{\theta}^{ix} + \frac{\partial}{\partial y} \overline{\rho \nu \theta}^{ix} + \frac{\partial}{\partial z} \overline{\rho w \theta}^{ix} = - \frac{\partial}{\partial z} \bar{F}^{ix}_{\text{HV}} - \frac{\partial}{\partial z} \bar{F}^{ix}_{\text{HHy}}.$$

If we take the vertical average of (1) we get another kind of two-dimensional model equation as

$$C_m \frac{\partial}{\partial t} \bar{\rho}^{iz} \bar{\theta}^{iz} + \frac{\partial}{\partial x} \overline{\rho u \theta}^{iz} + \frac{\partial}{\partial y} \overline{\rho \nu \theta}^{iz} = \bar{F}^{iz}_{\text{HV-B}} - \bar{F}^{iz}_{\text{HVT}} - \frac{\partial}{\partial x} \bar{F}^{iz}_{\text{HHx}} - \frac{\partial}{\partial y} \bar{F}^{iz}_{\text{HHy}}.$$

Here we have used the boundary condition that $W = 0$ at $Z = -B$ and $Z = B$.

If we take the average of (2a) over y , under the following boundary condition

$$F_{\text{HHy}} = V = 0, \text{ at } y \pm Y.$$

we get one kind of one-dimensional model equation

$$C_m \left(\frac{\partial}{\partial t} \bar{\rho}^{ixy} \bar{\theta}^{ixy} + \frac{\partial}{\partial z} \overline{\rho w \theta}^{ixy} \right) = - \frac{\partial}{\partial z} \bar{F}^{ixy}_{\text{HV}}.$$

Another kind of one-dimensional model equation is obtained if we take the vertical average of (2a) and write

$$C_m \frac{\partial}{\partial t} \bar{\rho}^{ixz} \bar{\theta}^{ixz} + \frac{\partial}{\partial x} \overline{\rho u \theta}^{ixz} = \bar{F}^{ix}_{\text{HV-B}} - \bar{F}^{ix}_{\text{HVT}} - \frac{\partial}{\partial x} \bar{F}^{ixz}_{\text{HHx}}.$$

We can write (4) separately for the ocean as

$$\begin{aligned} C_{m_1} \frac{\partial}{\partial t} \overline{\rho_1^{txz} \theta_1^{txz}} + C_{m_1} \frac{\partial}{\partial y} \overline{\rho_1 v_1 \theta_1^{txz}} \\ = \bar{F}_{HVB}^{tx} - \bar{F}_{HVO}^{tx} - \frac{\partial}{\partial y} \bar{F}_{HHy}^{txz}. \end{aligned} \quad (5)$$

For the atmosphere we can write the counterpart of (5) as

$$\begin{aligned} C_{m_2} \frac{\partial}{\partial t} \overline{\rho_2^{txz} \theta_2^{txz}} + C_{m_2} \frac{\partial}{\partial y} \overline{\rho_2 v_2 \theta_2^{txz}} \\ = \bar{F}_{HVO}^{tx} - \bar{F}_{HVT}^{tx} - \frac{\partial}{\partial y} \bar{F}_{HHy}^{txz}. \end{aligned} \quad (6)$$

Equations (5) and (6) describe many one-dimensional energy balance models. Now, if we take the average over x , y and z of (5) we get zero-dimensional model equations. We can write them as follows:

For the lower surface

$$\frac{\partial}{\partial t} \overline{C_{m_1} \rho_1 \theta_1^{txyz}} = \bar{F}_{HVB}^{txy} - \bar{F}_{HVO}^{txy}; \quad (7)$$

For the atmosphere

$$\frac{\partial}{\partial t} \overline{C_{m_2} \rho_2 \theta_2^{txyz}} = \bar{F}_{HVO}^{txy} - \bar{F}_{HVT}^{txy}; \quad (8)$$

And for the earth atmospheric system as a whole

$$\frac{\partial}{\partial t} \overline{C_m \rho \theta^{txyz}} = \bar{F}_{HVB}^{txy} - \bar{F}_{HVT}^{txy}. \quad (9)$$

Equations (7), (8) and (9) are the appropriate ones for zero-dimensional models.

In these equations the energy flux is the result of complex physical processes. For instance, the surface energy flux parameter F_{HV} is composed of five different components of significance. We can thus write

$$F_{HVO} = \sum F_{HVO}^i, \quad i = 1 \text{ to } 5,$$

where

$$F_{HVO}^1 = F_{SR} = \text{shortwave radiative net flux};$$

$$F_{HVO}^2 = F_{LW} = \text{longwave radiative net flux};$$

$$F_{HVO}^3 = F_{SH} = \text{sensible heat flux due to small scale turbulence. This is zero at the top of the atmosphere, } F_{HVT}^3 = 0;$$

$$F_{HVO}^4 = F_{LH} = \text{turbulent latent heat flux due to phase changes of water. This too is zero at the top of the atmosphere, } F_{HVT}^4 = 0; \text{ and}$$

feedback physically correctly in zero-dimensional models because of opposing baroclinicity effects. However, for completeness, we give below the parametrization of such an effect by Weare and Snell within the framework of zero-dimensional models:

$$\begin{aligned}\bar{\phi}_S^x &= 65.0 + 0.1(\bar{T}_0^{xy} - 288.0), \\ \bar{\phi}_N^x &= 65.0 + 2.0(\bar{T}_0^{xy} - 288.0), \\ \bar{\phi}^x &= 0.5(\bar{\phi}_S^x + \bar{\phi}_N^x), \\ \alpha_S &= (1 - \sin \bar{\phi}^x) \alpha_{ice} + (\sin \bar{\phi}^x) \alpha_{Nonice},\end{aligned}\tag{12}$$

where $\bar{\phi}_S^x$ is the southern hemispheric ice line, $\bar{\phi}_N^x$ the northern hemispheric ice line, $\bar{\phi}^x$ the mean ice line latitude, α_{ice} the ice albedo = 0.6, α_{Nonice} the non ice albedo = 0.07, T_0 the surface temperature and α_S the surface albedo. The large amount of empiricism of the above models is a common necessity in palaeoclimatic models and so we cannot hold this against these models as long as the parametrizations are done correctly.

The above models are examples of zero-dimensional energy balance models for the earth's surface and are purely radiative in character. Neither dynamics nor convective overturning of the atmosphere is considered. The next natural extension will be to include convective adjustment of the atmosphere also besides the radiative transfer. In effect such radiative convective models are parametrizing L , the sensible and latent heat transfer from the surface in a more realistic way by modelling the details of the atmospheric temperature structure. These models in general consider the most important radiatively active gases and aerosols. The radiative transfer calculations are performed by considering the absorption spectrum to various degrees of approximation. Convective adjustment is applied so that no part of atmospheric column is either dry adiabatically or moist adiabatically unstable. Because these models consider the atmospheric structure in detail in one dimension (vertical) we shall consider them as one-dimensional radiative convective energy balance models.

3.3 One-dimensional radiative convective energy balance models

Manabe and Moller model—One of the earliest of detailed radiative models is that of Manabe and Moller (1961). This was a test model to be later attached to a general circulation model for radiative transfer calculations. Moller and Manabe found that without convective adjustment computed tropospheric temperature were much too low because of lack of effective upward transfer of heat. Manabe and Strickler (1964) introduced convective adjustment. Manabe and Wetherald (1967) considered the same model which considers all radiatively important constituents. Their final results could reasonably get the correct lapse rate of 6.5° K/km. For details the reader may refer to the original papers.

For the radiative part of these models of this type, the fundamental equation considered is the Schwarzschild's radiative transfer equation for plane parallel atmosphere. This equation can simply be written as

$$\cos \theta \frac{dI_\nu}{d\tau_R} = -I_\nu + \mu^2(1 - a_\nu)B(\nu, T) + \frac{1}{4\pi} \int_0^{4\pi} \chi(\cos \gamma) a_\nu I_\nu d\omega. \quad (13)$$

We notice that (13) is an integro-differential equation for a beam of ν radiation in a plane parallel atmosphere. Thus the history of radiant energy after it enters a medium depends on ν , θ , K_ν , ρ , σ , μ , a_ν , $B(\nu, T)$ and χ . Indeed strictly complete radiative transfer computations are extremely difficult, even assuming we know $\chi(\cos \gamma)$, which is itself very difficult to obtain in real cases in nature.

When the single scattering albedo $a_\nu = 1$, we have pure scattering problem and when $a_\nu = 0$, we have pure absorption problem. Especially scattering computations as (13) reveals, will be most involved. So it is customary to simplify these computations by empiricism and assuming transmission and scattering coefficients. An example of such a procedure has already been given while discussing the Bryson and Dittberner model.

Scattering can be neglected for infrared calculations of the atmospheric radiative transfer. For short wave calculations, absorption is sometimes neglected while scattering is important. Hence at least for infrared calculations we deal with a simpler equation. Even for short wave region if we assume transmission and scattering coefficients, our equation will be much simpler than (13). Such methods are utilized in almost all atmospheric radiative calculations.

Now one important point remains to be clarified. Equation (13) is for a single ν . How can we solve such an equation for all relevant ν s? They will be enormous in number. However, because $B(\nu, T)$ varies slowly over short spectral wavenumber intervals (say 100 cm^{-1}) the problem is simplified for such short intervals. The next problem is to find the correct absorption coefficient or, absorptivities for this interval. About 20 such intervals are needed to model one absorption band, say 6.3μ , of water vapour (Ramanathan and Coakley 1978). Because K_ν varies rapidly with ν it is difficult to define a single average absorption coefficient for any band. Kondratyev suggested a generalized absorption coefficient usage which can reduce the number of K_ν s to be considered without introducing large errors. The method is based on a shift of emphasis from the frequency to the value of the absorption coefficient for the computation of averages. To get the average absorption coefficients we turn to spectroscopic theory and actual laboratory observations. It is found that K_ν depends on pressure and temperature also, and corrections have to be applied for these effects. To circumvent the detailed computational procedure briefly indicated above, empirical emissivities and absorptivities are generally utilized (Manabe and Moller 1961).

From the above discussion one can see that the total radiative calculations for the atmosphere in all their complexity can never be performed. Even if we perform line-by-line integration the problem of cloud effect with all its uncertainties remains. To consider the finiteness of a single cloud is itself a very challenging problem in a theoretical sense. If one has to consider a number of clouds of different structures and sizes the computational problem tends towards hopeless complexity if attempted purely by scattering theory. Therefore we can easily conclude that at one stage or other we have to take recourse to some level of empiricism in these computations. In such a case we must also know the sensitivity

further e.g. Sellers (1973). Such an astonishing result by sufficiently different models generated a great interest in these "educational toy" models. Therefore Budyko-Sellers type models were further investigated by a number of workers. Feagre (1972), Schneider and Gal-Chen (1973), Dwyer and Petersen (1973), Chylek and Coakley (1975), Held and Suarez (1974), Schneider and Dickinson (1974), North (1975a, b), Cess (1976), Su and Hsieh (1976), Ghil (1976), Cahalan and North (1979), North and Coakley (1979) and others examined the steady state behaviour, sensitivity, internal and structural stability of Budyko-Sellers type models and their minor variants. The temperature-dependent albedo formulation of all these models can be given in a generalized form (Ghil 1976) as

$$\alpha = \left| b(\phi) - C_1(\bar{\theta}_{1m}^{xyz} + (\bar{\theta}_1^{xz} - C_2 Z(\phi) - \bar{\theta}_{1m}^{xyz})) \right|_C \quad (21)$$

where $b(\phi)$ and $Z(\phi)$ are given functions of the latitude; C_1 and C_2 are empirical constants (0.009 and 0.0065 K⁻¹ respectively); C stands for cut-off; D is any dependent variable; $D^- = \min(D, 0)$; $D_C = 0.25$ if $D \leq 0.25$; $D_C = D$ if $0.25 < D < 0.85$; $D_C = 0.85$ if $0.85 \leq D$; and m stands for yearly average.

In the above type of formulation with $\bar{\theta}_1^{xz} - C_2 Z(\phi) > \bar{\theta}_1^{xyz}$ (when the temperature is greater than an average temperature), α is independent of the surface temperature because snow or ice cannot be expected at these latitudes above a certain critical temperature. Almost all the albedo formulations generally used are generalized by (21) by Ghil (1976). Cloud effect can be further included as was done by Mokhov (1979).

The most important results of further investigations into the nature of Budyko-Sellers models with the albedo formulation of the type (21) can be briefly stated as follows:

(i) Steady state behaviour. These models generally predict, for the present-day solar constant, three steady state solutions for the surface temperature at the equator in the range of 100 K to 300 K e.g. Ghil (1976), Frederiksen (1976). One of these solutions $\bar{\theta}_{1E}^{xyz1}$ corresponds to deep freeze. The second solution $\bar{\theta}_{1E}^{xyz2}$ corresponds to glacial and the third solution $\bar{\theta}_{1E}^{xyz3}$ corresponds to the present climate. The three steady state solutions obtained by a few authors are given below for comparison and to get an idea of the numbers involved.

In a time-dependent model such multiple states mean that the final solution is critically dependent on the initial conditions.

Looking at table 4, one can see great similarity in results. However, one cannot conclude that there is comfortable quantitative concurrence. Such differences, evidently due to differing parametrizations, highlight the need for greater observational work to parametrize the most important terms more physically accurately.

(ii) Stability behaviour. North (1975, 1979) and Su and Hsieh (1976) and Mokhov (1979) studied the stability properties of these models and rigorously established what is known as the slope stability theorem. According to this theorem the solution is stable or unstable if

$$\frac{dS}{dX_s} > 0 \text{ — stable; and } \frac{dS}{dX_s} < 0 \text{ — unstable,}$$

Table 4. Steady state solutions of Budyko-Sellers type models (global means)

Model	Authors	$\bar{\theta}_{1E}^{txz_1}$	$\bar{\theta}_{1E}^{txz_2}$	$\bar{\theta}_{1E}^{txz_3}$
		Deep freeze (K)	Glacial (K)	Inter glacial (K)
Sellers	Ghil (1976)	175.4	268.6	287.7
Faeger	Schneider and Gal-Chen (1973) Frederickson (1976)	175.0	287.0	302.0
Budyko	Frederickson (1976)	222.0	269.0	289.0

where S is the solar constant; and X_S the sine of latitude of the ice line.

These results were graphically depicted by Cahalan and North (1979) and reproduced in figure 5. It is seen that the ice free and deep freeze solutions are very stable. Even the present-day climate ($X_S = 0.85$) is quite stable. Very small ice cap solutions seem to be unstable.

Both Budyko and Sellers' models predict deep freeze for 1.5% reduction in solar radiation. We notice that both these models do not include cloud feedback as pointed out by Bryson and Snell or ice insulation feedback as pointed out by Newell (1974), Bunker (1976) and Saltzman (1978). One of the interesting models in the same class of Budyko-Sellers models is that of Saltzman (1978) (in that it is also an energy balance, nonlinear one-dimensional latitudinal model) which considers one of the important negative feedback mechanisms, the ice insulation feedback mechanism. We shall now deal with this model briefly.

Saltzman's oscillator model. Saltzman (1978) advanced a simple energy balance model with the ice insulation negative feedback having two predictive parameters (1) the mass mean ocean temperature θ_0 and (2) the sine of the latitude of ice edge η . Saltzman and Moritz (1980) further developed this model to include CO_2 feedback effects and more realistic turbulent heat transfer at the interface of the earth and the atmosphere. This model is still being developed.

The simplest physical system of Saltzman can be described if we take $y = \sin$ of latitude, as follows:

Ocean from $y = 0$ to $y = \eta = \text{melting ice edge}$;
 Melting ice from $y = \eta$ to $y = \eta + \Lambda = \text{frozen ice edge}$; and
 Ice from $y = \eta + \Lambda$ to $1 = \text{pole}$.

The thicknesses of ice and the active ocean are taken as constants I and D respectively. Frozen ice acts as an insulator so that the sensible heat-transfer cannot take place across it. Across the melting ice, however, the sensible heat-transfer can take place. With this physical picture, Saltzman writes two equations, one for the θ and one for η as follows

years. We shall now deal with Budyko's model with the following assumption introduced in (5) and (6):

$$\begin{aligned}\bar{F}_{-B}^{tx} &= 0; \\ \bar{F}_{HHy}^{txz} &= 0, \\ \overline{\rho v_1 \theta_1^{txz}} &= 0, \text{ and} \\ C_{m_2} \frac{\partial}{\partial y} \overline{\rho_2 v_2 \theta_2^{txz}} &= \bar{F}_{HVO}^{tx} - \bar{F}_{HVT}^{tx}.\end{aligned}$$

Budyko started with a simplified equation (in the light of the above assumption) corresponding to (5),

$$\begin{aligned}C_{m_1} \frac{\partial}{\partial t} \bar{\rho}_1^{txz} \bar{\theta}_1^{txz} + C_{m_2} \frac{\partial}{\partial y} \overline{\rho_2 v_2 \theta_2^{txz}} \\ = -\bar{F}_{HVT}^{tx} = \bar{F}_{HVT}^{tx_1} = \bar{F}_{HVT}^{tx_2}.\end{aligned}\quad (1)$$

Here $\bar{F}_{HVT}^{tx_1}$ and $\bar{F}_{HVT}^{tx_2}$ are fluxes at the top due to solar and infrared radiation respectively. Based on empirical computations over a large number of meteorological stations, Budyko assumed

$$\begin{aligned}\bar{F}_{HVT}^{tx_1} &= -S(1 - \alpha), \\ \bar{F}_{HVT}^{tx_2} &= a + B \bar{\theta}_1^{txz} - (a_1 + B_1 \bar{\theta}_1^{txz})n, \\ C_{m_2} \frac{\partial}{\partial y} \overline{\rho_2 v_2 \theta_2^{txz}} &= \beta(\bar{\theta}_1^{txz} - \bar{\theta}_1^{txyz}).\end{aligned}$$

Here α is the Albedo of the planet; β , an empirical constant $= 0.235 \text{ K cal cm month}^{-1} \text{ K}^{-1}$; S is the solar constant $= 1.92 \text{ cal cm}^{-2} \text{ min}^{-1}$; a , B , a_1 , B_1 and n are empirical constants equal to 14.0, -0.14 , -3.0 , -0.1 respectively and n = cloud cover. With these assumptions Budyko writes (14) as

$$\begin{aligned}C_{m_1} \frac{\partial}{\partial t} \bar{\rho}_1^{txz} \bar{\theta}_1^{txz} + \beta(\bar{\theta}_1^{txz} - \bar{\theta}_1^{txyz}) \\ = S(1 - \alpha) - a - \beta \bar{\theta}_1^{txz} + (a + B_1 \bar{\theta}_1^{txz})n\end{aligned}\quad (1)$$

with temperature-dependent albedo

$$\begin{aligned}\alpha &= 0.62 \text{ if } T < -10^\circ\text{C is covered} \\ &0.50 \text{ if } T = -10^\circ\text{C ice line} \\ &0.32 \text{ if } T > -10^\circ\text{C ice free}\end{aligned}\quad (1)$$

and the planetary temperature

$$\bar{\theta}_1^{txyz} = \frac{\bar{S}^y(1 - \bar{\alpha}^y) - a_1 + a_1 n}{B - B_1 n}.\quad (1)$$

Because of albedo (α) dependence given by (16) on temperature, (15) becomes nonlinear climate equation.

Sellers (1969) proposed a similar model differing from Budyko's model, especially in the parametrization of large scale eddy fluxes. Sellers takes the Austausch approach and writes

$$-\frac{\partial}{\partial y} \overline{v_2 \theta_2^{xz}} = \frac{\partial}{\partial y} K \frac{\partial}{\partial y} \bar{\theta}_2^{xz}, \quad (18)$$

where K is a macro turbulence coefficient of the order of $10^6 \text{ m}^2 \text{ sec}^{-1}$, taken as a constant.

The albedo in Sellers model was also a function of surface temperature and is given by:

$$\alpha = b - 0.009 \bar{\theta}_1^{xz}; \text{ if } \bar{\theta}_1^{xz} < 283.16 \\ b - 2.458; \quad \text{if } \bar{\theta}_1^{xz} > 283.16, \quad (19)$$

where b is a function of latitude only. The infrared radiative term in Seller model is nonlinear and is given by

$$\bar{F}_{\text{HVT}}^{xz} = \sigma (\bar{\theta}_1^{xz})^4 (1 - m \tanh \{19(\bar{\theta}_1^{xz})^6 10^{-16}\}), \quad (20)$$

where σ is the Stefan-Boltzman constant, and m is the Atmospheric attenuation coefficient = 0.50. One can see that both these models parametrize entire dynamics in simple ways. However, especially in the parametrization of large scale eddy flux and the long wave flux, these models differ considerably. Yet both these models gave similar results. Budyko's results are shown in figure 4. We can see that the ice line latitude decreases non-linearly with decreasing solar radiation. Once the ice line crosses a critical latitude, the ice spreads by self-development until it covers the whole earth. Similar result was obtained by Sellers also, who improved his model

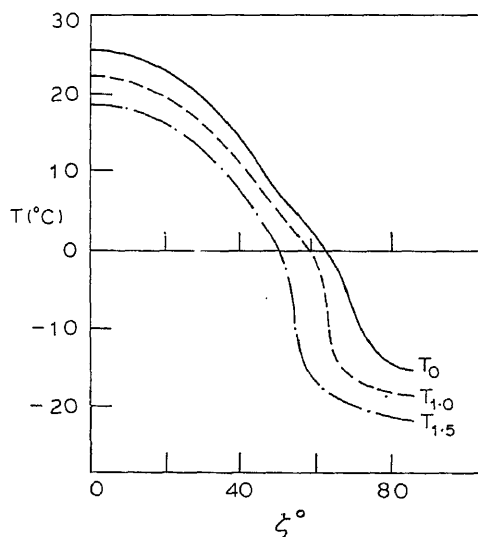


Figure 4. The dependence of temperature (T) distribution with latitude (ξ) on radiation amount. The suffixes for T indicate the percentual decrease in solar radiation (from Budyko, 1969).

of radiative transfer calculations to various empirical factors introduced. Therefore we need to perform highly theoretical computations at least for some simple cases and this is the justification for continuous sophistication in radiative computations.

Emissivity formulation is a very widely used empirical method in atmospheric radiative calculations. The results of Manabe and Moller (1961), using this method are given in figure 3. The following points can be noted:

- (i) H_2O and CO_2 have a cooling effect of the order of 1.5°K/day in the infrared.
- (ii) They both have a heating effect in the short wave region. H_2O heating effect is quite considerable, of the order of 0.5°K/day , in the lower troposphere where H_2O is maximum.
- (iii) In stratospheric regions O_3 is the most important constituent contributing to the heating effect of about 2°K .

Other radiative convective models suggested are similar to that of Manabe and Moller. In these models clouds are generally climatological and fixed. The distribution of CO_2 and O_3 are also fixed in many models. For H_2O it is sometime assumed that the relative humidity is only a function of surface relative humidity and pressure

$$\text{RH} = \text{RH}_* f(p),$$

convective adjustment is invariably applied.

One of the most important results to emerge from these models is that for the surface temperature of the planet, the hierarchy of importance in constituents is, clouds, H_2O , CO_2 , and O_3 in that order. This hierarchy varies for other temperatures at other altitudes. Climatic experiments have been performed by many workers with such one-dimensional models of radiative convective type. (Manabe and Strickler 1964; Manabe and Wetherald 1967; Rasool and Schneider 1971; Weare and Snell 1974; Schneider 1975; Ramanathan 1976; Wang *et al* 1976; Ramanathan and Coakley 1978). The most consistent results of such experimentation are given in table 3.

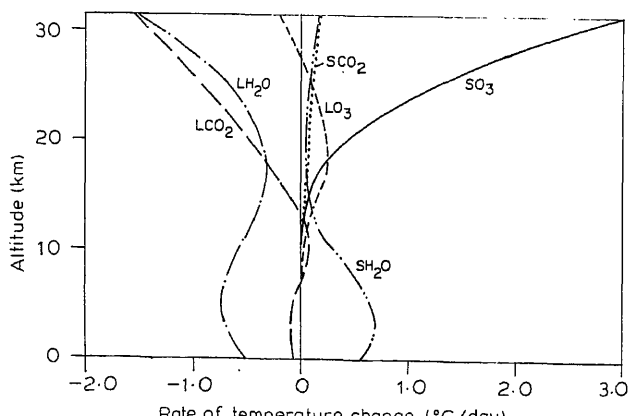


Table 3. Sensitivity results of some radiative convective models

Parameter	Change (%)	Surface effect (K)	Stratospheric effect (K)
CO ₂	Doubling	+ 2.0	- 8.0
O ₃	- 50	- 0.5	- 5.0
Clouds: Low	+ 1	- 0.82	
Medium	+ 1	- 0.39	
High	+ 1	- 0.17	
Solar constant	± 1	± 1.3 to ± 1.6	

Though the radiative convective models deal with detailed structure of the atmosphere in the vertical direction, it is difficult to consider feedbacks of great importance which have something to do with planetary albedo. This is because albedo arising mainly due to ice or snow cover and clouds is a function of latitude and longitude. Even if one tries to relate surface albedo and cloud cover to the surface temperature in these models, it will be physically not satisfactory. Suppose one relates the surface albedo to surface temperature such that cooler temperatures are associated with greater ice cover and hence greater albedo, it will not be correct without considering the baroclinicity and dynamical effects. It is because the large scale dynamical feedback mechanism is always a negative feedback as far as temperature gradients are concerned and so opposes the advance of ice cover. Greater the ice cover, greater the baroclinicity and hence the heat transport to the poles and consequently lesser the ice cover. Here is a powerful negative feedback mechanism which is due to the large scale dynamics.

In any case the vertical structure of the atmosphere is controlled by convection to a great extent. Hence the surface temperature fixes the temperature of the system. This surface temperature depends on the surface energy balance, which in turn depends on radiative transfer. However, it is felt that these energy balance parameters at the surface can be parametrized in terms of surface temperatures (without actually computing radiative fluxes in detail) with sufficient accuracy. Such a possibility allows one to construct one-dimensional models in the y direction (latitudinal direction) and consider the dynamical effects and the latitudinal variation of the albedo. Thus we are led to the latitudinal one-dimensional energy balance models. We shall now deal with such models.

3.4 Latitudinal one-dimensional energy balance models

Budyko-Sellers type models—Equations (4), (5) and (6) are the basic equations for these models. The models differ in parametrization of various terms in this equation. Budyko (1969) and Sellers (1969) proposed two simple models based on (2), (4) and (6) which are the basic equations for these models. Both

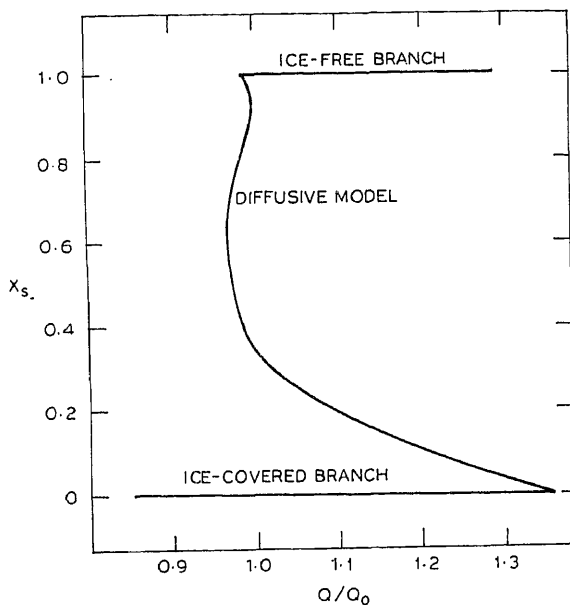


Figure 5. Graph of the sine of the latitude of the ice line X_s as a function of the solar constant Q (in units of the present value Q_0) obtained from a zonal model having diffusive heat transport (from Cahalan and North, 1979).

$$d\eta/dt = \frac{\Lambda}{2\rho_{ice}I} M(\eta) \quad (23)$$

where C_w is the specific heat of ocean waters; ρ_w is the density of ocean waters; L_f is the latent heat of fusion; ρ_{ice} is the density of ice; and $M(\eta)$ = rate of melting

$$= \frac{1}{L_f} \sum_{i=1}^5 \bar{F}_{HV,i}^x, \text{ at } y = \eta$$

After parametrizing for all the heat fluxes Saltzman gets an equation system in the simplest case as follows:

$$d\theta/dt = -\psi_1\eta - \psi_3\eta^2\theta, \quad (24)$$

$$d\eta/dt = \phi_1\theta - \theta_3\eta, \quad (25)$$

where ψ_1 , ψ_3 , ϕ_1 and ϕ_3 are positive constants. The system (24) and (25) has two basic components: (i) A damped harmonic oscillation due to ice insulator effect (negative feedback). As η increases, θ decreases due to greater sensible heat loss, and so η tends to decrease. (ii) A nonlinear restorative mechanism by $\eta^2\theta$ term.

The most important result of Saltzman *et al* (1980) for a particular variant of his original model that included CO_2 feedback is shown in figure 6. These results are for fixed structural and external parameters and are for arbitrary initial conditions which are small perturbation states from a reference state. $\theta_0 = 278$ and $\eta_0 = 0.85$ approximately. These results can be summarized as follows:

(i) The solution represents a periodic climate, exhibiting a limit cycle behaviour. The major period of oscillation is about 1300 years. The system is intransitive since

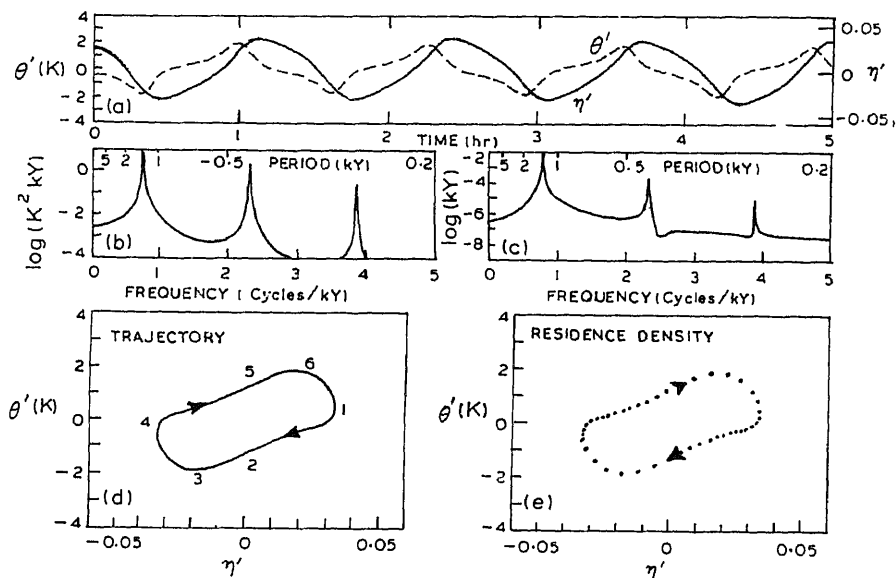


Figure 6. Sample solution for the deterministic system: a) the time evolution of η' (solid) and θ' (dashed), b) variance spectra for θ' , c) variance spectra for η' , d) phase plane trajectory and e) residence density of the solutions (dots are spaced at every 30 years) (from Saltzman *et al* 1981).

it never settles down to a single steady state.

(ii) The system has large residence densities in two states of extreme excursions of η . These two states can be looked upon as glacial and nonglacial climates. These states correspond to the two states of Budyko-Sellers type models (Cahalan and North).

(iii) η lags behind θ by about a hundred years.

(iv) The amplitudes of θ and η are about 1° K and 0.05 ($= 6^\circ$ latitude) respectively.

(v) We can say that the ice advances or retreats with a period of 1300 years about a mean position of 58° latitude. This is about the right period for inter glacial oscillations.

Besides this reference state, about which limit cycle oscillations occur, Saltzman *et al* found another steady state solution, an unstable saddle point state for large perturbation. This unstable state corresponds to $\eta = 0.6469$ (42° latitude) and $\theta = 277.67^\circ \text{ K}$.

Another important finding was that these two states merge at a bifurcation point as the solar radiation is reduced by about 1.6%. At this point deep freeze is a possibility. This result agrees with that of Budyko-Sellers model though Saltzman's model is different from Budyko-Sellers type models! Saltzman also obtained a deepfreeze solution for a decrease in CO_2 to about 215 ppm.

It was found by Saltzman that for the present-day solar constant, stochastic forcings of certain amplitude tend to keep the climate of the model in either of the two extreme excursion states (glacial or nonglacial) of large residence density for comparatively very long time skipping from one state to the other very fast. Hence

magnitude of the stochastic forcings in nature.

Here we shall take a detour and dwell a little on what are known as stochastic climate models. Hasselman (1976), Lemke (1977), Reynolds (1978), Ruiz and Lemke (1982) and others are experimenting with this approach. The basic idea here is that for small deviations from an equilibrium state the equation governing a climate variable, say y_s (may be SST, ice line etc) is governed by an equation of the type

$$dy_s/dt = -V(t)y_s(t) + W_s(t). \quad (26)$$

Here $W_s(t)$ is the random forcing by weather, $V(t)$ is a structural negative feedback parameter determined mainly by observations. This represents a resistance to change in $y_s(t)$. Equation (26) is known as Langevin type of equation. The main problem of these models is to fix correctly the nature of $V(t)$ and $W_s(t)$ from observations. These models are being successfully employed to predict SST over a time period of a few months to few years. The signature of $y_s(t)$, its amplitude and phase depends critically not only on $V(t)$ but also on $W_s(t)$. Thus from the signatures in nature we can reverse compute $V(t)$ and $W_s(t)$, having known the behaviour of this model for a variety of combinations of $V(t)$ and $W_s(t)$.

Now comparing Saltzman's results with those of Budyko-Sellers type model results, we note the following: (i) Both predict deep freeze for about -1.5% perturbation in the solar constant. (ii) Slope stability theorem holds for both the models (Saltzman 1980) (figure 11). (iii) For Budyko-Sellers models the present climate for the present solar constant is a stable one while for Saltzman model it is a nearby climate to a reference climate about which limit cycle oscillation, of period of 1500 years and an amplitude of 6° latitude for ice line exists. The amplitude of θ oscillation is only about 1° K.

The next natural variation of the climate models is to introduce either longitudinal dimension to include land ocean contrast effects also or to introduce the vertical dimension for zonally averaged models to include the atmospheric effects explicitly. In building up the climate models one must be able to keep the model simple enough by correct parametrization or else one will end up with a G.C.M. The real problem of parametrization is more difficult than direct numerical integration. For such parametrizations we must depend on nature and the G.C.Ms. These constraints were kept in mind by the modellers of two-dimensional palaeo-climate models. Now we shall deal with a few typical two-dimensional climate models.

4. Two-dimensional climate models

One of the unique two-dimensional models of its kind is due to Sellers (1976). The main equation of this model is (2b) which can be easily and separately written for ocean, atmosphere and land on which advective fluxes and storage term are neglected. (2b) was applied to a land-ocean atmospheric system by Sellers. The

main difficulty in applying (2b) is to parametrize the advective part. Sellers utilized the following few of the important assumptions to finally connect all the advective fluxes to the surface temperatures θ_s . Denoting atmosphere and oceans by subscripts 1 and 2 respectively Sellers writes for the atmosphere as

$$\begin{aligned}\theta_1 &= \theta_{1s}(p/p_s)^{0.19}; \\ u_1 &= u_{1s} - \frac{gz}{f\theta_{1s}} \frac{\partial \theta_{1s}}{\partial y}; \text{ and} \\ v_1 &= v_{1s}(2P - P_s - P_T)/(P_s - P_T)\end{aligned}$$

To compute u_0 s and v_0 s he considered the Ekman layer equations which also contain the pressure gradients in addition to u_0 s and v_0 s. These pressure gradients were expressed by Sellers in terms of surface temperature gradients. For this he utilized a parameter b which is purely empirical. Thus he writes

$$\frac{\partial p_s}{\partial y} = \frac{b}{a_2} \frac{\partial \theta_{0s}}{\partial y},$$

where b is the Ferrel cell in the middle latitudes. Fixing b is like fixing the geographical extent of the Ferrel Cell. For oceans he takes

$$\begin{aligned}v_2 &= v_{2s} e^{-az}(1 + az)C; \\ \theta_2 &= \theta_{2WB} + (\theta_2 - \theta_{2WB})e^{az}; \text{ and} \\ v_{2s} &= \pm C u_{1s}.\end{aligned}$$

Here θ_{2WB} is the prescribed subsurface temperature of the ocean.

$$\overline{v_1 \theta_1} = \overline{v_1} \cdot \overline{\theta_1} + \overline{v_1' \theta_1'} = \left(\overline{v_1 \theta_1} - K_1 \frac{\partial \theta_1}{\partial y} \right), \quad (27)$$

$$\overline{v_2 \theta_2} = \overline{v_2} \cdot \overline{\theta_2} + \overline{v_2' \theta_2'} = \left(\overline{v_2 \theta_2} - K_2 \frac{\partial \theta_2}{\partial y} \right). \quad (28)$$

Here overbar denotes $-txz$. u_1 is geostrophic in the atmosphere and hence $f_3(P)$ was taken consistent with $f_1(P)$; u_{1s} and v_{1s} were determined from Ekman equations for the atmospheric boundary layer with approximations. Note that $v_1 \cdot \theta_1$ and $v_2 \cdot \theta_2$ terms are explicitly taken by Sellers. This is important to include Hadley Cell effects properly in tropics e.g. Lindzen and Farrel 1977, Saltzman and Vernekar (1983).

After utilizing a few more assumptions in parametrizing the solar and infrared radiation fluxes, Sellers could integrate the final equation which is for the surface temperature, with a time step of one month.

Sellers conducted some of the standard palaeoclimatological sensitivity experiments with this model. Possible multiple states for the present-day solar constant are indicated by the experiments. These results are reproduced in figure 7. From this figure one can conclude that the final state reached is a function of initial conditions.

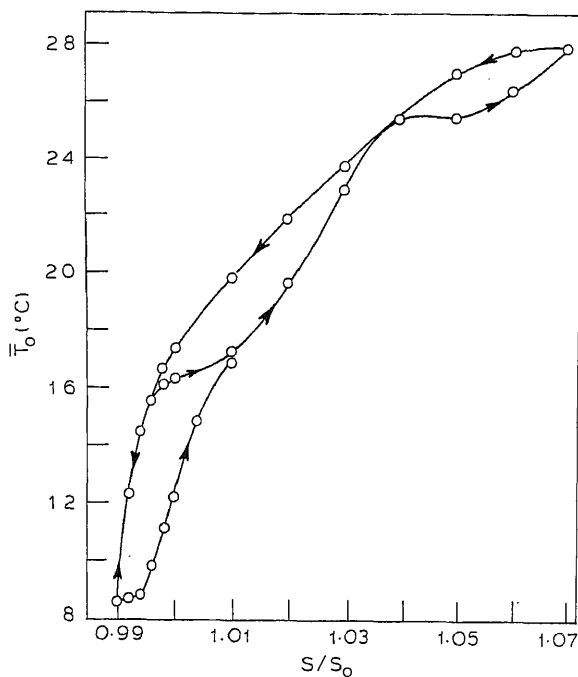


Figure 7. The average global sea level temperature in August as a function of the solar constant S , expressed as a function of its present value S_0 (1.94 ly min^{-1}). The arrows indicate the path followed in increasing or decreasing the solar constant (from Sellers, 1973).

Another typical two-dimensional model is the zonally averaged model. There are a great variety of zonally averaged models to explain the present-day climate e.g. Smagorinsky (1963), Saltzman and Vernekar (1971), Kurihara (1973), Sela and Wiin-Nielsen (1971) and a number of others. For brevity we deal only with the most important aspects of a few models only here.

In all these models parametrization of the large scale turbulent fluxes is the most challenging problem. It is nothing else but a problem in large scale turbulence. If we indicate as usual u , v , w , θ , ε and ϕ as dependent variables, the eddy flux matrix elements can be written as follows (table 5):

Table 5. Eddy flux parameters.

	u	v	w	θ	ε	ϕ
u	$\overline{u'^2}$					
v	$\overline{u'v'}$	$\overline{v'^2}$				
w	$\overline{u'w'}$	$\overline{v'w'}$	$\overline{w'^2}$			
θ	$\overline{u'\theta'}$	$\overline{v'\theta'}$	$\overline{w'\theta'}$	$\overline{\theta'^2}$		
ε	$\overline{u'\varepsilon'}$	$\overline{v'\varepsilon'}$	$\overline{w'\varepsilon'}$	—	$\overline{\varepsilon'^2}$	
ϕ	$\overline{u'\phi'}$	$\overline{v'\phi'}$	$\overline{w'\phi'}$	—	—	$\overline{\phi'^2}$

Here the primed quantities denote eddy fluxes. The most important of these fluxes, for parametrization problems are $\overline{v'^2}$, $\overline{v'u'}$, $\overline{v'\theta'}$, $\overline{v'\epsilon'}$, $\overline{w'\theta'}$, and $\overline{w'\epsilon'}$. We have seen that $\overline{v'\theta'}$ is generally parametrized by the Austausch approach. Stone (1973) showed how sensitive the results can be for such a parametrization. Sometimes constant Austausch assumption can be very misleading. His results are reproduced in figure 8. At this point we shall digress a little and dwell on the important aspects of the large scale eddy flux parametrization.

In almost all such parametrizations, the barotropic baroclinic instability theory is invoked in general. Baroclinic linear instability theory gives us the fastest growing mode, its rate of growth and its structure in the initial stages. Barotropic instability theory also does the same for barotropic case. In general, it is assumed that baroclinic instability analysis gives the unstable mode and this mode barotropically transfers momentum. It is also assumed, in general, that this unstable mode alone characterises all the transient eddies. Though the instability theory gives the gravest mode, growth rate, and structure, it leaves the amplitude arbitrary. But the amplitude is essential to compute the transient fluxes. Therefore, to fix the amplitudes generally, some closure assumptions are introduced as in turbulence theory. A number of workers, Arakawa (1961), Saltzman and Vernekar (1971), Smagorinsky (1964), Sela and Wiin-Nielsen (1971), Kurihara (1973), Egger (1975), Greene (1970), Sasamori (1978) and Branscome (1983) to name some, arrived at various parametrization formulae. A few of these parametrizations are given in table 6.

One can see that finally, especially for energy fluxes, Austausch-type parametrizations are arrived at. Such formulae may not hold good locally because the scale of the eddy must be small when compared with the scale of the mean field, for

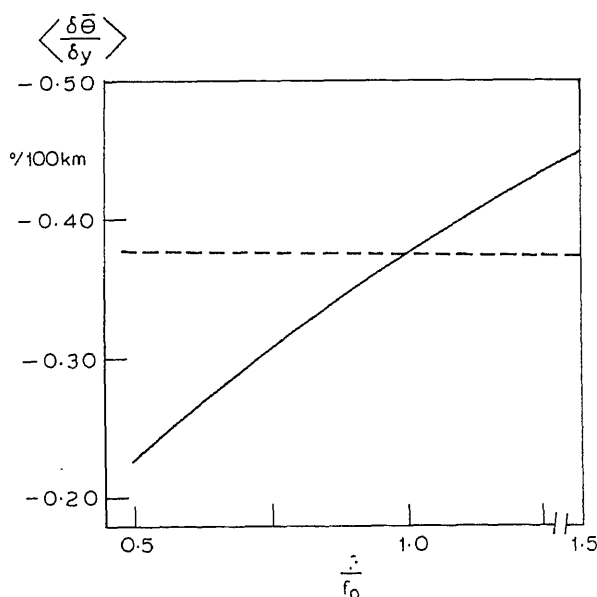


Table 6. Few large scale eddy flux parametrization schemes

Author(s) and Equations	Comments
Arakawa (1961) $\frac{\partial}{\partial t} \overline{u'^2}^{tx} + 2 \left(1 - \frac{2D^2}{L^2} \right) \frac{\partial \bar{u}^{tx}}{\partial y} \overline{u'v'}^{tx} = 0$ $\frac{\partial}{\partial t} \overline{v'^2}^{tx} + \frac{4D^2}{L^2} \frac{\partial \bar{u}^{tx}}{\partial y} \overline{u'v'}^{tx} = 0$ $\frac{\partial}{\partial t} \overline{u'v'}^{tx} + D^2 \frac{\partial^2 \bar{u}^{tx}}{\partial y^2} \frac{\partial}{\partial y} \overline{v'^2}^{tx} + \frac{\partial \bar{u}^{tx}}{\partial y} \left(\overline{v'^2}^{tx} + \frac{2D^2(\overline{u'^2}^{tx} - \overline{v'^2}^{tx})}{L^2} \right) = 0$	Barotropic vorticity equation which conserves the absolute vorticity is the basic equation. L and D, scale lengths of the disturbance in the x direction and that of $v' \partial \bar{u} / \partial y$ respectively are given by other considerations explicitly.
Saltzman and Vernekar (1971) $\overline{v'^2}^x = -\Lambda \bar{\eta} S_0^{-1} \frac{1}{a} \frac{\partial \bar{\theta}^x}{\partial \phi}$ $\overline{v' \theta'}^x = -K \frac{1}{a} \frac{\partial \bar{\theta}^x}{\partial \phi}$ $\overline{\omega' \theta'}^x = \Pi K \frac{1}{a} \frac{\partial \bar{\theta}^x}{\partial \phi} \left(\frac{\partial \bar{\theta}^x}{\partial p} \right)^{-1}$ $\overline{u'v'}^x = T \overline{v'^2}^x \cos \phi \frac{d\mu}{d\phi}$ $\mu = \frac{\bar{u}^x}{a \cos \phi} - \frac{a \cos \phi}{n_*^2} \left(\beta - \frac{1}{a^2} \frac{\partial}{\partial \phi} \frac{1}{\cos \phi} \frac{\partial}{\partial \phi} \bar{u}^x \cos \phi \right)$ $K \propto \frac{\partial \bar{\theta}^{tx}}{\partial p} \left(\frac{1}{a} \frac{\partial \bar{\theta}^{tx}}{\partial \phi} \right)^{-2}$	Barotropic and baroclinic instability theories are used. To determine K and T arcally averaged conservation of energy principle and zonally averaged conservation of angular momentum principles are utilised. The gravest mode is determined as baroclinically the most unstable one. Here θ is the potential temperature, Π a constant (0.75), and T a characteristic time for wave tilting. Λ is the constant of proportionality between the gravest mode growth rate and the kinetic energy of the unstable eddy. K is determined by requiring that the upward heat flux below the level must balance the heat loss above. So is a static stability parameter proportional to $(\partial \theta_0 / \partial p)^{1/2}$.
Greene (1970) $\overline{v' S'}^x = -K_{vy} \frac{\partial \bar{S}^x}{\partial y} - K_{vz} \frac{\partial \bar{S}^x}{\partial z}$ $\overline{v' S'}^x = -K_{zy} \frac{\partial \bar{S}^x}{\partial y} - K_{wz} \frac{\partial \bar{S}^x}{\partial z}$ $\frac{\partial}{\partial y} (\rho_0 \overline{u'v'}^x) = \rho_0 K_{vy} \frac{\partial}{\partial y} (f + \xi) - \frac{f \rho_0}{B} \frac{\partial \phi}{\partial y} \frac{\partial}{\partial z} K_{vy} - f \frac{\partial}{\partial z} \rho_0 K_{vz}$ $\frac{\partial}{\partial y} (\rho_0 \overline{u'v'}^x) = f \frac{\partial}{\partial z} \rho_0 \overline{v \phi}^x - \frac{\rho_0}{B} \rho_0 \overline{v' Q'}^x$	Here Q = potential vorticity. For momentum transfer the barotropic theory of conservation of geostrophic absolute vorticity was used. An equation connecting $\overline{Q'v'}^x$ of $\overline{u'v'}^x$ which was utilized by Arakawa (1961) and Charney & Stern (1962) was utilized. Here K_{vy}, K_{wy} and K_{wz} are proportional to K_{vy}. Kinematic consideration that $\overline{v' Q'}^x$ is maximum at quarter period of the baroclinically unstable wave, was utilized. Here Q is defined as

$$Q' = f \frac{\partial \phi'}{\partial z} + B\xi + \frac{f}{\rho_0} \frac{\partial \rho_0}{\partial z}$$

$$K_{vy} = -1.5 \alpha (g/B)^{1/2} (\Delta \phi)^2 / (\partial \phi / \partial y)$$

B a static stability parameter. S is any conservative parameter, $\phi = \log \theta$, and θ is potential temperature.

Branscome, L. E.

$$\overline{v' T'^{ix}} = 0.6 \frac{N d^2}{\alpha g} \left(\frac{\partial \bar{u}^{ix}}{\partial z} \right)^2 e^{-(z/d)}$$

$$d = \frac{hH}{h+H}$$

$$h = \frac{f^2 \frac{\partial \bar{u}^{ix}}{\partial z}}{\beta N^2}$$

$$\overline{w' T'^{ix}} = \frac{f}{N^2} \frac{\partial \bar{u}^{ix}}{\partial z} \left(\frac{z}{d} - \frac{z^2}{4d^2} \right) \overline{v' T'^{ix}}$$

Baroclinic instability theory used with (variation of coriolis parameter β) effect explicitly included. d is the characteristic depth of the unstable eddy, N the Brunt Vaisala frequency, α the coefficient of thermal expansion and H the density scale height. This parametrization holds good for both deep ($h \rightarrow H$) and shallow ($h \rightarrow 0$) waves. Note that $\overline{v' T'^{ix}}$ depends on stability parameter N , just as in Saltzman-Vernekar equations it depends on $(\partial \theta_0 / \partial p)$ through K .

Sasamori

$$\overline{v' T'^x} = -K_y \frac{\partial \bar{T}^x}{\partial y}$$

$$K_v = -\frac{0.067 \alpha_3 R \partial \bar{T}}{f \cdot l_e \partial y}$$

$$l_e = \frac{f}{[R \sigma_{1/2} (R/C_p) + 1]^{1/2}}$$

Geostrophic turbulence theory of Charney (1971) which shows that the large scale turbulence below a critical scale of Rossby radius of deformation, if properly scaled in the vertical, is analogous to the two-dimensional turbulence, was utilised. l_e is the baroclinically unstable excitation wavelength.

Lorenz

$$\overline{v' \dot{T}'^x} = -K \frac{\partial \bar{T}^x}{\partial y} + r$$

After studying real data extensively Lorenz (1979) suggested this type of formulae where r is a random parameter.

Austausch approach to be correct locally. It was observationally found by many workers that such parametrizations are not good enough over certain regions (Shaw 1966; Srivatsangam 1978; and Lorenz 1979). Nevertheless these parametrizations are based on physical theoretical knowledge and can be considered as definite advances in a problem which is no less difficult than the problem of turbulence.

It must be noted that we have given only a few typical parametrization schemes. Other schemes which are equally important are not given here for the sake of brevity. With this brief digression we now return to the zonally averaged two-dimensional models. A typical model is that of Saltzman and Vernekar (1971).

Zonally-averaged two-dimensional steady state model of Saltzman and Vernekar: Saltzman and Vernekar (1971) start with the primitive equations in the p system given symbolically as

$$d\Phi_i/dt = X_i; \quad i = 1, 4,$$

$$\Phi_1 = u, \quad \Phi_2 = v, \quad \Phi_3 = \theta, \quad \Phi_4 = \varepsilon,$$

$$\partial\phi/\partial p = -\frac{R\Phi_3}{P} \left(\frac{P}{P_s} \right)^\kappa; \quad (\text{Here } \phi \text{ is geopotential and } \kappa = R/C_p)$$

$$\frac{1}{a \cos \phi} \frac{\partial \Phi_1}{\partial \phi} + \frac{1}{a \cos \phi} \frac{\partial}{\partial \phi} \Phi_2 \cos \phi + \frac{\partial \omega}{\partial p} = 0.$$

From this set they get another set describing the zonally symmetric time averaged state. Steady state equations of such a set were the starting point.

The parametrization of the radiative and small scale turbulent fluxes by Saltzman and Vernekar can be compared to that of Sellers. However, their parametrizations of large scale fluxes are different and are given in table 6. They solved the equations just as Sellers did. Though the vertical structure is assumed, this model is considered as the two-dimensional model; because the atmosphere and ocean are considered separately and the model can be viewed as a two-layer system model. They finally arrive at an equation governing the vertically integrated temperature of the atmosphere. This is a disguised diffusion equation (having advection clubbed with forcing) given by

$$\frac{d}{\cos \phi \partial \phi} K_\times \cos \phi \frac{\partial \bar{\theta}^{xz}}{\partial \phi} - \lambda(\phi) \bar{\theta}^{xz} = -F_0(\phi). \quad (29)$$

Here $K_\times$ is the nonconstant Austausch coefficient, $\lambda(\phi)$ involves atmospheric radiative parameters and $F_0(\phi)$ is the forcing due to radiative fluxes and fluxes due to zonally averaged and zonally asymmetric motions. In (29), $K_\times$ is not a constant but a function of surface winds and temperature. For surface parameters they arrive at another system that gives u_{0S} , v_{0S} and θ_{0S} . To arrive at the surface parameters the authors utilised the absolute angular momentum balance due to frictional forces and large scale momentum flux convergences.

Saltzman and Vernekar solved such a system iteratively within about 100 iterations. The amount of success of such a method can be seen from figure 9. The authors applied this model to predict the temperature of the earth atmospheric system due to realistic changes in solar radiation to test the theory of Milankowich (1930) and found only little variation. Including cloud albedo feedback later they found the system similar but more stable than the Budyko-Sellers system.

With a diffuse cloudiness feedback mechanism, Temkin and Snell (1976) worked with a similar zonally symmetric model utilizing Austausch approach for eddy flux terms. The response of this model for the standard test runs, doubling of CO_2 and reduction of solar constant by 1%, was found to be more than that of global model but lesser than that of other workers. We give a part of their results in table 7. Even in this model we see from table 7 that the diffuse cloudiness feedback makes the

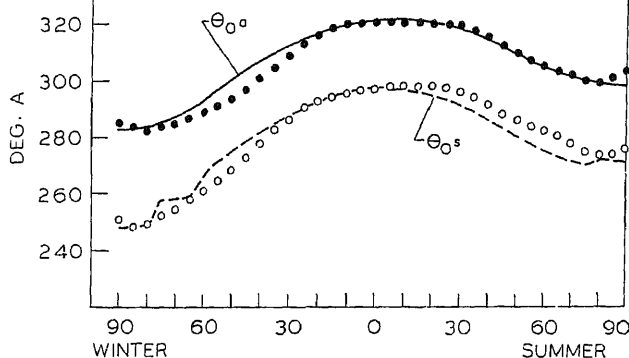


Figure 9. Mean latitudinal distribution of the surface potential temperature θ_{0s} (lower curve) and vertically averaged potential temperature θ_{0a} (upper curve). Black and white circles represent the observed values (from Saltzman and Vernekar, 1971).

Table 7. Global annual sea level temperature change (K) for given environmental variation.

Variation	Zonal model	Global model	Budyko (1969)	Sellers (1973)	Manabe and Weatherald (1975)	Rasool Schneider (1971)
2CO ₂	1.7	0.7	>4	1.32	2.36	0.8
0.99 S	-1.0	-0.7	-5	-5.00	-1.30	

system less sensitive. These results clearly bring about the importance of cloudiness feedback even over very long climatic scales.

In the above models of Sellers, Saltzman and Vernekar and Temkin and Snell, the atmosphere is explicitly considered. But the integrations could be performed with almost a time step of one month. The atmospheric equations contain gravity waves and one may wonder how such integrations were possible. In these models not only geostrophy is utilized consistently so that the gravity waves do not exist but also the diffusion number corresponding to the Austausch coefficient, $K_* \approx 3 \times 10^6 \text{ m}^2 \text{ sec}^{-1}$ and the grid spacing of about 10^6 m , allows Δt of that order. Thus these models could be utilized for palaeoclimatic integrations for thousands of years.

Adem (1963, 1964, 1965), developed similar thermodynamic models for the forecast of climate on monthly basis, and these models are being used for palaeoclimatic integrations. The basic idea of parametrizing entire dynamics was mooted by Adem as early as sixties. The diffusive model of Adem (1981) could give results similar to those of Gates (1976).

Our discussion of the energy balance models will not be complete without mentioning a highly interesting idea due to Paltridge (1975). Working with the energy balance models Paltridge found that the earth-atmospheric climate system reaches a steady state such that the global net rate of production of entropy (which is the sum over all latitude zones of the ratio of the net radiant energy input to the

effective emission temperature of the zone) is minimized. The model based on this principle shows greater stability than Budyko–Sellers original models. Very limited work is reported on these variational type of models. (North *et al* 1979).

We have seen that most of the palaeoclimatic models are highly parametrized. If the parametrizations are exactly true, it will be ideal. However, if they are not yet perfected it is important to check the predictions of these simple toy models with full three-dimensional general circulation models in which structural parameters are minimum. However, the CFL criterion prohibits palaeoclimatic calculation with these models until a super computer is developed. But these models can be utilized to test the effect of the changed boundary conditions and changed composition of the atmosphere, though they cannot be utilized to simulate very long time scale phenomena such as the onset of a “White Earth” because of the huge cost involved. We shall now deal again very briefly with a few results of few experiments with GCMs.

5. Three-dimensional GCM type models

The GCMs currently operating at various centres such as GLAS, (essentially the UCLA Arakawa-Mintz model is further developed here by Shukla, Halem, Kalney and others), GFDL, (the model developed by Smagorinsky, Manabe, Miyakoda, Bryan, Kurihara and others is used here), NCAR (a model developed by Kurihara and Washington is in operation here), etc. are of two kinds basically, depending on how the horizontal advection and diffusion terms are treated. Besides these complex models, there are many other models such as that of Webster and Lau (1977) which are ocean atmosphere models that are less complex. The two kinds of GCMs are (1) discrete grid type and (2) spectral type. In these models, there are basically five important parts: (i) the skeleton advection diffusion equation part, (ii) radiation part, (iii) boundary layer part, (iv) convective parametrization part and (v) ocean part and hydrology part. Even in these models, especially in parts (ii), (iii) and (iv) there is a large amount of parametrization involved. All these models are almost equally successful in explaining the present-day climate. As a sample of the capability of these models, in the detail that they can provide, we give in figure 10, the zonal structure of the atmosphere given by some of these models in some experiments.

The most important aspects of GCMs when compared with any other climate model are the following, regarding each of the above-said five parts in that order:

(i) The dynamics is considered explicitly by the well-known fluid mechanical equations based on conservation of momentum, mass and energy principles. The scales explicitly considered are truncated only by the hydrostatic limit. Even the finite difference approximations to these equations are constrained to conserve mass, momentum (in some models), enstrophy and energy, globally. Because of this fundamental approach the advection of energy in these models is the result of natural evolution of hydrostatic meteorological systems and so is definitely more physically satisfactory. However, because of this approach these models are most complex and computer time-consuming too. For instance, at present the

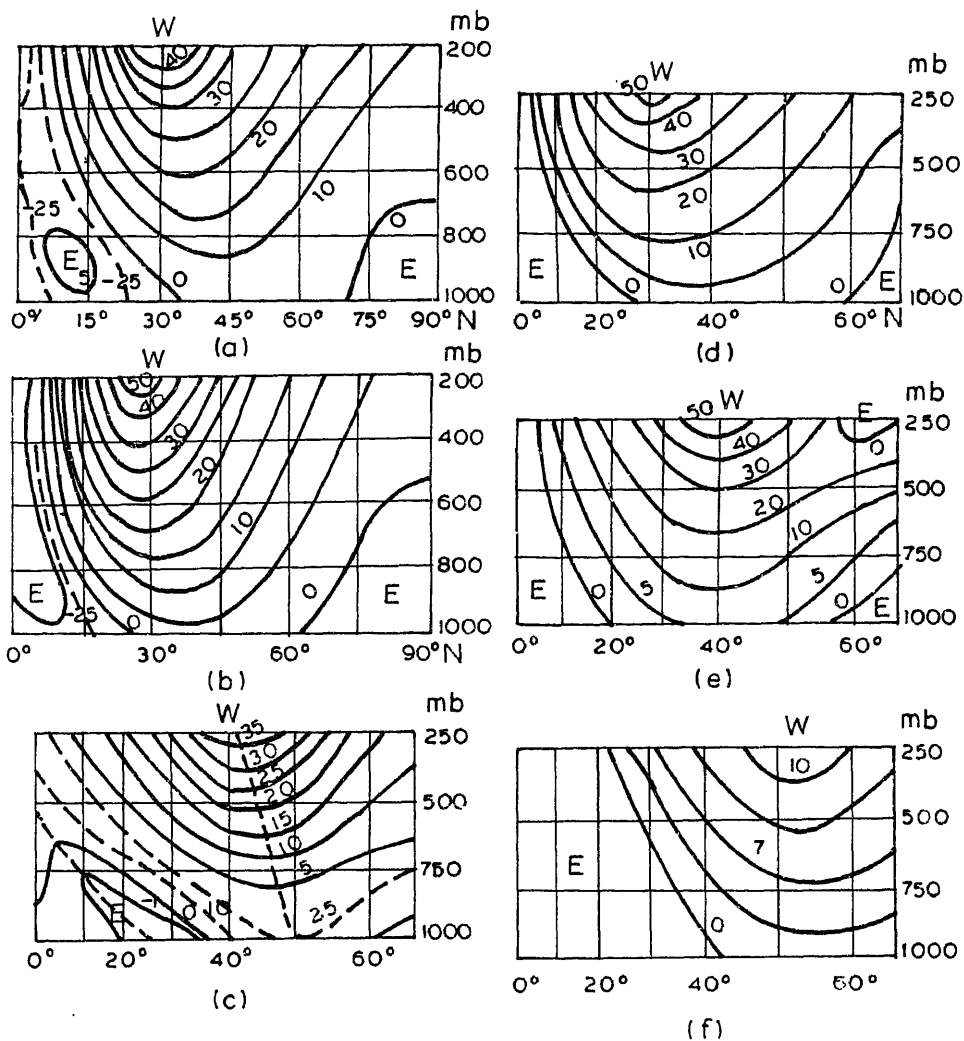


Figure 10. Distribution of the mean zonal wind in the Northern Hemisphere a) the observed mean for winter, b) mean for 30 days - Mintz, c) mean for the period from the 17th to the 39th day - Smagorinsky, d) mean for 26 days - Charney, e) mean for the period from the 8th to the 30th day - Phillips, f) mean for winter - Adem (from Gavrillin 1965).

for the richest country in the world. The time step used is of the order of about 10 min to 1 hr for 500 km resolution and about 2000 points have to be considered over the globe. If the number of variables is 6 and the vertical levels about 10 in general, the computer time required can easily be imagined.

(ii) In radiation computations, the radiative transfer of CO_2 , O_3 and H_2O is considered in great detail. The vertical resolution as already indicated is about 10 levels. CO_2 and O_3 are generally fixed in time and space. Because H_2O varies in time in these models which consider the conservation of water vapour equation also, the surface temperature humidity feedback is automatically taken care of.

Because of the high cost of radiative computations, these are performed only once in every five hours and the radiative heating is kept constant in between. In many models at present, clouds too change with time. The clouds considered are in general three types; low, medium and high and are black in the infra-red. Short wave cloud albedo is a function of the cloud type and is taken from observations. (iii) The planetary boundary layer processes in GCMs are parametrized in equally detailed manner. To take care of the transfer processes during stable, neutral and unstable conditions semi-empirical theories are utilized.

In the surface layer at the interface the fluxes are determined, in general, by the aerodynamic type formulae in many of the models. For instance, the evaporation depends on the specific humidity difference between the lower boundary and the lowest model layer. The heat flux depends on the temperature difference. The momentum flux depends on the velocity difference. The roughness length appears in all the formulae.

(iv) The precipitation is generally considered to be of two types: (a) large scale precipitation—this occurs when the grid point of the model is supersaturated. The precipitation is proportional to the difference between the specific humidity at the grid point and the saturation specific humidity. In many cases, saturation is deemed to occur when relative humidity is about 80%. The rainfall depends on such thresholds. These thresholds are generally tuned to observations. (b) Convective precipitation—This occurs mainly in tropics due to what is known as the convective instability of the second kind (CISK). Because of large humidity in tropical boundary layer, the atmosphere if lifted to a level (lifting condensation level), becomes unstable due to release of latent heat. This lifting can occur due to Ekman frictional convergence, which in turn can occur due to cyclonic system above the boundary layer. When such lifting occurs, large amount of moisture from the boundary layer goes into the free atmosphere because of instability, precipitates there and heats the atmosphere. This process can also transfer momentum. It is found that especially for tropical motions this phenomenon is extremely important. Hence in GCMs, this convection is parametrized in different ways. Arakawa-Schubert parametrization is the most complex of all such parametrizations. It computes a spectrum of clouds. Kuo's parametrization is much simpler. Finally, the convective adjustment method of GFDL model is the simplest. It adjusts the whole column such that there is no more instability and in that process, precipitates some water substance. The model climate depends very much on tropical climate. If the convection is strong, the Hadley circulation will be strong and this in turn effects the whole global circulation. One can see that this convective parametrization not only redistributes heat, moisture and momentum in the vertical but also effects the surface heat, moisture and momentum fluxes via the precipitation induced dynamics.

(v) In many of the models ocean is treated as a swamp without motion. Some ocean-coupled models have the surface layer of the ocean driven by the wind stress. In ocean-coupled models, ocean model is integrated with larger time steps than the atmosphere. Atmospheric forcing is kept constant for each such large step. In general, the depth of the snow cover, and the amount of soil moisture over continents are computed by detailed balance requirements. Snow depth increases by snowfall and decreases due to ablation. The amount of ablation is computed by heat balance requirements when snow melt conditions are fulfilled. Difference

Thus we can easily conclude that the GCMs are the most sophisticated tools for climate study but are still very costly for the very long term phenomena.

A number of experiments on the sensitivity of the climate on a time scale of a month, to various external and internal anomalies are being conducted with these models. One of the most important factors of influence on this scale is the SST according to some observations e.g. Bjerknes (1966), Rowntree (1972, 1976), Namias (1964). A number of experiments were conducted with these models, to understand the effect of SST anomaly (SSTA) on the climate on this time scale. The main difficulty is the fact that the SST anomalies (SSTA) are of the order of 1°C and their effect in general is submerged in the noise of these GCMs. However when experiments were performed with stronger forcings ($\text{SSTA} > 2^{\circ}\text{C}$ to 3°C), the models predict significant changes in the climate on this time scale e.g. Shukla (1975). To circumvent the noise problem linear models are being utilized e.g. Egger (1977), Webster (1972). The linear models have the problem of resonance for certain scales. However the overall impression that one gets after perusing the results of all these SSTA observations and experiments is that there cannot be any doubt about the effect of SSTA on climate on this time scale. These effects need not confine to the region of SSTA and in many cases they can propagate to higher latitudes by complicated teleconnecting dynamical processes such as induced propagating Rossby waves e.g. Hoskins and Karoly (1981).

A number of palaeoclimatic boundary conditions and composition sensitivity experiments are being conducted with some of these models e.g. Williams (1974), Williams *et al* (1974), Wetherald and Manabe (1975), Gates (1976). The boundary conditions for these experiments correspond to 18000 YBP, when a glacial age took place and for which there is enough evidence. These palaeoclimatically reconstructed data indicate that the ocean was several degrees colder during such an ice age (CLIMAP). The great ice sheets covered the continents upto nearly 50° latitude from the poles. The computations are generally made with such boundary conditions. We note that these types of computations do not give us any idea of how such boundary conditions resulted in the first place. Instead they tell us how the atmosphere was balanced for those boundary conditions.

A typical example of such a palaeoclimatic experiment with a simple GCM is that of Gates. Many interesting results were obtained when one looks at the details of fluxes, energetics and fields. Here we give only one result, that of the mean zonal temperature obtained for such boundary conditions, in figure 11. One can see that everywhere the ice age temperature is colder. This only reflects the boundary temperature conditions. Such results indicate that one can treat the atmosphere as a quick response system to the lower boundary (Saltzman). In the polar regions, there is an indication of slight increase in baroclinicity. All these results depend on the tropical ocean temperature which controls baroclinicity.

Manabe and Wetherald (1975, 1980) conducted CO_2 and cloud sensitivity experiments with the GCM of the GFDL, which includes the detailed hydrological cycle. Two surprising results emerge from these studies: (1) The poleward transport of moist static energy is independent of the solar radiation variations between $+2\%$ to -4% . (2) The variable clouds have insignificant effect when compared with fixed clouds on the total net radiation into the system. Both the

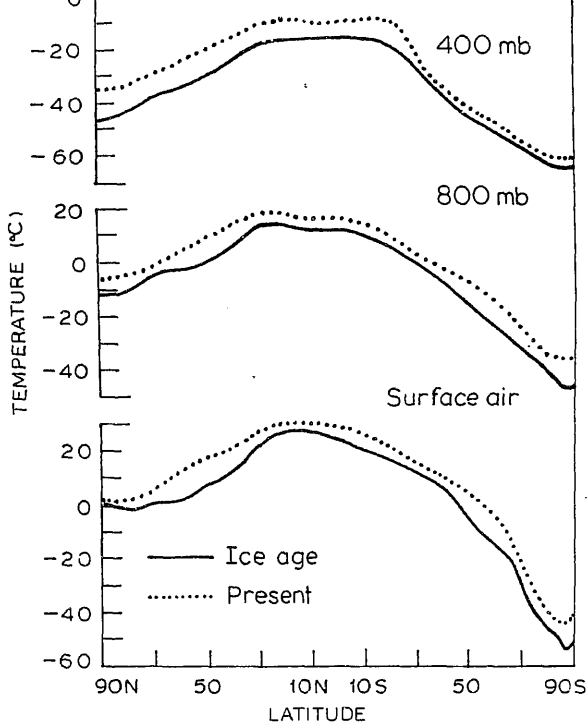


Figure 11. The zonally averaged 400 mb, 800 mb and surface air temperature simulated for the ice age and present July (from Gates, 1976).

above results are the result of compensating effects. However, these results have to be checked with other models because such results critically depend on parametrizations. For instance, Shukla and Sud (1981) report significant effect of variable cloudiness on seasonal climate.

Held and Suarez (1978) are experimenting with simple highly truncated GCMs for climatic sensitivity. Their results show that the inclusion of only one unstable wave results in reasonably realistic zonal mean fields e.g. figure 12. Inclusion of one or two more waves resulted in marginal changes such as the destruction of polar jet. These results along with other interesting results obtained by the authors indicate that the large scale eddy flux parametrization that depends on only one gravest mode that is unstable baroclinically can be realistic. No wonder these parametrizations are remarkably successful.

6. Conclusions

The calculations by a widely different spectrum of models indicate that the earth atmospheric system is quite stable for small perturbations in all parameters including solar radiation. At least a 2% decrease in solar radiation is necessary to produce a white earth. The decrease can be anywhere between 2% and 20% e.g. Warren and Schneider (1979). The hierarchy of importance in the parameters at

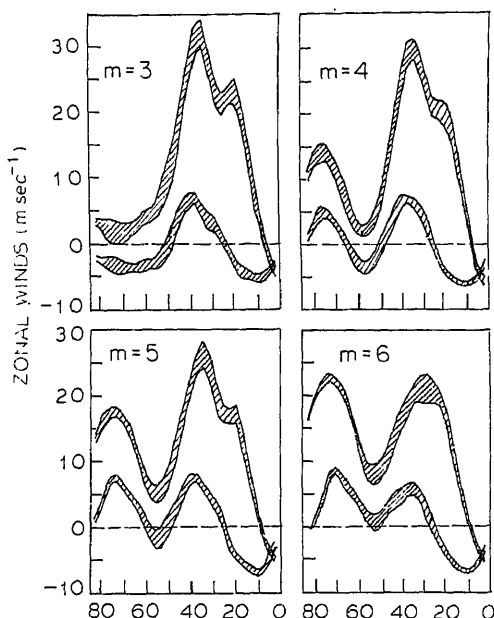


Figure 12. Upper and lower layer zonal winds produced by moist 1-wave models with $m = 3, 4, 5$ and 6 . Estimated standard deviations of 200-day averages are shown in each case (from Held and Suarez, 1978).

present seems to be (i) solar radiation (ii) CO_2 (iii) Clouds and (iv) Aerosols. However the hierarchy is not yet very clear.

Can such changes occur in the sun? If so can we predict them well? These are questions that meteorologists may have to ponder in future. Can such effective decrease in solar radiation reaching the ground occur due to spreading of dust either by volcanic eruptions or by nuclear wars is another question. Next in importance is the cloud. The future climate models must be able to parametrize the cloud feedback mechanism realistically. To arrive at correct results the physics of clouds must be continuously understood for this reason. The aerosols take the last place in importance. However, over a long time scale these may be of great importance, especially in upper troposphere and stratosphere. In introducing the effects of aerosols such as sulphates, dust, salts and organic compounds, into climatic models, their natural cycles must be studied observationally, understood and then incorporated in future. Their scattering properties, their effects on precipitation and cloud albedo must be studied observationally for a long time to come to understand their direct and indirect importance for climate changes.

There seem to be natural oscillations in the earth atmospheric system according to some models (Saltzman). According to one simple model with only three components, deep continental ice sheet mass, marine ice mass and the mean world ocean temperature, Saltzman *et al* (1984) could get 100 ky cycles, without any external forcing having such a long period. The nonlinear system behaviour is involved here. The understanding of such oscillation is as important as it is to understand a steady state. With the astronomical forcings, the structure, the variance spectrum and the phase of the quaternary glaciations were obtained.

Imbrie (1980) also obtained with a highly simplified model some support for the astronomical theory of climate on these scales. However, it is too early to come to firm conclusions. By reconstructing the geologic history of the earth atmospheric system we might be able to collect more observational evidence regarding such oscillations. Therefore this must be our endless goal. With the help of these models we can start at one epoch and predict correctly the next observed epoch in such a set of observation will be the next question.

Thus we see that two broad hypotheses of climate change exist today. The first one, the Milankowichian hypothesis says that the perturbation in the earth's orbital parameters due to astronomical causes, changes the zonal distribution of the incoming solar radiation though not its total global magnitude. Warm summers and mild winters over the critical ice latitudes result in deglaciation and vice versa. The periods of such perturbations are in the same range as the ice ages and so the hypothesis is plausible. The second one, the internal oscillation hypothesis, probes the idea that the system oscillates due to internal feedbacks. The time periods of such oscillations are also in the same range as the ice ages. The tussle goes on and the final judgement is not in sight yet.

Thus the overall feeling we get at this juncture is that we have a long way to go before we understand correctly significant climatic changes, though we are not in the immediate danger of a deep freeze unless some large catastrophic change, on the earth or in the sun, occurs.

Appendix A

a, C	: empirical constants derived and used by Sellers
$-B$	: suffix indicating lower boundary of the ocean layer
C_m	: mean specific heat
C_{m1}, C_{m2}	: specific heats of the atmosphere and oceans resp.
$\frac{d}{dt}$	: total derivative
a_v	: albedo of single scattering radiation
ε	: water vapour mixing ratio
F_{HH}	: horizontal heat flux due to subgrid scale turbulent conduction
F_{HV}	: vertical heat flux due to small scale turbulent conduction
F_{HHx}, F_{HHy}	: x and y components of horizontal heat flux
F_{HVO}^i	: component of the vertical heat flux at the interface
$\bar{F}_{HVi}^\alpha$	: component of the vertical heat flux, average over t & x
γ	: scattering coefficient
Φ	: geopotential
ϕ	: latitude
K_v	: absorption coefficient
L_v	: radiation intensity

μ	: refractive index
ν	: frequency of radiation
O	: suffix indicating the earth-atmosphere interface
P	: pressure
P_2	: 500 m pressure
P_s	: surface pressure = 1000 m
ρ	: density
S	: suffix for surface
σ	: Stefan-Boltzman constant
T	: Temperature
t	: time
θ	: Temperature of potential temperature
$\bar{\theta}_{iE}^{xz1}$	: equatorial equilibrium temperature of the ocean, the first solution
$\bar{\theta}_{iE}^{xz2}$	: equatorial equilibrium temperature of the ocean, second solution
$\bar{\theta}_{iE}^{xz3}$	: equatorial equilibrium temperature of the ocean, third solution
$-t$	: average over time
$-tx, -tz$	: average over t and x and t and z
$-txz$	: average over t, x and z
τR	: optical depth
u, v	: velocity components towards east and north resp.
V	: velocity vector
W, ω	: vertical velocity components in z and b -systems
x, y	: co-ordinates towards east and north
z	: co-ordinate in the vertical direction
ξ	: vorticity
χ	: scattering indicatrix

Appendix B

Each climate model is like a system in which advective heat transfer is a subsystem. If two or more than two types of parametrizations for one subsystem result in the same major output of the total system (such as white earth for about 1.6% decrease in solar radiation) we do not really know whether we should feel happy or disturbed. Similar results abound in meteorology. For instance, different parametrizations for surface fluxes give almost similar results for the mean state. Similarly differing cumulus parametrization do not give comparatively different results for the steady state. Such results seem to mean at least one of the following:

- (i) The subsystem is unimportant. We may rule this out because we do not include it unless we know it is important.
- (ii) The subsystem models, though they appear to be different, are not really different.
- (iii) The subsystem models are really different, but the final output is independent of them, because of some deeper reason or because of tuning.
- (iv) Though the final system output is the same, the actual history by which such a result is attained, may be quite different because the subsystem models are

different. Not only that. Depending on the level of sophistication of the questions we ask not only the models may differ from each other in their answers, but also they may differ widely from the real nature, though they are all tuned to give the right answer for a particular question.

(v) The total system, in other words, the master model, is simply not suited to introduce the variations in the parametrizations of the subsystem. For instance, Cumulus parametrization is concerned with the vertical redistribution of heat and momentum. If the master model is not well resolved in the vertical, it cannot bring out the differences in the parametrization schemes, in the final model output.

Budyko parametrization corresponds to Rayleigh friction whereas Sellers parametrization is like the Navier–Stokes viscous effect. For smooth temperature profiles they are identical qualitatively if the diffusion coefficient is inversely proportional to the square of the wave number. For wave number one, the difference can only be in the actual values taken. For all wave numbers the Budyko time constant

$$\left(\beta / C_p \int_0^H \rho \, dz \right)$$

is approximately 3 days. This corresponds to a scale of 500 km for Sellers diffusion ($K_H = 10^6 \text{ m}^2 \text{ sec}^{-1}$). In the symmetric problem, North showed that wave number one $P_2(x)$ is the dominant one for the temperature most probably because of the net radiational forcing. On this scale there will be a large difference between, the time constants of Budyko and Sellers, the later one being much larger. Therefore, the transport mechanism is much weaker in the case of Sellers. We expect thus, much faster growth of ice and the onset of white earth in the case of Sellers model. In the case of Saltzman's model the temperature is a smooth function of latitude. This only means that much faster heat transport and so Saltzman's model is nearer to Budyko model. However, the sensible heat flux depends on baroclinicity in the Saltzman's model. In effect this might be looked at as a heat transport mechanism in which K_H depends on temperature gradient very strongly. To the best of author's knowledge, the above aspects were seldom clarified in the literature.

This is only an attempt to compare certain aspects of these models. Evidently, it is a difficult problem to be further examined. From this point of view it appears to be more advantageous to build the subsystems within the framework of an already existing total system (instead of building ever new total systems) if it can be considered reasonably realistic.

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A review of stochastic dynamic method of weather prediction

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Abstract. The stochastic dynamic method of weather prediction (SDP) has been suggested recently for better understanding of the numerical weather prediction. The SDP is described using a simple one-dimensional advection equation. The salient features of the method, its scope and limitations, are discussed.

Keywords. Stochastic dynamic prediction; weather prediction.

1. Introduction

The deterministic method of weather prediction (usually referred as NWP models) requires a knowledge of the initial state of the atmosphere. Time evolution of this state, governed by the dynamical laws, is essentially the aim of these models. In the absence of any contrary evidence the dynamical equations of fluid mechanics are extended to atmospheric systems with apparent modification and parametrization of certain physical features whose explicit inclusion in the dynamical laws is neither possible for want of adequate data nor desirable for want of economy of computational time (e.g. sub-grid scale processes). The dynamical laws are of highly nonlinear nature. Even if we assume that dynamical laws are known perfectly, the knowledge of the initial atmospheric state puts considerable constraints on the weather prediction, because of a significant uncertainty involved in prescribing the atmospheric state uniquely. To start with, the observations of meteorological variables are available at some discrete points which contain some degree of systematic and random errors. Subjective and objective methods of analysis are used to derive the initial state of the atmosphere at regularly spaced grid points from the discrete observations. In the subjective methods, given the same observations, different analysts may arrive at somewhat different values at the grid points. Similarly when different objective methods are utilized to analyze the same data, differences among the grid point values may result due to the interpolation schemes or weighting functions. The differences among the analyses depend to a large extent on the density of the observations available for analyses. Experiments have been reported by several workers for the estimation of the differences among different objective analysis schemes (contours and winds) as applied over mid latitude regions (Kruger 1969, Otto-Bliesner *et al* 1977, Gustafsson 1981, etc.). Vincent and Borenstein (1980) also conducted experiments concerning variability among subjective analysis schemes over the North American mid-latitudes. Over the Indian tropical region, the objective analysis schemes used follow the successive correction methods (Sikka and Ramanathan

1970; Mukherjee and Datta 1972; Ramanathan *et al* 1973). The variability that exists in the tropical wind analyses was discussed by Sikka and Joshi (1983).

Since each analysis (subjective or objective) is consistent with the corresponding set of observed data, there is no *a priori* method available to say that a particular state is the 'true' initial state of the atmosphere. The non-linear dynamics when applied to prediction problem with two nearly similar initial states may lead to two different states as integration in time advances. It can be seen in some typical nonlinear problems that (Lorenz 1976; 1982) if there is some uncertainty about the initial state, then in finite time this uncertainty may blow upto such a large magnitude that the prediction to satisfactory accuracy becomes impossible. Thus besides the limitation of forecast model, the variability among the so-called imperfect initial state may also impact on the perfectness of the forecast.

The stochastic dynamic method of weather prediction (Epstein 1969a) explicitly considers this inherent limitation. Instead of a single initial state we consider an infinite ensemble of initial states where each of the analyzed state is a member of this ensemble. The time evolution of this ensemble in phase space consistent with dynamical laws then depicts the real representation of the weather prediction. The ensemble is characterized by a probability density function in phase space. The probabilistic initial state so obtained makes the forecast also probabilistic in nature hence the name 'stochastic dynamic' given by Epstein. In this scheme the statistics are used only in describing the uncertainty in the initial state and the evolution of the atmospheric state is governed by deterministic physical laws.

Our objective in this study is to review the stochastic-dynamical weather prediction approach with its possible application to the tropics. Section 2 discusses the basic equations and the methodology of the stochastic-dynamic prediction (SDP). In §3 we illustrate the stochastic dynamic prediction method by using a simple example of one dimensional advection equation. Section 4 briefly summarizes the work done by various workers using SDP for both the barotropic and the baroclinic atmospheres. The limitations of the SDP and the possible improvement of these through the Monte Carlo methods are discussed in §§5 and 6 respectively. The conclusions are summarized in section 7.

2. Methodology of stochastic dynamic prediction

2.1 Stochastic dynamic equations

Suppose N parameters $X_1, X_2, \dots, X_n$ are needed to describe the atmospheric state. These parameters may be the coefficients of the orthogonal functions in the spectral representation of concerned meteorological field or may be the values of meteorological field at different grid points. Thus the atmospheric state can be represented as a point in the N -dimensional phase space. Let us represent this point by the tip of the vector $\mathbf{X}$ in phase space. The ensemble of initial states will consist of a cloud of points in the phase space. The probability density of the ensemble can be represented as $\phi(\mathbf{X}, t_0)$. On the basis of the total probability being always one, the following continuity equation could be easily obtained (Gleeson 1968, 1970):

$$\frac{\partial \phi}{\partial t} + \sum_{i=1}^N \frac{\partial}{\partial X_i} (X_i \phi) = 0. \quad (1)$$

The solution of (1) is physically not possible for atmospheric system and hence Epstein suggested the following procedure by which one may make use of (1) in prediction problems.

The full probability function contains much more details than of usual interest. Therefore, instead of full function one can study the various moments of this function. The expectation value of a stochastic variable is defined as:

$$E[f(\mathbf{X})] = \int f(\mathbf{X}) \phi(\mathbf{X}, t) d\mathbf{X}, \quad (2)$$

where integration is over full phase space. The first and second moments of the distribution are defined as:

$$\mu_i(t) = \int X_i \phi(\mathbf{X}, t) d\mathbf{X} \quad (3)$$

$$\rho_{ij}(t) = \int X_i X_j \phi(\mathbf{X}, t) d\mathbf{X}. \quad (4)$$

Instead of the second moment expressed by (4), one is generally more concerned with:

$$\sigma_{ij} = \rho_{ij} - \mu_i \mu_j = E[(X_i - \mu_i)(X_j - \mu_j)] \quad (5)$$

which are variances ($i = j$) and the covariances ($i \neq j$). Similarly other moments about the mean, viz. τ_{ijk} (third), λ_{ijkl} (fourth) etc. can be defined. Taking the time derivative of (2), making use of (1) one can show that:

$$\frac{d}{dt} E[f(\mathbf{X})] = E\left[\frac{df(\mathbf{X})}{dt}\right]. \quad (6)$$

Consequently:

$$\dot{\mu}_i = E(\dot{X}_i), \quad (7)$$

$$\dot{\rho}_{ij} = E(X_i \dot{X}_j + \dot{X}_i X_j), \quad (8)$$

$$\dot{\sigma}_{ij} = E(\dot{X}_i X_j + X_i \dot{X}_j) - \dot{\mu}_i \mu_j - \mu_i \dot{\mu}_j. \quad (9)$$

Also, Lorenz (1963) has shown that every forced dissipative system of which the atmosphere may be presumed to be one, is typified by prognostic equations of the form

$$\dot{X}_i = \sum_{j,k} a_{ijk} X_j X_k - \sum_j b_{ij} X_j + c_i \quad (10)$$

where

$$\sum_{i,j,k} a_{ijk} X_i X_j X_k = 0 \text{ and } \sum_{i,j} b_{ij} X_i X_j \geq 0. \quad (11)$$

Obviously the forms of a , b , c depend on the dynamics of the atmosphere used. The

represents the forcing. Using (10) Epstein derived the equations for the different moments as follows:

$$\frac{d\mu_i}{dt} = \sum_{j,k} a_{ijk}(\mu_j\mu_k + \sigma_{jk}) - \sum_j b_{ij}\mu_j + c_i \quad (12)$$

$$\begin{aligned} \frac{d\sigma_{ij}}{dt} = & \sum_{k,l} [a_{ikl}(\mu_k\sigma_{jl} + \mu_l\sigma_{jk} + \tau_{jkl}) \\ & + a_{jkl}(\mu_k\sigma_{il} + \mu_l\sigma_{ik} + \tau_{ikl})] \\ & - \sum_k (b_{ik}\sigma_{jk} + b_{jk}\sigma_{ik}), \end{aligned} \quad (13)$$

$$\frac{d\tau_{ijk}}{dt} = \dots \text{etc.} \quad (14)$$

These equations are the basic stochastic dynamic equations.

2.2 Moment closure problem

Equations (12)–(13) reveal that the prognostic equation for each moment contains a moment of next higher order. These equations do not form a closed finite set, which is required of a set of prognostic equations containing nonlinear terms. Even to predict the mean of the distribution, all the moments of the distribution must be known. Accordingly one is forced to make certain assumptions for the higher order moments to close the system. The following assumptions are in use.

2.2a *Third-moment discard*: Epstein (1969a) observed the following points

- (i) Expressing $\mu_i(t)$ as a Taylor series in time, one finds that the higher order moments enter only in the terms containing higher powers of $(t - t_0)$. Hence the higher order moments may not be important for short range prediction.
- (ii) For many operational purposes the first two moments are generally required. Further a knowledge of the first two moments may be a reasonable limit as to what one can expect to derive with confidence, from the observational data.

On the basis of his earlier work, Epstein (1969b) suggested that we can neglect the third moments without sacrificing too much. More specifically he assumed:

$$\sum_{k,l} (a_{jkl}\tau_{ikl} + a_{ikl}\tau_{jkl}) = 0. \quad (15)$$

With this assumption (13) reduces to:

$$\begin{aligned} \dot{\sigma}_{ij} = & \sum_{k,l} [a_{ikl}(\mu_k\sigma_{jl} + \mu_l\sigma_{jk}) \\ & + a_{jkl}(\mu_k\sigma_{il} + \mu_l\sigma_{ik})] \\ & - \sum_k (b_{ik}\sigma_{jk} + b_{jk}\sigma_{ik}) = 0 \end{aligned} \quad (16)$$

2.20 *Quasi-normal eddy damped closure.* Fleming (1971a,b) examined in detail the effects of third moment discard. He did not agree with the assumption that the third moments, which are initially small or zero would not significantly alter the forecast of the first moment until the former becomes large. He suggested that the sum effect of many small third moments may be significant and also the third moments directly affect the second moments. In order to overcome this difficulty Fleming retained the third moments equation and closed the set of equations by making assumptions analogous to the quasi-normal theory used in turbulence studies (Orszag 1970). Assuming initial values are multivariate normally distributed, one finds:

$$\lambda_{klpq} = \sigma_{kl}\sigma_{pq} + \sigma_{kp}\sigma_{lq} + \sigma_{kq}\sigma_{lp} . \quad (17)$$

In a multivariate normal distribution, the third moments are identically zero. However in the approach suggested by Fleming, the third moment equations have been retained and normality has only been assumed for expressing the fourth order moments in terms of the second order moments. Hence the name 'quasi-normal' has been used. However there is always some problem in verifying the validity of the closure schemes. Fleming compared the results of different closure schemes with that obtained by considering a great many samples in a Monte Carlo calculations as used by Epstein (1969a); obviously the Monte Carlo estimates are treated as standard solutions. The quasi-normal closure scheme also showed a significant change from the Monte Carlo solutions after few days. Taking analogy from turbulence theories, Fleming considered the eddy-damping of third moments i.e. the prognostic equation for third moments is written in the form:

$$\dot{\tau}_{jkl} = . . . + [\text{quasinormal}] - \kappa \tau_{jkl} \quad (18)$$

where κ , the damping coefficient is a positive constant. The damping coefficient would be a function of time, energy density and other statistical quantities; κ was taken as an empirical constant by Fleming. This scheme was found to give solutions comparable to those expected from Monte Carlo estimates.

Though the retention of third moments has been necessitated by Fleming's study, most of the studies for the short range prediction until now have been made using the third moment discard, mainly because of avoiding excessive computations.

2.3 *Stochastic analysis of meteorological fields*

The SDP technique has the potential of combining the present and the past information in depicting the 'truer' atmospheric state. The forecasts based on SDP technique for a particular time give the moments of probability density function—called 'prior estimates'. These forecasts are obviously made on the basis of past data. The observation at the initial time of the atmospheric state also gives the estimates of first two moments. The forecast estimates can be revised on the basis of observed estimates through the use of Baye's theorem (Epstein 1962). Epstein

and Pitcher (1972) named this procedure as the stochastic analysis. The steps involved are as follows (Pitcher 1977):

If β is a row vector of parameters of dimensions N (implying N terms are retained in the orthogonal expansion), the atmospheric state can be written as

$$Y = \beta X, \quad (19)$$

where X is a $P \times N$ matrix of functions (implying observations are made at P points). The SD-prediction precisely gives

$$d = E(\beta) \quad (20)$$

$$S = [(\beta - d)' (\beta - d)], \quad (21)$$

where a dash represents the transpose of the matrix. These we call as the prior mean and the covariance matrix for β . The true values and their unbiased estimates are denoted by β and $\hat{\beta}$ respectively. The revised judgements, in view of the information contained in observations, called posterior means $\hat{d}$ and covariance $\hat{S}$ can be obtained with the help of Baye's theorem (Epstein 1962). Baye's theorem reads that the posterior probability distribution function is proportional to the product of the likelihood function and the prior probability distribution function. The likelihood function is a conditional probability of the occurrence of the observed data given β . The likelihood function can always be linked with the probability density of observations errors for which a normal distribution is quite befitting. However, Baye's theorem requires full knowledge of prior probability distribution and only the first two moments of them are known. The normal distribution has the unique property that it can be described by its first two moments. Thus, if only first two moments are considered, the prior distribution, which because of non-information of higher moments, could be taken as the normal distribution. Epstein and Pitcher (1972) argued that with the knowledge of only two moments, we cannot distinguish the 'prior' distribution from a normal distribution. Taking prior also as multivariate normal distribution, it is seen that (for proof see Pitcher 1977):

$$\hat{d} = (dS^{-1} + YX' \sigma^{-2}) \hat{S}, \quad (22)$$

$$\hat{S} = (S^{-1} + \sigma^{-2} XX')^{-1}, \quad (23)$$

where

$$\sigma^2 = \frac{s^2}{P-N}, \quad s^2 = (y - X\beta)' (y - X\beta). \quad (24)$$

Thus if $P \geq N$

$$\begin{aligned} \hat{d} &= \hat{S}(S^{-1}d + S_{\hat{\beta}}^{-1}\hat{\beta}) \\ S_{\hat{\beta}} &= [(\beta_0 - \hat{\beta}) (\beta_0 - \hat{\beta})'] \end{aligned} \quad (25)$$

i.e. the inverses of the covariance matrices are direct measures of the precision associated with d and β . In stochastic analyses these matrices become the appropriate weighting factors based on the stochastic weighting factor and the

particular arrangement of stations in the observing network. If no stochastic forecast is available, then the prior is considered to be vague, or non-informative and S^{-1} is taken as the zero matrix. In this case $\hat{d}$ and $\hat{S}$ reduce to simple least squares solutions.

By making simulated observations of a predefined atmosphere (Epstein and Pitcher 1972) and the real data (Pitcher 1977) it was found that the uncertainty in analyzed values is substantially less than either the uncertainty in the forecast or in the observation. Thus the stochastic analysis offers a unique way of making use of both the past and the present available information for determining the 'true' state of the atmosphere.

2.4 Procedures of calculations

The various steps involved in SDP are as follows:

- (i) The set of dynamic equation is converted in the generalized canonical form (equation (10)) through some spectral representation.
- (ii) Using one of the above mentioned closure approximations, the corresponding stochastic dynamic equations are derived.
- (iii) With some suitable numerical schemes the SDP equations are integrated. However for the integration the initial values of the first and the second moments are required. This is one of the difficult problems of SDP. Most of the calculations (Pitcher 1977) are made by using these moments obtained through least squares procedures by fitting the spectral decomposition of equation for the variables to the corresponding observed fields.

The results obtained by various workers are described in the following sections.

3. A simple illustration of SDP

From the above discussion it is obvious that in SDP the number of equations is considerably increased in comparison to deterministic prediction. One of the simplest prediction equation in meteorological problems has been the use of one dimensional advection equation (Platzman 1964). Wiin-Nielsen (1979) recently studied the asymptotic behaviour (large time) of the stochastic dynamic solutions of this equation, by taking a Gaussian function for characterizing the ensemble of initial states. Joshi (1981) compared the SD solutions as given by Wiin-Nielsen with the deterministic solutions of the advection equation for illustrating the salient features of SDP. In the following the methodology of SDP is explained using these examples.

The one-dimensional advection equation without the forcing and the dissipation terms is as follows

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} = 0, \quad (26)$$

where u is the meteorological variable of interest. For the purpose of SDP, the above equation is transformed into spectral domain as follows:

$$u(x,t) = \sum_{n=1}^{\infty} U_*(n,t) \sin nKx; \quad K = 2\pi/L, \quad (2)$$

where an interval $0 \leq x \leq L$ and the boundary conditions $u = 0$ at $x = 0$ and $x = L$ have been assumed. Using

$$\begin{aligned} U(n) &= U_*(n)/(2E_0)^{1/2} \\ T &= (2E_0)^{1/2} Kt \end{aligned} \quad (2)$$

$$2E_0 = \sum_{n=1}^{\infty} U^2(n)$$

and also retaining only two components in the expansion i.e. $X_u = U(1)$ and $Y_u = U(2)$, the prognostic equation for X_u and Y_u are found to be

$$\frac{dX_u}{dT} = \frac{1}{2} X_u Y_u, \quad (2)$$

$$\frac{dY_u}{dT} = -\frac{1}{2} X_u^2. \quad (3)$$

The variables X_u and Y_u become the parameters defining the atmospheric state and the above equations as the predictive equations. These equations have analytical solutions in terms of hyperbolic functions. The stability of the deterministic solutions of the equations (29)–(30) is very much sensitive to the initial conditions (X_{u_0}, Y_{u_0}). With different initial values of X_{u_0} and Y_{u_0} and $\sigma_{11} = \sigma_{22} = 0$, $\sigma_{12} = 0$ the different phase space trajectories have been derived. The results are shown in figure 1 which also gives the deterministic solutions. The SD solutions correspond to the exact solution of the SD equations.

Figures 1(a) and 1(c) show that the two trajectories are very nearly the same. Thus, as far as the first order moments are concerned, no new information over the deterministic ones has been obtained. In such cases the advantage of the stochastic dynamic prediction lies in the prediction of the second order moments (which have not been shown in the figure) i.e. the uncertainties in the prediction variables are also obtained. However, the stochastic trajectories deviate very much from the deterministic ones (figures 1b and 1d). The deterministic trajectories show that both the X_u and Y_u components are equally significant for most of the time (i.e. the energy is nearly equally distributed in wave number one and two) whereas the stochastic dynamic trajectory clearly shows that for most of the time Y_u component (wave number two) is predominant. Since X_u and Y_u are further used in predicting physical variables, errors in X_u and Y_u will also lead to errors in the physical variables.

Since the stochastic dynamic solutions have been obtained by considering a small spread (uncertainty) in the initial state, which in actual cases is always present, the SD solutions are expected to be more realistic than the deterministic solution. Thus from the above simple example, it is evident that in some circumstances the deterministic solutions may give misleading information after some time interval.

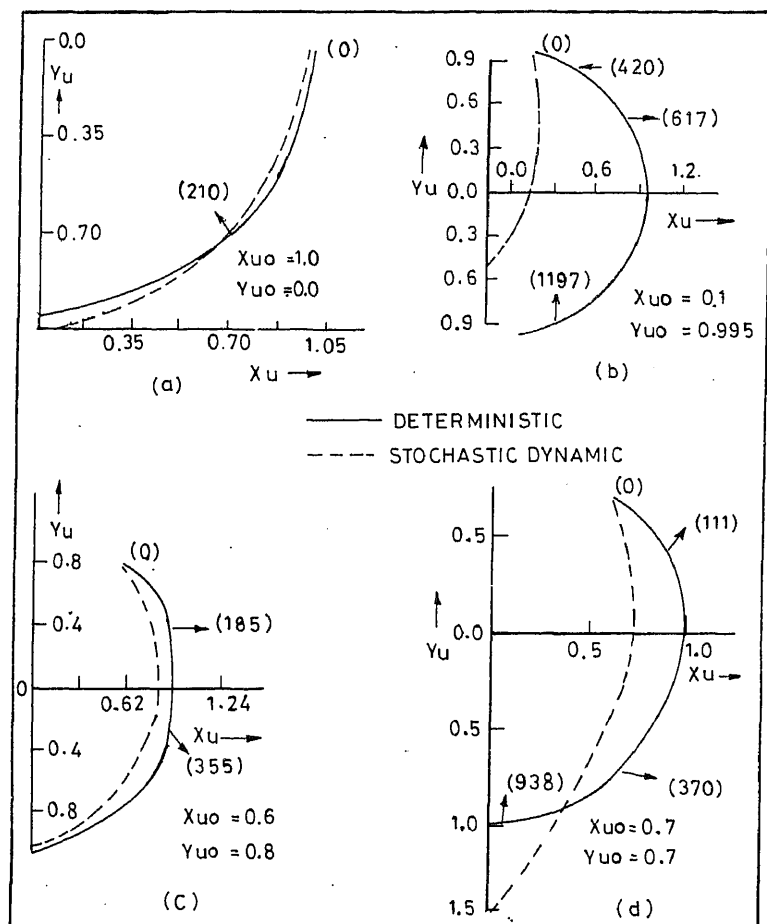


Figure 1. Deterministic and stochastic dynamic trajectories in the phase space for different initial conditions. The entries in the bracket with arrow-mark show the time in hours.

Such situations occur more prominently (Epstein 1969a) for the unstable initial conditions. Interestingly these unstable situations are of greater importance in meteorological phenomena. It should also be noticed that the time interval is crucial. For short times the differences between SDP and deterministic values will be small for a good forecast model and there would be no advantage in using SDP.

4. Review of other results with the SDP

4.1 Barotropic atmosphere

4.1a Synthetic data The first illustration of SD prediction and their utility was made by Epstein (1969a) by taking the Lorenz's minimum equations (Lorenz 1960) for the prediction and with pre-assigned values of the model parameters. This

an equation is closer to the atmospheric prediction equations with several assumptions and simplifications. However, even after a decade of initiation of SD scheme, most of the studies in SD prediction use these minimum assumptions, mainly because they are simplest in nature and their deterministic solutions are available in the analytical forms.

Using a simplified barotropic atmosphere and a spatial domain of dimension $(2\pi/l)$ over which the double Fourier expansion of vorticity field has been

$$\nabla^2\psi = A_1 \cos ly + A_2 \cos kx + 2A_3 \sin ly \sin kx \quad (31)$$

obtained the following prognostic equations for the parameters

$$\frac{dA_1}{dt} = c_1 A_2 A_3; \quad \frac{dA_2}{dt} = c_2 A_1 A_3; \quad \frac{dA_3}{dt} = c_3 A_1 A_2. \quad (32)$$

The first term in (31) represents the zonal flow and the second and the third terms represent the eddies superimposed over this. These equations are called Lorenz's equations. Solutions of these equations are elliptic functions. Epstein made the stochastic dynamical calculations (with third moment discard) for these equations with two extreme situations—one in which initial conditions are stable and the other in which they are unstable. The stable and unstable conditions are determined by relative magnitudes of different coefficients i.e. A_1 , A_2 and A_3 . For a stable condition $A_1 = 0.12$, $A_2 = 0.24$, $A_3 = 0.0$ as initial condition with $\sigma_{ij}^{-4} (3 \text{ hr})^{-1}$ and $\sigma_{ij} = 0.0i \neq j$, the results are shown in figure 2. Monte Carlo solution with a sample size of $N = 500$ is treated as standard to compare the deterministic solutions. On the scale of the graph it is not possible to distinguish between the Monte Carlo and Monte Carlo solutions. The deterministic solution is also very close to these solutions. In this example, the initial uncertainties were relatively small, i.e. they did not have severe effects on the forecasts of the parameters. In

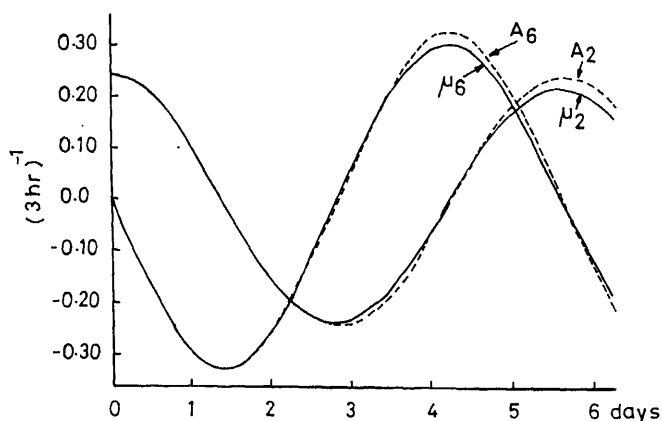


Figure 2. SDP solutions for μ_2 and μ_3 and the deterministic solutions for A_2 and A_3 for

such cases the utility of the stochastic procedure lies in the information about the uncertainties.

For an unstable initial condition $A_1 = 0.12$, $A_2 = 0.00$, $A_6 = 0.66$ and all covariances as zero and variances as earlier, the representative result for the parameter A_1 is shown in figure 3. The sign of A_1 becomes negative after three days. The SDP solution μ_1 remains positive throughout the period. The differences between the two stochastic solutions (i.e. SD and Monte Carlo) are certainly real after the third day. The difference may be attributed to the third moment discard. The deterministic solution shows departures from the Monte Carlo solution after 24 hr and is very misleading after about 3 days. Epstein also showed that the solutions for $A_2(\mu_2)$ and $A_3(\mu_3)$ tend to emphasize these differences. It should be noted here that the implication here is not this that the forecast is wrong, only that it is a poor forecast.

Assuming that for sufficiently small times the deviations from the deterministic results are small, Laurman (1978) linearized the SDP equations using small perturbation approximations. This approximation is applied to both the cases of Epstein (i.e. applying to Lorenz's minimum equations) and it is seen that a substantial reduction in computational time is possible. However, as described earlier, for smaller times SDP hardly gives any new information over deterministic formulation.

4.1b Real data

An attempt for the real atmospheric data was first made by Pitcher (1977). For predicting the height of 500 mb level he used a simple barotropic vorticity equation

$$\frac{\partial}{\partial t} \left(\zeta + \frac{r^2}{a_0^2} \psi \right) = \mathbf{V} \cdot \nabla (\zeta + f + ch), \quad (33)$$

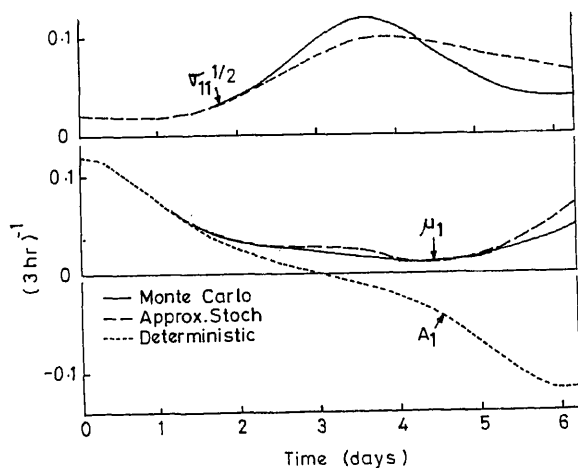


Figure 3. SDP solutions for μ_1 and $\sigma_{11}^{1/2}$ and the deterministic solution for A_1 for

where ψ = stream function

ζ = vertical component of relative vorticity

f = coriolis parameter

$\mathbf{V}$ = horizontal wind

h = the smoothed terrain height

c = parameter measuring the effectiveness of the orographically induced vertical velocity

r = parameter to reduce retrogression of long waves

a_0 = earth's radius

The vorticity equation is transformed into spectral form by expanding ψ and h in terms of spherical harmonics

$$\psi(\phi, \theta, t) = \sum_{m=-J}^J \sum_{n=|m|}^{|m|+J} a_0^2 \psi_n^m(t) Y_n^m(\phi, \theta) \quad (34)$$

and similarly for $h(\phi, \theta)$, where ψ_n^m is an expansion coefficient, θ the longitude, ϕ the latitude, Y_n^m are spherical harmonics, a is the radius of the earth and J is the wave number at which expansion has been truncated. Restricting the studies to the northern hemisphere, Pitcher has taken the period of 1–4 December 1969. Geopotential heights were available from approximately 450 radiosonde stations. Less than 10% of the data was located south of 20°N. The method of least squares was used to extract the moments directly from observations. In order to stabilize the least squares fit in the tropical region, it was found necessary to insert artificial data there. Approximately twenty points were added and assigned climatological values appropriate for the month of December. The expansion was terminated at $J = 10$. This limited the spectral coefficients to 105. Thus a total of 5670 numbers, the parameters and their variance-covariance information, formed the initial conditions for a stochastic prediction. The height fields were presented as the least squares estimated, the stochastically forecasted and the stochastically analyzed (as described in §2.3) fields for all the four days. It was found that the uncertainty in the stochastically analyzed height fields was much less than the observed height field. The standard error of the forecast (second moment) was greater than the corresponding uncertainty associated with the observed expected state. Qualitatively, this would indicate that the forecast expected state would be given less weight than the least-squares estimations in arriving at a stochastically analyzed expected state. They also found that at some points the uncertainty in the forecasted field was less than the uncertainty in the initial field. This result is possible and is a manifestation of the detailed influence of the correlations between the spectral amplitudes in the model. The uncertainty of each model parameter is not necessarily a monotonically increasing function of time throughout the forecast period. The examination of the standard error field and the observed data field after few days also revealed that some times the subsequent analysis placed more credence in the forecast expected state. They concluded that the validity of the analyzed uncertain information depends crucially upon the extent to which the

Fleming (1971a,b) made the SD calculations using a baroclinic model by considering a simple two-layer model having 28 degrees of freedom.

The two-level model used was taken from Lorenz (1965). In this model the basic equations are the quasi-geostrophic vorticity equation and the thermal equation that could be written with x, y as horizontal coordinates and pressure p as the vertical coordinate. The equations of the model in spectral form are:

$$\dot{\psi}_i = \sum_{\substack{j,k=1 \\ k>j}}^{\infty} [a_i^{-2}(a_j^2 - a_k^2)c_{ijk}(\psi_j\psi_k + \theta_j\theta_k)] - \kappa'(\psi_i - \theta_i) \quad (35)$$

$$\begin{aligned} \dot{\theta}_i = & \sum_{\substack{j,k=1 \\ k>j}}^{\infty} \left[c_{ijk} \frac{\sigma_0(a_j^2 - a_k^2) + 1}{\sigma_0 a_i^2 + 1} \theta_j\psi_k \right. \\ & + c_{ijk} \frac{\sigma_0(a_j^2 - a_k^2) - 1}{\sigma_0 a_i^2 + 1} \psi_j\theta_k \left. \right] + \frac{\kappa' \sigma_0 a_i^2}{\sigma_0 a_i^2 + 1} \psi_i \\ & - \frac{\sigma_i a_i^2 (\kappa' + 2\kappa'') + h}{\sigma_0 a_i^2 + 1} \theta_i + \frac{h}{\sigma_0 a_i^2 + 1} \theta_i^* \end{aligned} \quad (36)$$

where ψ and θ are the non-dimensional dependent variables representing the mean wind and mean potential temperature and C_{ijk} are constants depending upon the choice of the orthogonal functions used to represent the space domain. Qualities had been made dimensionless by appropriate scale factors including a length L and a time f^{-1} (f is the coriolis parameter assumed constant here). A frictional drag with coefficient $2\kappa'$ is introduced at the lower surface, proportional to the velocity in the lower layer. A frictional drag at the boundary separating the two layers with coefficient κ'' is introduced proportional to the difference between the temperature of the lower layer and a preassigned temperature field θ^* , characterized by a coefficient $2h$. A coefficient for heat exchange between the layers cancels out when σ (a measure of static stability) is considered constant. The choice of the functions depends upon the geometry of the space domain in which the model equations are applied. In the present case the functions were given over an infinite channel of width πL having walls at the surfaces $y = 0$ and $y = \pi L$ where the flow in the channel is periodic along the length with a specified fundamental wavelength. The model had 28 variables (14 orthogonal functions for ψ and θ).

Fleming studied the stochastic dynamic effects by considering the energetics of the atmosphere. For the deterministic model referred above the kinetic energy K and the available potential energy (hereafter referred to as APE) A could be derived as:

$$K = \frac{S}{2} \sum_i a_i^2 (\psi_i^2 + \theta_i^2), \quad (37)$$

$$A = \frac{s}{2\sigma} \sum \theta_i^2. \quad (38)$$

However if we apply the stochastic dynamic equations to the above model, the energies are given by

$$K = \sum_i \frac{a_i^2}{2} [\psi_i^2 + \theta_i^2 + \text{variance}(\psi_i) + \text{variance}(\theta_i)], \quad (39)$$

$$A = \sum_i \frac{1}{2\sigma} [\theta_i^2 + \text{variance}(\theta_i)], \quad (40)$$

Thus it appears that the stochastic dynamic equations provide for a partitioning of the energy. There is a 'certain' energy associated with the expected components of the ensemble and 'uncertain' energy associated with the variances. The initial uncertain energy may represent, in part, some of the actual energy of the atmosphere; that is, energy is lost by smoothing the data or turbulent energy, or any small-scale energy unresolved by the deterministic initial conditions. Thus the quantity $\sum_i \frac{1}{2} a_i^2 (\psi_i^2 + \theta_i^2)$ is the certain kinetic energy; $\sum_i \frac{1}{2} a_i^2 [\text{variance}(\psi_i) + \text{variance}(\theta_i)]$ is the uncertain kinetic energy, $\sum \theta_i^2 / 2\sigma$ is the certain APE; and $\sum \text{Variance}(\theta) / 2\sigma$ is the uncertain APE. It is to be mentioned here that ψ_i and θ_i in (37) and (38) are deterministic stream function and potential temperature whereas they refer as ensemble mean values in (39) and (40).

Fleming derived the expressions for different energy parameters viz. the zonal APE, the eddy APE, the zonal K.E. and eddy K.E. with both 'certain' and uncertain components as well as generation, dissipation of different kinds of energies and also the various conversions from one kind of energy to the other. His results showed that the third moments do not affect energy conservations but they affect energy conversions between uncertain components. The change in 'uncertain components' eventually lead to altering the forecast of the mean. Considering various stable and unstable initial conditions and their certain and uncertain components of energies, Fleming found that the third moment discard stochastic dynamic forecast of the mean of the stream function and potential temperatures are different from more accurate Monte Carlo calculations. As an example the stream function coefficient corresponding to wave number 4 and its prediction is shown in figure 4. Here the SDP corresponds to third moment discard approximation. The deviations between Monte Carlo and the SDP for this component start becoming important after 13 days. Figure 5 shows the results for the same component using eddy-damped quasnormal scheme described in §2.2. The calculations of SDP now

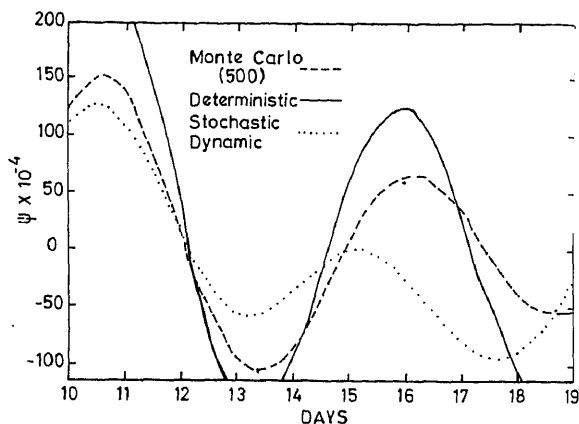


Figure 4. Comparison of SDP (third moment discard) with the Monte Carlo calculations in a baroclinic atmosphere for a stream function component corresponding to wave number 4 (after Fleming 1971a)

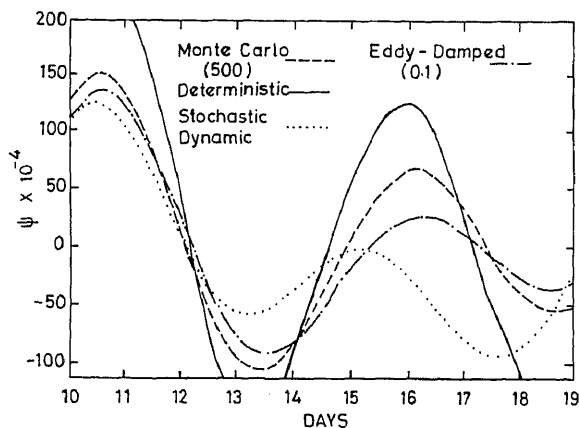


Figure 5. Same as in figure 4 but with eddy damped quasi-normal closure scheme in SDP (after Fleming 1971a)

5. Limitations of SD predictions

Except for the study of low order systems, the approach outlined above has the following rather severe limitations if application to current NWP models is contemplated.

(i) Foremost among these is the excessive computation time required over a conventional forecast for a system with reasonably large number of parameters. In

- (ii) The above formulation is limited to quadratically nonlinear systems and while this appears to be the dominant nonlinearity in atmospheric prediction models, a reformulation taking into account higher order nonlinearities, as are present in certain local physical processes would pose some severe mathematical difficulties and an extra computational burden.
- (iii) SDP models ought to be spectral rather than grid point to reduce the total number of covariance quantities.
- (iv) The SDP equations require some closure approximation. Various closure approximations made to solve the system may not be valid for some situations.

6. Monte Carlo forecasting

Many of the practical difficulties associated with the solution of the SD equations may be substantially alleviated, if not eliminated entirely, by utilising a Monte Carlo approach. Leith (1974, 1980) advocated such an alternative and investigated the theoretical skill of the Monte Carlo procedure by application to a two-dimensional turbulence model. Instead of predicting the moments of the forecast ensemble directly, one draws a sample of m members from the initial ensemble and integrates each member forward in time with the governing dynamical equation. Later, the m members continue to serve as a sample drawn from the evolved ensemble and the ensemble moments may be estimated from averages over the sample still consisting of original m members. Of course such a calculation could be repeated with a different sample, thereby obtaining a different set of statistical estimates. The hope, however, is that we may choose, within the limits of computer resources, a sample size m sufficiently large to produce stable statistics. Thus we have

$$\mu_i = \bar{X}_i, \quad \sigma_{ij} = \frac{m}{m-1} \overline{(X_i - \mu_i)(X_j - \mu_j)}, \quad (41)$$

where overbar denotes an average over the sample of m members.

The total computation involved is proportional to mN , but if $m \ll N$ we may capture the advantage of SDP without the penalty of excessive arithmetic.

Other limitations of the stochastic-dynamic system of equations discussed above are not present with the Monte Carlo procedure. However, in the procedure advocated the problem of moment closure is replaced by the sampling problem in deciding what value to choose for m , which is done essentially by experimentation.

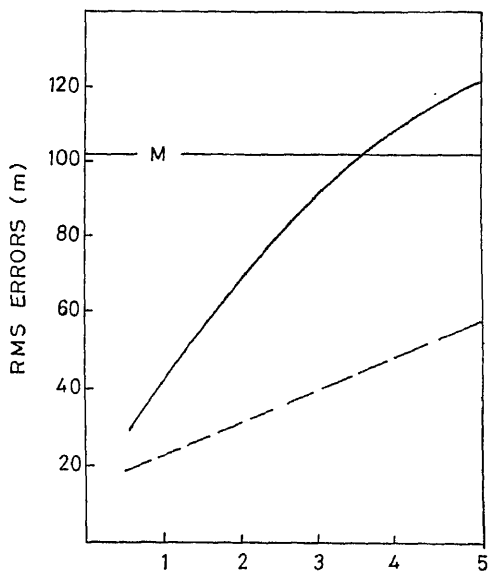
Pitcher (1979) investigated the design of the Monte Carlo forecasting technique by taking a case study of generating an initial sample. He considered a relatively straightforward manner of generating an initial sample that possess a statistical structure reflecting inhomogeneity and spatial correlations among analysis errors. Assuming existence of an unbiased initial state vector $\mathbf{X}$, the initial sample was generated by choosing $\mathbf{X}_i = \mathbf{X} + \boldsymbol{\varepsilon}_i$ ($i = 1, 2, \dots, m$); where $\boldsymbol{\varepsilon}_i$ were chosen at random. Choosing spherical harmonics as basis functions the error field $\boldsymbol{\varepsilon}_i$ were represented as

$$\varepsilon = \sum_{m,n} (A_n^m \cos m\lambda + B_n^m \sin m\lambda) P_n^m(\mu) \quad (43)$$

$$= \sum_{m,n} C_n^m \cos(m\lambda - \theta_n^m) P_n^m(\mu), \quad (44)$$

where $A_n^m = C_n^m \cos \theta_n^m$ and for $B > 0$ $B_n^m = C_n^m \sin \theta_n^m$. If θ_n^m and C_n^m are assigned at random, then permits the calculation of random vector field. A series of five-day Monte Carlo forecasts were made for a three-month winter period. The forecasts were for the 500 mb height field. The three-month period thus consisted of 18 pentads. The results were given by averaging over these 18 independent calculations. The dynamical component was a spectral barotropic model with triangular truncation at wave number eighteen. A choice of $m = 10$ was found to give relatively stable statistical estimates of second moment quantities. The results are shown in figure 6. Figure 6 gives the RMS error of sample mean and the sample standard deviation for 18 Monte Carlo forecasts. The difference between the respective curves is indicative of the external error growth for this model, as SDP predicts only the internal error growth. As pointed out by Pitcher the model used here was a simple one, however dramatic improvements are not expected even if we use existing operational forecasting models. After a few days the error growth, because of model imperfections, vitiated the advantage obtained by SDP.

It is a common experience of meteorologists that atmospheric flow patterns under certain situations become rather stable from one day to the other



(quite probably) over a limited region and time, the predictability of local synoptic features gets enhanced under special atmospheric conditions. For example, the blocking situation which tends to persist over a region for a few weeks is easily distinguishable from the situations in which short-waves follow each other in quick succession. The former situation becomes a stable flow pattern and is thus *a priori* more predictable than the less predictable situation characterised by the later situation. Leith (1979) applied the Monte Carlo technique to provide an estimate of the local synoptic scale predictability. He conducted a numerical experiment in which random perturbations for a sample of $m = 100$ perturbations, were imposed on the vorticity field and the effect of these perturbations in the forecast through a non-divergent spectral barotropic model was investigated. He found that the error sensitivity on the forecast was higher in the region of maximum wind speed which was expected from the error advection effects.

The Monte Carlo alternative for the solutions of SD equations suggests a promising alternative and further investigation in this direction may lead to its applications to realistic forecasts. Recently another technique called Lagged Average Forecasting (LAF) has been proposed by Hoffman and Kalnay (1983) as an alternative to Monte Carlo forecasting also. The utilization of LAF in stochastic dynamic prediction may make it more feasible for practical forecasting problems.

7. Conclusions

The SDP technique is based on some sound principles and has the capability of quantifying the uncertainty in the forecast field caused by the uncertainty in the initial field. However after nearly 15 years since the technique was first suggested, it has been applied only to a single realistic case study and all other studies are based mainly on the applications to the Lorenz's minimum equations. The major factor responsible for this has been the excessive computational time. The enhanced computational time arises because of the simultaneous calculations of large number of covariance parameters. Greater effort is required, even with the simplified situations, to find ways for reducing the computational time so that it may be used for operational purposes.

Depending on the numerical methods used, the simplicity the dynamical models has been put into, the incorrect mathematical representations of certain physical processes, the imperfect parameterisation of sub-grid scale processes, a substantial contribution to the growth in forecast error is unavoidable. This is generally referred as external growth. The SDP technique predicts the internal error growth. The quantitative estimation of uncertainty by SDP may get overmasked by the external error growth also. Hence any attempt in making use of SDP for operational use will have to assess the external growth also.

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SUBJECT INDEX

- Absorption**
 Computation of synthetic seismograms for plane layered earth models including absorption and dispersion phenomena 321
- Accretion**
 Effect of waves in the redistribution of sediments along the Konkan coast 237
- Air-sea exchange**
 Estimation of evaporation rates over the Arabian Sea from satellite data 417
- Alkali granites**
 Radioelement geochemistry of alkali granites of the Kerala region, south-west India 103
- Alpha-recoil**
 Preferential solution of ^{234}U from recoil tracks and $^{234}\text{U}/^{238}\text{U}$ radioactive disequilibrium in natural waters 245
- Ambalavayal**
 Geochemistry of coexisting hornblende and biotite from the Ambalavayal granite, Kerala 91
- Amplitude and phase gradients**
 Wind gradients at meteor heights over mid-latitude stations 427
- Anomalous enhancement**
 Seasonal and solar cycle variations of nighttime anomalous enhancements in total electron content 409
- Anorthosite**
 The petrology and geochemistry of massif anorthosites and gabbros of the Bavali fault zone, North Kerala 117
- Base metal**
 Base metal exploration by gravity surveys in a part of the Cuddapah basin (India) — a case study 131
- Basic magnetism**
 Lineament analysis of Cannanore district and their implication on the tectonic evolution of the area 363
- Bavali fault**
 The petrology and geochemistry of massif anorthosites and gabbros of the Bavali fault zone, North Kerala 117
- Bengal basin**
 Evolution of the landscapes in the Calcutta-Sundarban coastal areas of South Bengal, India 143
- Bhaskara II**
 Estimation of evaporation rates over the Arabian Sea from satellite data 417
- Biotite**
 Geochemistry of coexisting hornblende and biotite from the Ambalavayal granite, Kerala 91
- Boiling**
 Physico-chemical conditions of ore deposition in Malankhand copper sulphide deposit 209
- Breakdown pressure**
In situ stress measurements using hydraulic fracturing method 311
- Capillary hysteresis model**
 Numerical hysteresis model for intermittent studies in unsaturated soils 153
- Chemical discriminant**
 Palaeo-tectonic environment of the Precambrian basaltic mafic dykes on the application of chemical discriminants 351
- China clay**
 Origin and depositional environment of the china clay deposits of South Kerala 285
- Climate model**
 An overview of climate models 447
- Compton stripping coefficients**
 Natural radioactivity distribution studies in Trivandrum district, Kerala, India 397
- Computer programs**
 Computation of synthetic seismograms for plane layered earth models including absorption and dispersion phenomena 321
- Contour sonagrams**
 Spectral characteristics of low latitude Pc3 geomagnetic pulsations in south-east Australia 25
- Cosmic ray fluxes**
 Cosmogenic isotope and track records in meteorites and their implications to cosmic ray fluxes 183
- Cosmic rays**
 Radionuclide depth profiles in Dhajala chondrite 169
- Cosmogenic isotopes**
 Cosmogenic isotope and track records in meteorites and their applications to cosmic ray fluxes 183

- Cosmogenic nuclides
 Radionuclide depth profiles in Dhajala chondrite 169
- Cuddapah basin
 Base metal exploration by gravity surveys in a part of the Cuddapah basin (India) — a case study 131
- Delta
 Evolution of the landscapes in the Calcutta-Sundarban coastal areas of South Bengal, India 143
- Deltalobe
 Evolution of the landscapes in the Calcutta-Sundarban coastal areas of South Bengal, India 143
- Dhajala
 Radionuclide depth profiles in Dhajala chondrite 169
- Differentiation
 The petrology and geochemistry of massif anorthosites and gabbros of the Bavali fault zone, North Kerala 117
- Diogenite
 On the thermal and redox history of the Yamato diogenite Y-74013 381
- Dispersion
 Computation of synthetic seismograms for plane layered earth models including absorption and dispersion phenomena 321
- Distribution coefficient
 Geochemistry of coexisting hornblende and biotite from the Ambalavayal granite, Kerala 91
- Diurnal and semidiurnal components
 Wind gradients at meteor heights over mid-latitude stations 427
- Drought
 Global features of sea-level pressure distribution in April and July associated with contrasted situations of Indian summer monsoon 299
- Electric-field
 Some theoretical aspects of electric field and precipitation growth in a finite thunder cloud 293
- Erosion
 Effect of waves in the redistribution of sediments along the Konkan coast 237
- Eustatic change
 Evolution of the landscapes in the Calcutta-Sundarban coastal areas of South Bengal, India 143
- Exploration
 Base metal exploration by gravity surveys in a part of the Cuddapah basin (India) — a case study 131
- fH_2O
 Geochemistry of coexisting hornblende and biotite from the Ambalavayal granite, Kerala 91
- Field line resonance
 Wave packet structure and phase jumps in low latitude Pc3 geomagnetic pulsation 13
- Filtering
 The effect of filtering on drift and anisotropy parameters determined by full correlation analysis 83
- Flood-plains
 Evolution of the landscapes in the Calcutta-Sundarban coastal areas of South Bengal, India 143
- Fluid inclusion
 Physico-chemical conditions of ore deposition in Malanjkhanda copper sulphide deposit 209
- Fluvial environment
 Origin and depositional environment of the china clay deposits of South Kerala 285
- fO_2
 Geochemistry of coexisting hornblende and biotite from the Ambalavayal granite, Kerala 91
- Fourier transform
 Magnetic anomalies over thin plates and their analysis 331
- Fugacity
 Physico-chemical conditions of ore deposition in Malanjkhanda copper sulphide deposit 209
- Full correlation analysis
 The effect of filtering on drift and anisotropy parameters determined by full correlation analysis 83
- Gabbro
 The petrology and geochemistry of massif anorthosites and gabbros of the Bavali fault zone, North Kerala 117
- Geochemistry
 Geochemistry of coexisting hornblende and biotite from the Ambalavayal granite, Kerala 91
- Geochronologic correlation
 Radioelement geochemistry of alkali granites of the Kerala region, south-west India 103

Index

- Geomagnetic parameters
 - Effects of linear trend and mean value on maximum entropy spectral analysis 201
- Global sea-level pressure
 - Global features of sea-level pressure distribution in April and July associated with contrasting situations of Indian summer monsoon 299
- GOSSTCOMP charts
 - Estimation of evaporation rates over the Arabian Sea from satellite data 417
- Granite
 - Geochemistry of coexisting hornblende and biotite from the Ambalavayal granite, Kerala 91
- Gravimagnetic interpretation
 - Analysis of gravimagnetic anomalies by horizontal gradients 343
- Gravity
 - Base metal exploration by gravity surveys in a part of the Cuddapah basin (India) — a case study 131
- Gravity anomalies
 - Gravity anomalies of a trapezoidal model with quadratic density function 275
- Grey scale dynamic spectra
 - Spectral characteristics of low latitude Pc3 geomagnetic pulsations in south-east Australia 25
- Geomagnetic pulsations
 - Spectral characteristics of low latitude Pc3 geomagnetic pulsations in south-east Australia 25
- High periodicity
 - Effect of linear trend and mean value on maximum entropy spectral analysis 201
- Homogenization
 - Physico-chemical conditions of ore deposition in Malankhand copper sulphide deposit 209
- Horizontal cylinder
 - Analysis of gravimagnetic anomalies by horizontal gradients 343
- Horizontal disturbance vector
 - An interpretation of induced magnetic variations near Shillong and Gulmarg 373
- Horizontal gradients
 - Analysis of gravimagnetic anomalies by horizontal gradients 343
- Hornblende
 - Geochemistry of coexisting hornblende and biotite from the Ambalavayal granite, Kerala 91
- Hydrometeors
 - Some theoretical aspects of electric field precipitation growth in a finite thundercloud 1
- Induction
 - An interpretation of induced magnetic variations near Shillong and Gulmarg
- Induction coil magnetometers
 - Spectral characteristics of low latitude Pc3 geomagnetic pulsations in south-east Australia 25
- In situ* stresses
 - In situ* stress measurements using hydraulic fracturing method
- Intermittent conditions
 - Numerical hysteresis model for intermittent studies in unsaturated soils
- Intertropical convergence zone
 - Sub-seasonal scale fluctuations of the ITCZ over the Indo-Pacific region during the summer monsoon season. Part I: Features over Indian region
- Inverse problem
 - Inverse problem for the determination of bounds on the mantle heat flow
- Ionospheric drift
 - The effect of filtering on drift and anisotropic parameters determined by full correlation analysis
- Ionospheric radio wave absorption
 - Absorption of radio waves during a solar eclipse 1
- ISMEX-73
 - Sub-seasonal scale fluctuations of the ITCZ over the Indo-Pacific region during the summer monsoon season. Part I: Features over Indian region
- Jeep mounted gamma ray spectrometer
 - Natural radioactivity distribution studies in Trivandrum District, Kerala, India
- Kaolinite
 - Origin and depositional environment of china clay deposits of South Kerala
- Khondalite
 - Origin and depositional environment of china clay deposits of South Kerala
- Konkan coast
 - Effect of waves in the redistribution of sediments along the Konkan coast

- Leptynite
Origin and depositional environment of the china clay deposits of South Kerala 285
- Lineament analysis
Lineament analysis of Cannanore District and their implication on the tectonic evolution of the area 363
- Linear programming
Inverse problem for the determination of the bounds on the mantle heat flow 223
- Linear trend
Effects of linear trend and mean value on maximum entropy spectral analysis 201
- Low frequency
Effects of linear trend and mean value on maximum entropy spectral analysis 201
- Mafic dykes
Palaeo-tectonic environment of the pre-cambrian basaltic mafic dykes on the application of chemical discriminants 351
- Magma source
The petrology and geochemistry of massif anorthosites and gabbros of the Bavali fault zone, North Kerala 117
- Magnetic interpretation
Magnetic anomalies over thin plates and their analysis 331
- Magnetic variation
An interpretation of induced magnetic variations near Shillong and Gulmarg 373
- Magnetosphere
Wave packet structure and phase jumps in low latitude Pc3 geomagnetic pulsation 13
- Major element
The petrology and geochemistry of massif anorthosites and gabbros of the Bavali fault zone, North Kerala 117
- Mantle heat flow
Inverse problem for the determination of the bounds on the mantle heat flow 223
- Marquardt algorithm
Gravity anomalies of a trapezoidal model with quadratic density function 275
- Maximum cloud zone
Sub-seasonal scale fluctuations of the ITCZ over the Indo-Pacific region during the summer monsoon season. Part I: Features over the Indian region 47
- Maximum compression
In situ stress measurements using hydraulic fracturing method 311
- Effects of linear trend and mean value on maximum entropy spectral analysis 201
- Meteorites
Radionuclide depth profiles in Dhajala chondrite 169
Cosmogenic isotope and track records in meteorites and their implications to cosmic ray fluxes 183
- Meteor winds
Wind gradients at meteor heights over mid-latitude stations 427
- Mid-latitudes
Wind gradients at meteor heights over mid-latitude stations 427
- Mineralization
Base metal exploration by gravity surveys in a part of the Cuddapah basin (India) — a case study 131
- Monazite
Natural radioactivity distribution studies in Trivandrum District, Kerala, India 397
- Monex-79
Sub-seasonal scale fluctuations of the ITCZ over the Indo-Pacific region during the summer monsoon season. Part I: Features over the Indian region 47
- Natural barriers
Effect of waves in the redistribution of sediments along the Konkan coast 237
- Natural planes of weakness
In situ stress measurements using hydraulic fracturing method 311
- North Indian Ocean
Sea surface temperature variability over North Indian Ocean — A study of two contrasting monsoon seasons 435
- Nuclear tracks
Radionuclide depth profiles in Dhajala chondrite 169
- Onset and mid-season revival of monsoon
Sub-seasonal scale fluctuations of the ITCZ over the Indo-Pacific region during the summer monsoon season. Part I: Features over the Indian region 47
- Paleo-climatic scale
An overview of climate models 447
- Palaeo-tectonic environment
Palaeo-tectonic environment of the pre-cambrian basaltic mafic dykes on the application of chemical discriminants 351

- Pelagic suspension
Origin and depositional environment of the china clay deposits of South Kerala 285
- Periodicity peaks
Power spectrum analysis of geomagnetic indices 1
- Phase jump
Wave packet structure and phase jumps in low latitude Pc3 geomagnetic pulsation 13
- Physiographic units
Evolution of the landscapes in the Calcutta-Sundarban coastal areas of South Bengal, India 143
- Power spectrum analysis
Power spectrum analysis of geomagnetic indices 1
- Precipitation growth
Some theoretical aspects of electric field and precipitation growth in a finite thundercloud 293
- Preferential leaching
Preferential solution of ^{234}U from recoil tracks and $^{234}\text{U}/^{238}\text{U}$ radioactive disequilibrium in natural waters 245
- Preeonset
Sub-seasonal scale fluctuations of the ITCZ over the Indo-Pacific region during the summer monsoon season. Part I: Features over the Indian region 47
- Pressure departures
Global features of sea-level pressure distribution in April and July associated with contrasting situations of Indian summer monsoon 299
- Pseudo-magnetic potential
Magnetic anomalies over thin plates and their analysis 331
- Quadratic density function
Gravity anomalies of a trapezoidal model with quadratic density function 275
- Quasi specular reflection
Quasi-specular sea scatter with modified reflection coefficient 75
- Radar altimeter
The effect of whitecaps and foam on wind speed extraction with a pulse limited radar altimeter 265
- Radioactive disequilibrium
Preferential solution of ^{234}U from recoil tracks and $^{234}\text{U}/^{238}\text{U}$ radioactive disequilibrium in natural waters 245
- Radioactive elements
Natural radioactivity distribution studies in Trivandrum District, Kerala, India 397
- Radio-element exploration
Radioelement geochemistry of alkali granites of the Kerala region, south-west India 103
- Radioelement geochemistry
Radioelement geochemistry of alkali granites of the Kerala region, south-west India 103
- Radiometric survey
Natural radioactivity distribution studies in Trivandrum District, Kerala, India 397
- Reactivation
Lineament analysis of Cannanore District and their implication on the tectonic evolution of the area 363
- Redox history
On the thermal and redox history of the Yamato diogenite Y-74013 381
- Refract pressure
In situ stress measurements using hydraulic fracturing method 311
- Sand bar
Effect of waves in the redistribution of sediments along the Konkan coast 237
- Satellite microwave radiometer
Estimation of evaporation rates over the Arabian Sea from satellite data 417
- Scanning curves
Numerical hysteresis model for intermittent studies in unsaturated soils 153
- Scattering
Quasi-specular sea scatter with modified reflection coefficient 75
- SEASAT
Quasi-specular sea scatter with modified reflection coefficient 75
The effect of whitecaps and foam on wind speed extraction with a pulse limited radar altimeter 265
- Seasonal variations
Seasonal and solar cycle variations of night-time anomalous enhancements in total electron content 409
- Sea surface temperature
Sea surface temperature variability over North Indian Ocean — A study of two contrasting monsoon seasons 435
- Sea surface wind speed
Quasi-specular sea scatter with modified reflection coefficient 75
- Sediment movement
Effect of waves in the redistribution of sediments along the Konkan coast 237
- Shut-in pressure
In situ stress measurements using hydraulic fracturing method 311
- Solar cycle variations
Seasonal and solar cycle variations of night-time anomalous enhancements in total electron content 409

Solar eclipse		Trace element	
Absorption of radio waves during a solar eclipse	193	The petrology and geochemistry of massif anorthosites and gabbros of the Bavali fault zone, North Kerala	117
South equatorial trough		Trapezoidal model	
Sea surface temperature variability over North Indian Ocean — A study of two contrasting monsoon seasons	435	Gravity anomalies of a trapezoidal model with quadratic density function	275
Southwest monsoon		Two-layer media	
Global features of sea-level pressure distribution in April and July associated with contrasting situations of Indian summer monsoon	299	The effect of whitecaps and foam on wind speed extraction with a pulse limited radar altimeter	265
Spaced receiver method		Unsaturated soils	
The effect of filtering on drift and anisotropy parameters determined by full correlation analysis	83	Numerical hysteresis model for intermittent studies in unsaturated soils	153
Specular point model		Uranium isotopes	
The effect of whitecaps and foam on wind speed extraction with a pulse limited radar altimeter	265	Preferential solution of ^{234}U from recoil tracks and $^{234}\text{U}/^{238}\text{U}$ radioactive disequilibrium in natural waters	245
Stochastic dynamic prediction		Vertical fault	
A review of stochastic dynamic method of weather prediction	485	Analysis of gravimagnetic anomalies by horizontal gradients	343
Sub-seasonal scale fluctuations		Warkalli beds	
Sub-seasonal scale fluctuations of the ITCZ over the Indo-Pacific region during the summer monsoon season. Part I: Features over the Indian region	47	Origin and depositional environment of the china clay deposits of South Kerala	285
Synthetic seismograms		Wave packet	
Computation of synthetic seismograms for plane layered earth models including absorption and dispersion phenomena	321	Wave packet structure and phase jumps in low latitude Pc3 geomagnetic pulsation	13
Tectonic evolution		Weather prediction	
Lineament analysis of Cannanore District and their implication on the tectonic evolution of the area	363	A review of stochastic dynamic method of weather prediction	485
Tectonic lineaments		White earth	
Natural radioactivity distribution studies in Trivandrum District, Kerala, India	397	An overview of climate models	447
Tensional fractures		Wind speed	
Lineament analysis of Cannanore District and their implication on the tectonic evolution of the area	363	The effect of whitecaps and foam on wind speed extraction with a pulse limited radar altimeter	265
Thermal history		Yamato-74013 meteorite	
On the thermal and redox history of the Yamato diogenite Y-74013	381	On the thermal and redox history of the Yamato diogenite Y-74013	381
Thin plate		Zonal anomaly	
Magnetic anomalies over thin plates and their analysis	331	Sea surface temperature variability over North Indian Ocean — A study of two contrasting monsoon seasons	435
Total electron content			
Seasonal and solar cycle variations of night-time anomalous enhancements in total electron content	409		

- | | | | |
|---|--------|---|-----|
| Agarwal A K | | Ganguly Aniruddha | |
| An interpretation of induced magnetic variations near Shillong and Gulmarg | 373 | <i>see</i> Das K R Dilip | 143 |
| Ahmed M I | | Gowd T N | |
| <i>see</i> Devara P C S | 427 | <i>In situ</i> stress measurements using hydraulic fracturing method | 311 |
| Ansari I A | | Gujar A R | |
| Wave packet structure and phase jumps in low-latitude Pc3 geomagnetic pulsation | 13 | <i>see</i> Rajamanickam G Victor | 237 |
| Spectral characteristics of low latitude Pc3 geomagnetic pulsations in south-east Australia | 25 | Hingane L S | |
| | | <i>see</i> Kumar K Rupa | 299 |
| Babu V Ramesh | | Hussain N | |
| <i>see</i> Rao M Venkateswara | 417 | Preferential solution of ^{234}U from recoil tracks and $^{234}\text{U}/^{238}\text{U}$ radioactive disequilibrium in natural waters | 245 |
| Balan N | | Iyer K N | |
| Seasonal and solar cycle variations of nighttime anomalous enhancements in total electron content | 409 | <i>see</i> Balan N | 409 |
| Banerjee M | | Jaireth S | |
| Numerical hysteresis model for intermittent studies in unsaturated soils | 153 | Physico-chemical conditions of ore deposition in Malanjkhanda copper sulphide deposit | 209 |
| Bhandari N | | Jani K G | |
| <i>see</i> Potdar M B | 169 | <i>see</i> Patel D B | 193 |
| Cosmogenic isotope and track records in meteorites and their implications to cosmic ray fluxes | 183 | Joshi P C | |
| | | A review of stochastic dynamic method of weather prediction | 485 |
| Chandra H | | Kane R P | |
| <i>see</i> Vyas B M | 83 | Power spectrum analysis of geomagnetic indices | 1 |
| Chandrasekhar G | | Effect of linear trend and mean value on maximum entropy spectral analysis | 201 |
| <i>see</i> Devara P C S | 427 | Kotadia K M | |
| Chary K B | | <i>see</i> Patel D B | 193 |
| <i>see</i> Gowd T N | 311 | Kumar G R Ravindra | |
| Das K R Dilip | | The petrology and geochemistry of massif anorthosites and gabbros of the Bavali fault zone, North Kerala | 117 |
| Evolution of the landscapes in the Calcutta-Sundarban coastal areas of South Bengal, India | 143 | <i>see</i> Raju G K | 397 |
| Deshpande V R | | Kumar K Rupa | |
| <i>see</i> Sikka D R | 47 | Global features of sea-level pressure distribution in April and July associated with contrasting situations of Indian summer monsoon | 299 |
| Devara P C S | | Kumar K Vijaya | |
| Wind gradients at meteor heights over mid-latitude stations | 427 | <i>see</i> Santosh M | 103 |
| Fraser B J | | Kumar M R Ramesh | |
| <i>see</i> Ansari I A | 13, 25 | Sea surface temperature variability over North Indian Ocean — A study of two contrasting monsoon seasons | 435 |
| Gairola R M | | | |
| The effect of whitecaps and foam on wind speed extraction with a pulse limited radar altimeter | 265 | | |

tion coefficient	75	Thiruvandur District, Kerala, India	397
Lal D		Raju M V Rama	351
<i>see</i> Hussain N	245	Ramadas G	
Lele P D		Base metal exploration by gravity surveys in a	
<i>see</i> Patel D B	193	part of the Cuddapah basin (India) — a case	
		study	131
Machado Terry		Analysis of gravimagnetic anomalies by hori-	
<i>see</i> Soman K	285	zontal gradients	343
Majumdar P Jay		Ram Avadh	
<i>see</i> Das K R Dilip	143	<i>see</i> Yadav K N S	321
Mathai John		Rambabu H V	
<i>see</i> Raju G K	397	Magnetic anomalies over thin plates and their	
McLean K J		analysis	331
<i>see</i> Ansari I A	13, 25	Rao D Bhaskara	
Mohammed A Raja		Gravity anomalies of a trapezoidal model with	
<i>see</i> Ramadas G	343	quadratic density function	275
Mohan N L		Rao L V Gangadhara	
<i>see</i> Ramadas G	131	<i>see</i> Rao M Venkateswara	417
Mujumdar V R		<i>see</i> Kumar M R Ramesh	435
<i>see</i> Sikka D R	47	Rao M Sankar	
Mukherjee Amalbikash		An overview of climate models	447
On the thermal and redox history of the		Rao M Venkateswara	
Yamato diogenite Y-74013	381	Estimation of evaporation rates over the	
		Arabian Sea from satellite data	417
Naik H B		Rao P S	
<i>see</i> Agarwal A K	373	<i>see</i> Balan N	409
Nair N G K		Rao S M Varaprasada	
<i>see</i> Raju G K	397	<i>see</i> Ramadas G	131
Narayanaswamy		<i>see</i> Ramadas G	343
<i>see</i> Raghavan Venkatesh	363	Rao S V Srirama	
		<i>see</i> Gowd T N	311
Pandey P C		Rao T Ramamohana	
<i>see</i> Gairola R M	265	Palaeo-tectonic environment of the pre-	
Patel D B		Cambrian basaltic mafic dykes on the applica-	
Absorption of radio waves during a solar eclipse	193	tion of chemical discriminants	351
Paul D K		Rummel F	
<i>see</i> Sikka D R	47	<i>see</i> Gowd T N	311
Potdar M B		Santosh M	
Radionuclide depth profiles in Dhajala	169	Geochemistry of coexisting hornblende and	
chondrite		biotite from the Ambalavayal granite, Kerala	91
Puranik P V		Radioelement geochemistry of alkali granites	
<i>see</i> Sikka D R	47	of the Kerala region, south-west India	103
Raghavan Venkatesh		Sardesai D V	
Lineament analysis of Cannanore District and		<i>see</i> Vyas B M	83
their implication on the tectonic evolution of	363	Sarkar Abhijit	
the area		<i>see</i> Kumar Raj	75
Rai R K		Sastry J S	
<i>see</i> Vyas B M	83	<i>see</i> Rao M Venkateswara	417
Rajamanickam G Victor		Sathyendranath S	
Effect of waves in the redistribution of sedi-	237	<i>see</i> Kumar M R Ramesh	435
ments along the Konkan coast		Sharma M	
Raju B V Satyanarayana	351	<i>see</i> Jaireth S	209
<i>see</i> Rao T Ramamohana			

Sikka D R		Varshneya N C	
Sub-seasonal scale fluctuations of the ITCZ over the Indo-Pacific region during the summer monsoon season. Part I: Features over the Indian region	47	<i>see</i> Singh Pratap	293
<i>see</i> Joshi P C	485	Vasseur G	
Singh Pratap		<i>see</i> Singh R N	223
Some theoretical aspects of electric field and precipitation growth in a finite thundercloud	293	Verma T S	
		<i>see</i> Singh Pratap	293
Singh R N		Vethamony P	
Inverse problem for the determination of the bounds on the mantle heat flow	223	<i>see</i> Rajamanickam G Victor	237
Sinha G D J Sivakumar		Viswambharan N K	
<i>see</i> Rambabu H V	331	<i>see</i> Kumar M R Ramesh	435
Soman K		Viswanath T A	
Origin and depositional environment of the china clay deposits of south Kerala	285	<i>see</i> Mukherjee Amalbikash	381
Suthar K M		Vyas B M	
<i>see</i> Potdar M B	169	The effect of filtering on drift and anisotropy parameters determined by full correlation analysis	83
Trivedi N B		Vyas G D	
<i>see</i> Kane R P	201	<i>see</i> Vyas B M	83
		Yadav K N S	
		Computation of synthetic seismograms for plane layered earth models including absorption and dispersion phenomena	321
